1st Semester 1436/1437	Final Exam	King Saud University
(without calculators)	Time allowed: 3 hours	College of Science
Monday 24-3-1437	240 Math	Math. Department

Q1: (a) Find the inverse of $A=\begin{bmatrix}1 & 2 & 3\\ 2 & 5 & 3\\ 1 & 0 & 8\end{bmatrix}$. (4 marks)

(b) Use Cramer's rule to solve the following linear system:

$$\begin{aligned} x_1 + 2x_3 &= 6 \\ -3x_1 + 4x_2 + 6x_3 &= 30 \\ -x_1 - 2x_2 + 3x_3 &= 8 \end{aligned}$$

(4 marks)

Q2: (a) Find a **subset** of the vectors $v_1=(1,-2,0,3)$, $v_2=(2,-5,-3,6)$, $v_3=(0,1,3,0)$, $v_4=(2,-1,4,-7)$ and $v_5=(5,-8,1,2)$ that forms a basis for the space **spanned** by these vectors. (4 marks)

(b) Find bases for the eigenspaces of the matrix $A=\begin{bmatrix}3 & 0\\ 8 & -1\end{bmatrix}$ (4 marks)

Q3: (a) Let $\mathbb{R}^4$ have the Euclidean inner product. Find the cosine of the angle θ between the vectors **u**= (4,3,1,-2) and **v**=(-2,1,2,3).

$-8+3+2-3$ -9
6

Moreover, if P_2 has the inner product $\langle p,q\rangle = \int_{-1}^{1} p(x)q(x)dx$, then show that the vectors x and x^2 are orthogonal. (4 marks).

(b) Assume that the vector space $\mathbb{R}^3$ has the Euclidean inner product. Apply the Gram-Schmidt process to transform the basis vectors (1,1,1), (0,1,1), (0,0,1) into an orthonormal basis. (4 marks)

Q4: (a) Consider the basis $S=\{v_1=(1,1,1), v_2=(1,1,0), v_3=(1,0,0)\}$ for $\mathbb{R}^3$. Let $T:\mathbb{R}^3 \to \mathbb{R}^2$ be the linear transformation for which $T(v_1)=(1,0)$, $T(v_2)=(2,-1)$ and $T(v_3)=(4,3)$. Find a formula for $T(x_1,x_2,x_3)$, and then use that formula to compute T(2,-3,5). (4 marks)

(b) Let $T:\mathbb{R}^3 \to \mathbb{R}^3$ be the linear operator defined by the formula:

$$T(x_1,x_2,x_3) = (3x_1 + x_2, -2x_1 - 4x_2 + 3x_3, 5x_1 + 4x_2 - 2x_3)$$

Determine whether T is one-to-one; if so, find $T^{-1}(x_1, x_2, x_3)$. (4 marks)

Q5: Prove the following statements:

(a) If **u** and **v** are orthogonal vectors in an inner product space, then $\|u+v\|^2 = \|u\|^2 + \|v\|^2$. (2 marks)

(b) If $T: V \to W$ is a linear transformation, then the range of T is a subspace of W. (2 marks)

(c) If $T: V \to W$ is a linear transformation, then T is one-to-one if and only if ker(T)={0}. (2 marks)

(d) If $T_1: U \to V$ and $T_2: V \to W$ are two linear transformations, then $(T_2 \circ T_1): U \to W$ is also a linear transformation. (2 marks)