King Saud University Second Semester 2012/13
Department of Mathematics
College of Sciences

Integral Calculus (M-106), Serie N: 2

Exercise 1:

Evaluate the sum.

1) $\sum_{j=1}^{4}(j^2+1)$

2) $\sum_{j=1}^{4}(2^j+1)$

3) $\sum_{j=1}^{4}j(j-1)$

4) $\sum_{j=1}^{1000}2$

Exercise 2:

Express the sum in terms of n

1) $\sum_{j=1}^{n}(j^2-5j+1)$

2) $\sum_{j=1}^{n}(j^3+2j^2-j+4)$

Exercise 3:

Express in summation notation.

1) $1+5+9+13+17$

2) $\frac{1}{2}+\frac{2}{5}+\frac{3}{8}+\frac{4}{11}$

3) $1+x+\frac{x^2}{2}+\frac{x^3}{3}+...+\frac{x^n}{n}$

Exercise 4:

Let $f(x)=\sqrt{x}$, and let R be the region under the graph of f from 1 to 5. Approximate the area A of R using:

1) an inscribed reclangular polygon with $\Delta x = 0.1$
2) a circumscribed rectangular polygon with $\Delta x = 0.1$.

Exercise 5:

Let A be the area under the graph of the given function $f(x) = x^2 + 1$ from 1 to 3. Approximate A by dividing the interval $[a, b]$ into subintervals of equal length Δx using:

1) A_{IP} : Area of an inscribed recrangular polygon

2) A_{CP} : Area of a circumscribed rectangular polygon

Exercise 6:

Let us consider $f(x) = x^3$,

$a)$ Find the area under the graphs of f from 0 to b for any $b > 0$, by subdividing the interval $[0, b]$ into n equal parts, using an inscribed reclangular polygon.

$b)$ Find the area under the graph of f corresponding to the interval $[1, 3]$ by using $a)$.

Exercise 7:

Find the Riemann sum R_P for the given function $f(x)$ on the indicated inteval with a regular partition P of the size n by choosing on each subinterval of P (a) The left-hand endpoint, (b) the right-hand endpoint and (c) the midpoint.

1) $f(x) = x^3$, $[-2, 6]$, $n = 6$

2) $f(x) = x^2\sqrt{\cos x}$, $[0, 1]$, $n = 5$

Exercise 8:

Verifiy the inequality without evaluating the integrals.

1) $\displaystyle\int_1^2 (3x^2 + 4)dx \geq \int_1^2 (2x^2 + 5)dx$

2) $\displaystyle\int_{2}^{4}(x^2-6x+8)dx \leq 0$

3) $\displaystyle\int_{2}^{4}(5x^2-x+1)dx \geq 0$

4) $\displaystyle\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}}(\sec x - 2)dx \leq 0$

Exercise 9:

The integral $\int_a^b f(x)dx$ of the continous function f over the interval $[a,b]$ can be evaluted. $a)$ Find a number z that satisfies the conclusion of the mean value theorem and $b)$ Find the average value of the function f on $[a,b]$, where:

1) $\displaystyle\int_{-2}^{1}(x^2+1)dx = 6$

2) $\displaystyle\int_{-1}^{8}3\sqrt{x+1}dx = 54$

3) $\displaystyle\int_{-2}^{-1}\frac{8}{x^3}dx = -3.$