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Simulation of Crowd Behavior and Movement: Fundamental Relations and Application

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A model of crowd behavior and movement in the Jamarat system, which is a critical bottleneck during the Hajj, the Muslims' annual pilgrimage to Makkah, Saudi Arabia, is described. The model consists of a set of partial differential equations that are solved numerically following a discretization of time and space. Mathematical relations are defined for three fundamental processes: (a) radial movement, (b) lateral movement, and (c) stoning process. These relations are developed and calibrated using actual measurements taken at the site. As a result, a bidirectional speed-concentration model is developed to describe radial movement, revealing that the impedance to movement from facility users going in the same direction is about twice that from those going in the opposite direction. The overall model is applied to the evaluation of possible design and control strategies aimed at improving the efficiency and throughput of the system.

The efficient and safe operation of heavily crowded facilities used in special circumstances by unfamiliar users is a major concern to those responsible for the design and control of such public places and events. Crowd disasters in which people are seriously injured, sometimes fatally, can occur, and have occurred at sports events, religious gatherings, and rock music concerts (*1*).

A remarkable example of pedestrian overcrowding is that of the Hajj, or Muslims' pilgrimage to Makkah, Saudi Arabia. The Hajj consists of a set of prescribed rites at specific hours and days in assigned locations in and around the city of Makkah. Each year, over two million pilgrims converge on Makkah to perform this religious duty at the same time, resulting in a crowded event of extraordinary magnitude. Every hour, there are hundreds of thousands of pilgrims walking between the various rites of the Hajj (*2*). Bottlenecks can turn into disasters, and one of the most difficult of such bottlenecks is the relatively small Jamarat system. One of the pillars of the Hajj is the throwing of (seven) small pebbles at each of the three stone monuments located at a place called Jamarat. In performing this ritual, many pilgrims suffer considerable hardships because of congestion and overcrowding in the tight Jamarat area, because of space and time constraints. For example, in the 1983 Hajj season, 64 fatalities were reported in this area because of overcrowding (*3*).

Little scientific work has been directed at the characterization of crowd behavior and movement, leaving considerable gaps in the knowledge base and methodological capabilities pertinent to this problem. The limited body of the available studies related to this subject can be classified, for convenience, into two broad categories: (a) those that deal with steady state pedestrian flows under normal conditions, and (b) those that address crowding conditions, which correspond to the higher end of the concentration spectrum, and generally involve multidirectional flow patterns with strong interactions.

The first class of studies deals primarily with streamlined and orderly pedestrian flows encountered on a daily basis in high traffic areas such as passageways, stairways, plazas, and sidewalks (*4–11*). It appears that the theory of pedestrian traffic flow for orderly movement under normal conditions has been sufficiently developed to reflect the actual behavior of foot traffic flows in such contexts, and to provide an adequate basis for design and control purposes (*11*).

The second category addresses the behavior of large crowds under nonroutine circumstances, often involving extremes of human emotion, such as excitement, fear, religious fervor, anger, or exaltation. Movement is therefore far less orderly, multidirectional interactions are strong, and dynamic effects are of greater importance than in the previous case. Some studies have qualitatively discussed the sociopsychological aspects of crowd behavior, rather than its physical aspects (*12,13*). Another group of studies has sought to develop models of individual behavior in a crowd, recognizing the constructed as well as the behavioral environments (*14–21*).

These studies do not, individually or collectively, provide a sufficient technical basis for management of crowd movements at special events. For example, no speed-concentration models exist for crowd movement with multidirectional flow patterns with strong interactions. Similarly, limited research has been directed at crowd behavior and movement during the Hajj (*2,22–27*). Few studies have attempted to characterize pilgrims' behavior in the Jamarat system (*3,28–30*). One exception is a study by the Hajj Research Center (*3*) that successfully characterized, in qualitative terms, the problems encountered in the Jamarat area. However, no models appear to have been developed to characterize pilgrims' behavior in that context.

