5.1 :

7 characteristic eq^n is $|\lambda I - A| = 0$

$$\Leftrightarrow \begin{vmatrix} \lambda-4 & 0 & -1 \\ 2 & \lambda-2 & 0 \\ 2 & 0 & \lambda-1 \end{vmatrix} = 0$$

$$\Leftrightarrow (\lambda-4)(\lambda-1)^2 - (-2(\lambda-1)) = 0$$

$$\Leftrightarrow (\lambda-4)(\lambda-1)^2 + 2(\lambda-1) = 0$$

$$\Leftrightarrow (\lambda-1)\left[(\lambda-4)(\lambda-1) + 2\right] = 0$$

$$\Leftrightarrow (\lambda-1)\left[\lambda^2 - 5\lambda + 6\right] = 0$$

$$\Leftrightarrow \lambda = 1, \quad \lambda = 2, \lambda = 3$$

For eigen vector :

* at $\lambda = 1$:

$$I - A = \begin{bmatrix} 3 & 0 & -1 \\ 2 & 0 & 0 \\ 2 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -\frac{1}{3} \\ 0 & 0 & 2/3 \\ 0 & 0 & 2/3 \end{bmatrix}$$

$$\Rightarrow x_3 = 0, \quad x_1 = 0, \quad x_2 = t \in \mathbb{R}$$

$$\therefore (x_1, x_2, x_3) = (0, t, 0) = t(0,1,0)$$

Basis for eigen vector corresponding $\lambda = 1$ is $\{(0,1,0)\}$

* at $\lambda = 2$: $2\lambda - A = \begin{bmatrix} -2 & 0 & -1 \\ 2 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$

$$\Rightarrow x_1 + \tfrac{1}{2} x_3 = 0$$

$$x_2 - x_3 = 0$$

$$x_3 = t \Rightarrow x_2 = t, \quad x_1 = -\tfrac{1}{2} t$$

$$(x_1, x_2, x_3) = t\left(-\tfrac{1}{2}, 1, 1\right)$$

$\Rightarrow$ Basis is $\{(-\frac{1}{2}, 1, 1)\}$

* at $\lambda = 3$: $3I - A = \begin{bmatrix} -1 & 0 & -1 \\ 2 & 2 & 0 \\ 2 & 0 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & -2 \\ 0 & 0 & 0 \end{bmatrix}$, $x_1 + x_3 = 0 \rightarrow x_1 = -x_3$

$x_2 - x_3 = 0 \rightarrow x_2 = x_3$

$\therefore (x_1, x_2, x_3) = (-t, t, t) = t(-1, 1, 1)$ if $x_3 = t \Rightarrow x_1 = -t$, $x_2 = t$

Basis for eigen vector corresponding $\lambda = 3$ is $\{(-1,1,1)\}$

5.2

$$\begin{bmatrix} 2 & 0 & -2 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$|\lambda I - A| = 0 \Leftrightarrow \begin{vmatrix} \lambda-2 & 0 & 2 \\ 0 & \lambda-3 & 0 \\ 0 & 0 & \lambda-3 \end{vmatrix} = 0 \Leftrightarrow \lambda = 2, 3, 3$$

* at $\lambda = 2$: $2I - A = \begin{bmatrix} 0 & 0 & 2 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} = 0$

$\rightarrow x_3 = 0,\ x_2 = 0,\ x_1 = t$

$\therefore (x_1, x_2, x_3) = (t, 0, 0) = t(1, 0, 0)$

$\therefore p_1 = (1, 0, 0)$

* at $\lambda = 3$: $3I - A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0 \quad x_1 + 2x_3 = 0 \rightarrow x_2 = t$

$x_3 = s \rightarrow x_1 = -2s$

$\Rightarrow (x_1, x_2, x_3) = (-2s, t, s)$

$= s\underbrace{(-2, 0, 1)}_{P_3} + t\underbrace{(0, 1, 0)}_{P_2}$

$$\therefore P = \begin{array}{c} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ \begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ p_1 & p_2 & p_3 \end{array} \end{array}$$

شيماء الحمين - نورة الحريم

16

$$A = \begin{bmatrix} 1 & -3 & -1 & -5 \\ 0 & 3 & 3 & 3 \\ 1 & 7 & 9 & 5 \\ 1 & 1 & 3 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & -3 & -1 & -5 \\ 0 & 3 & 3 & 3 \\ 0 & 10 & 10 & 10 \\ 0 & 4 & 4 & 4 \end{bmatrix}$$

$$\begin{array}{cccc} c_1 & c_2 & c_3 & c_4 \end{array}$$

$$\sim \begin{bmatrix} \boxed{1} & -3 & -1 & -5 \\ 0 & \boxed{1} & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{array}{cccc} \boxed{w_1} & \boxed{w_2} & w_3 & w_4 \end{array}$$

the Basis is $\{c_1, c_2\}$

\# $(-1,1,0,0) = \overset{2}{\alpha_1}(1,0,0,0) + \overset{1}{\alpha_2}(-3,1,0,0)$

$\Longleftrightarrow$

$\boxed{(-1,3,9,3) = 2(1,0,1,1) + (-3,3,7,1)}$ $\quad \alpha_2 = 1$, $\alpha_1 = 2$

$(-5,1,0,0) = \underset{-2}{\alpha_1}(1,0,0,0) + \overset{1}{\alpha_2}(-3,1,0,0)$

$\alpha_2 = 1$

$\alpha_1 - 3\alpha_2 = -5$

$\Leftrightarrow (-5,3,5,-1) = -2(\quad) + (\quad)$ $\quad \rightarrow \alpha_1 = -5+3 = -2$