JAMARAT SYSTEM CONFIGURATION

The Jamarat system is composed of three stone monuments symbolizing the devil, which are supposed to be stoned (i.e., pebbles thrown at each of them) by pilgrims in a given sequence, starting with the first, Small, second, Middle, and

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then the third, Big Jamarah, as shown in Figure 1. On the first day of stoning, only the Big Jamarah is stoned by pilgrims over a period extending from sunrise to sunset. However, on each of the other days of stoning all three jamarat are stoned in sequence, over a period extending from noon until sunset. Each monument (called a Jamarah in the Arabic language) is enclosed by a ring of about 16 m in diameter, referred hereafter as the Jamarah ring, to collect pebbles. Pilgrims squeeze through the crowd to get their pebbles into the ring, and, preferably, to hit the monument. At the same time, others who have finished stoning squeeze and push their way out. On each of the three days of stoning, pilgrims arrive at about the same time, creating extremely crowded conditions. Peak crowding occurs at the same time every year (*3*). To meet the demands of the enormous mass of pilgrims, an upper level was added to the Jamarat system in 1975, so that there are now two levels from which the stoning of the Jamarat can be performed (Figure 1).

THE JAMARAT MODEL: AN OVERVIEW

Previously, the authors (*31*) developed a general crowd behavior and movement model, which consists of a set of simultaneous partial differential equations describing the principal processes that govern the dynamics of the system. The system of governing differential equations can be solved numerically by discretizing in time and space. Such solution yields a profile of the system's evolution and allows the computation of various performance measures and figures of merit. The application of this general model to the Jamarat system of the Hajj is described here, focusing on the processes taking place on the upper level of the Jamarat bridge around an individual Jamarah ring. Pilgrims' behavior and movement there are characterized by a set of fundamental relations among the principal underlying variables. In the following subsections, an overview of the principal elements of the model is given. The details of the individual model elements are reported elsewhere (*31*).

Space Discretization Scheme

Because Pilgrims' major movements around the Jamarah ring are mainly in the radial direction toward and away from the ring, the physical space around the ring is divided into a number of concentric rings, NRING, each of a width of 1 m. Each ring is divided into a number of cells, NCELL, by taking equally spaced radials from the center of the Jamarah (stone monument) to the outer perimeter of the external ring (i = NRING + 1), as shown in Figure 2. Each cell is identified by its (i,j) index ($i = 2, 3, \ldots,$ NRING $+ 1$; $j = 1, 2, \ldots,$ NCELL; $i = 1$ identifies the Jamarah ring itself). The areas of cells in the same ring are equal, whereas the cell area increases with Ring Index i.

Facility Users' Classification and Movements

A fundamental concept in the model is the classification of the pilgrims in the Jamarat system into two major classes: Class 1 users, who have the intention of performing stoning (or Rajm), and Class 2 users, who intend to leave the Jamarah after having completed stoning. Accordingly, $K(i,j)$, the total concentration of users (in persons/m^2) in Cell (i,j), is the sum of the respective concentrations of the two classes; i.e.,

$$K(i,j) = K^1(i,j) + K^2(i,j) \tag{1}$$

For each class of facility users, two types of movement can be identified: in the radial direction (forward and backward), and in the circumferential direction (clockwise and counterclockwise). However, it is assumed that those who have yet to perform the Rajm (Class 1) do not go backward and those who have completed it (Class 2) no longer move towards the

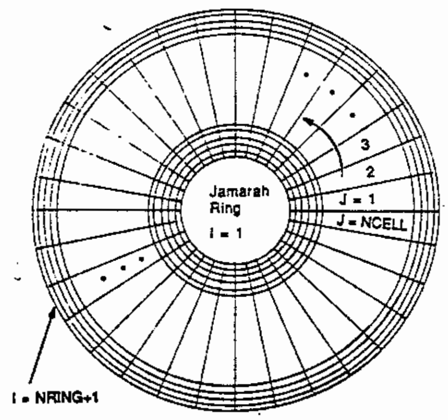

FIGURE 2 Space discretization around the Jamarah ring.

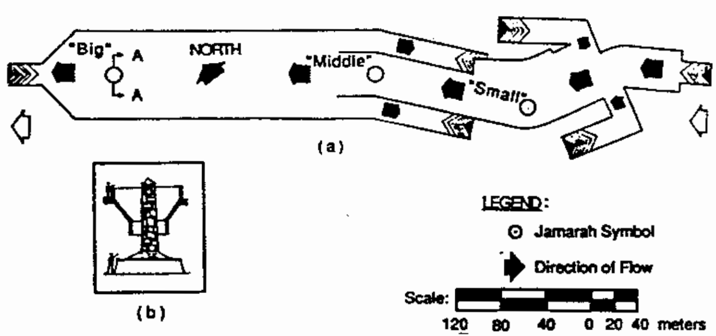

FIGURE 1 The Jamarat system: (a) plan view, (b) section A-A.

ring. Correspondingly, the following speeds of movement are defined:

U_{ij}^{1F} = Speed of Class 1 users, in Cell (i,j), moving towards the Jamarah ring (i.e., forward radial direction), in meters per hour.
U_{ij}^{2B} = Speed of Class 2 users, in Cell (i,j), moving away from the Jamarah ring (i.e., backward radial direction), in meters per hour.
U_{ij}^{mLR} = Speed of Class m users (m = 1, 2), in Cell (i,j), moving in the clockwise lateral direction, in meters per hour.
U_{ij}^{mLL} = Speed of Class m users (m = 1, 2), in Cell (i,j), moving in the counterclockwise lateral direction, in meters per hour.

Conservation of Mass Law

For a given Cell (i,j), the general balance states that the rate of change of facility users in the cell equals the net creation rate of users there plus the net rate at which users flow into the cell across its boundaries. The continuity equations used in the numerical discrete time simulation approach, as applied to the Jamarat system, take the following difference forms (Figure 3):

For Class 1 users:

$$A(i,j) * K^1(i,j)_{t+\Delta t} = A(i,j) * K^1(i,j)_t + [Q^{1F}(i+1,j) - Q^{1F}(i,j) - SW(i,j) + Q^{1LR}(i,j+1) - Q^{1LL}(i,j) + Q^{1LL}(i,j-1) - Q^{1LR}(i,j)]_t \quad (2)$$

for i = 2, 3, . . . , NRING + 1;

j = 1, 2, . . . , NCELL

For Class 2 users:

$$A(i,j) * K^2(i,j)_{t+\Delta t} = A(i,j) * K^2(i,j)_t + [Q^{2B}(i-1,j) - Q^{2B}(i,j) + SW(i,j) + Q^{2LR}(i,j+1) - Q^{2LL}(i,j) + Q^{2LL}(i,j-1) - Q^{2LR}(i,j)]_t \quad (3)$$

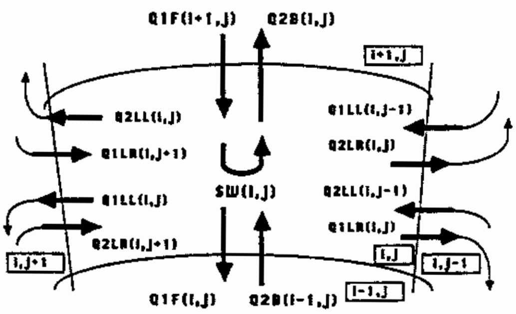

FIGURE 3 Concentration fluxes operating simultaneously in Cell (i,j).

for i = 2,3, . . . ,NRING + 1;

j = 1,2, . . . ,NCELL

where

$K^m(i,j)$ = Concentration of Class m users (m = 1, 2) in Cell (i,j), in persons/m².
$A(i,j)$ = area of Cell (i,j), in m².
$Q^{1F}(i,j)$ = net flux of Class 1 users, in Δt, from Cell (i,j) in the radial forward direction, in persons.
$Q^{2B}(i,j)$ = net flux of Class 2 users, in Δt, from Cell (i,j) in the radial backward direction, in persons.
$Q^{mLR}(i,j)$ = net flux of Class m users, in Δt, from Cell (i,j) in the lateral clockwise direction, in persons.
$Q^{mL1}(i,j)$ = net flux of Class m users, in Δt, from Cell (i,j) in the lateral counterclockwise direction, in persons.
SW(i,j) = number of Class 1 users switching into Class 2 users, in Cell (i,j) in Δt, after finishing stoning. This quantity acts as a sink term for Class 1 and a source term for Class 2 users.

Equation 1 indicates that three principal processes are identified as governing crowd behavior and movement at the Jamarat: (a) movement in the radial direction towards the Jamarah ring by pilgrims seeking to perform stoning (Class 1), and away from it by those leaving the system after completing the stoning (Class 2); (b) movement in the circumferential (lateral) direction around the ring of both classes of pilgrims in their effort to escape crowd pressure by moving to a less-congested space around the ring; and (c) the mechanism by which Class 1 pilgrims switch to the other class after completing stoning. Each of these principal processes is discussed hereafter, with the corresponding underlying variables identified, and relations among them established.

Radial Movement of Pilgrims

The radial movements of pilgrims at the Jamarah are the dominant ones relative to the lateral movements. Their flow rates were modeled by the mathematical product of the prevailing radial speed and the corresponding concentration. For example, the flux $Q^{1F}(i+1,j)$ of Class 1 pilgrims, in a given time step Δt from Cell $(i+1,j)$ to Cell (i,j), is given by

$$Q^{1F}(i+1,j) = U^{1F}(i+1,j) * K^{1R}(i+1,j) * L1(i+1,j) * \Delta t, \quad (4)$$

where

$K^{1R}(i+1,j)$ = concentration of Class 1 users who are moving radially in cell $(i+1,j)$(persons/m²), and
L1$(i+1,j)$ = length of the interior edge of Cell $(i+1,j)$ in meters.

The average radial speed of a given class of users in Cell (i,j) is a function not only of the concentration of users of the

same class in the cell, but the concentration of both classes of users. The following calibrated relation is a central component of the Jamarat model:

$$U^{in} = 1{,}907 * [1 - (K^{in} + 0.38 * K^{opp})/K_{jam}] \quad (5)$$

where U^{in} is the average speed, in meters per hour, of users of a given class, moving with a stream of pedestrians of the same class, with concentration K^{in} (persons/m²), and against another stream of pilgrims from the other class with a corresponding concentration level K^{opp} (persons/m²). The jam concentration K_{jam} was estimated to be 5.68 persons/m². (In the simulation runs, it is assumed that movement continues at a shuffling speed of 200 m/hr at jam concentration.) The parameters of this relation were estimated using field data, in the form of time lapse photographs, which were taken for this purpose at the Jamarat system. The details of the observational work are reported elsewhere (*31*). The calibrated relation revealed some interesting features of the process. For instance, the impedance, on the speed of a given class of facility users, caused by people moving in the same direction is more than twice that caused by those moving in the opposite direction. Another important aspect of this relation is that the marginal impedance of each of the two types of concentrations involved, on the speed of movement, is independent of the level of the other stream's concentration. That is, there is no interaction term between the two types of concentration in the estimated speed-concentration model.

Lateral Movement of Pilgrims

By analogy with the treatment of Gazis et al. (*32*) of interlane concentration oscillations in multilane highway traffic, it is assumed that the exchange of pedestrians between two neighboring cells (in the same ring) is proportional to the difference of their respective concentrations, and only when this difference exceeds a certain threshold, that is:

$$Q_{ij}^{mL} = \begin{cases} \alpha[\Delta K(i,j)]^{\beta}, & \text{if } \Delta K(i,j) \geq \Delta K_{min}^{m}, \\ 0 & \text{otherwise} \end{cases} \quad m = 1, 2 \quad (6)$$

where

- Q_{ij}^{mL} = net lateral flux of Class m users (m = 1, 2) in Δt, from Cell (i,j) to the neighboring cell, in persons;
- $\Delta K(i,j)$ = concentration differential between the two cells;
- ΔK_{min}^{m} = Class m (m = 1, 2) concentration differential threshold; and
- α, β = parameters.

Analysis of limited relevant data has suggested that the threshold level ΔK_{min}^{m} is about 1.0 persons/m² for each class of users, and yielded parameter estimates of α = 850 pilgrims/hr and β = 0.005 for purposes of the present illustration of the behavior of the Jamarat model (*31*).

Switching Mechanism (Stoning Process)

The number SW(i,j) of Class 1 users switching to Class 2 in Cell (i,j), in a given Δt depends on two principal factors: (a) the fraction FR(i,j) of Class 1 users in the cell who perform stoning from that cell, and (b) the time TRAJM(i,j) it takes an individual to perform Rajm. The relation between SW(i,j), FR(i,j), and TRAJM(i,j) can be expressed as follows:

$$\text{SW}(i,j) = [A(i,j) * K^{1}(i,j) * \text{FR}(i,j) * \Delta t]/[\text{TRAJM}(i,j)] \quad (7)$$

The fraction FR(i,j) of Class 1 users switching to Class 2 depends on two main factors: the distance from the rim of the Jamarah ring, in meters, and the prevailing pilgrim concentration level in Cell (i,j), in persons/m², that is,

$$\text{FR}(i,j) = \begin{cases} 1, & \text{if } r_i \leq 1 \\ 0, & \text{if } r_i > R_{max} \text{ or } K(i,j) < K_{FR} \\ 1.005\, r_i^{-0.793}\, [K(i,j)]^{2.341}, & \text{otherwise} \end{cases} \quad (8)$$

where R_{max} is the maximum throwing distance, estimated to be 11 m from the perimeter of the Jamarah ring; and K_{FR} is the concentration threshold, estimated to be 3 persons/m², below which no switching takes place (*31*). Equation 6 indicates that the fraction is initially equal to unity at the rim of the Jamarah ring, where all Class 1 users have to switch regardless of the prevailing concentration level. Beyond the first ring, the fraction is expressed as a function that decreases with distance from the rim until it equals zero beyond the maximum throwing distance from the Jamarah ring, R_{max}. Furthermore, the fraction switching increases with the prevailing concentration beyond a threshold concentration, K_{FR}, below which pilgrims would prefer to proceed further to perform stoning from a closer ring. The values of the parameters reported in Equation 7 are based on measurements taken on site (*31*).

Regarding the other principal factor, TRAJM, the observational evidence did not support the hypothesis that it is a function of either the distance from the ring or the prevailing concentration level; it was estimated to have an average value of 27 sec (*12*).

Choice of Solution Time Step and Space Discretization Scheme

Finally, the choice of time step, Δt, and space discretization scheme (cell width in the radial direction, Δr), when using numerical methods to solve partial differential equations, has important implications for the properties of the solution. Of course, it is generally desired that both Δr and Δt be small enough to approximate the assumed underlying smooth function of $K(i,j)$ with respect to r and t. In addition, the ratio $\Delta r/\Delta t$ should be greater than the mean free speed U_{max}, to ensure that no pilgrim transfer is possible from one cell to another nonadjacent cell in any time step of the numerical solution. The step sizes used in the solution of this model are Δr = 1 m, and Δt = 10^{-4} hr, for a ratio of 10 km/hr (>> U_{max}).

MODEL APPLICATION

In this section, the results of several numerical simulations are presented to illustrate the Jamarat system model and its

application to the investigation of the system's properties under several possible crowd control and management schemes. The system throughput is examined first, followed by an investigation of the effect of several design and operational strategies on system performance.

Loading Strategies and Throughput Analysis

The Jamarat system was simulated under different spatial and temporal pilgrim loading patterns. Two different spatial loading patterns are considered here, as shown in Figure 4. In the first pattern, pilgrims approach the Jamarah from all surrounding directions; in the second pattern, they load from the ring's half side that first faces the pilgrims approaching from the main entrance (i.e., from the southeast, Figure 1). For a given spatial pattern, a variety of temporal loading patterns can be simulated. In the present exploration of maximum system throughput, the system was loaded continuously at a constant rate (for a given spatial pattern) until a steady state was attained. The system's maximum throughput was obtained by performing a series of simulations under different values of the loading rate and observing the behavior of the system.

Figures 5a and 5b show the maximum throughput as a function of the loading rate when loading from all directions and from one side only, respectively. For the first spatial loading pattern, and using the parameter values and relations described earlier, the maximum system throughput achieved is about 74,000 pilgrims/hr. Loading at rates higher than the optimum caused the system to break down. Similarly, the maximum throughput of the system achieved for the case of semiuniform loading (from one side only) was found to be about 44,000 pilgrims/hr. However, loading at higher rates caused the throughput to decline accordingly to lower but stable values.

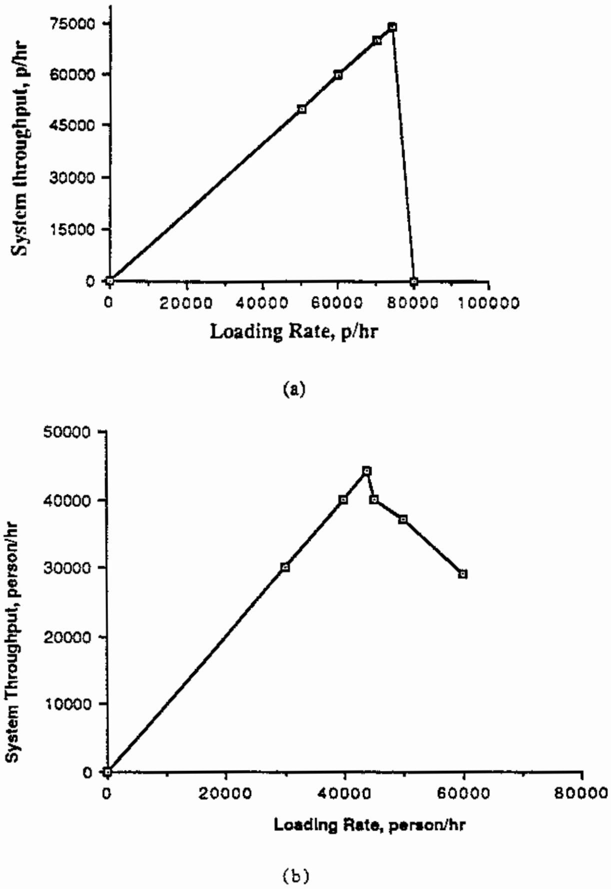

FIGURE 5 Maximum throughput of the Jamarah when loading uniformly from (a) all directions around the ring, and (b) one side only.

Overall system behavior in both cases appears reasonable. In the first case, the system gets locked up beyond a maximum loading rate that it can absorb, because masses of each class of users eventually block the others' progress. In the second case, one-half of the Jamarah is used for loading whereas the other half functions as an evacuation outlet; the decline in throughput at rates higher than optimum is caused by the higher concentration levels, especially of Class 1 pilgrims, which results in more impedance to those who have already finished the stoning (Class 2) and are therefore seeking to leave the system.

It is interesting to point out that a field study conducted independently at the Jamarat measured the maximum throughput of the system at 69,000 pilgrims/hr (only at the top level of the Jamarat bridge) for a case that approximates that of loading from all directions (during the first day of stoning when pilgrims are required to stone only the Big Jamarah, which can be approached from any direction), and at 43,000 pilgrims/hr for loading only from one-half of the Jamarah (during the second and third days of stoning) (*28*). The findings are in remarkably close agreement with these measurements even though the measurement procedure by the Hajj Research Center (*28*) is not directly comparable to the manner in which throughput is calculated in the simulation.

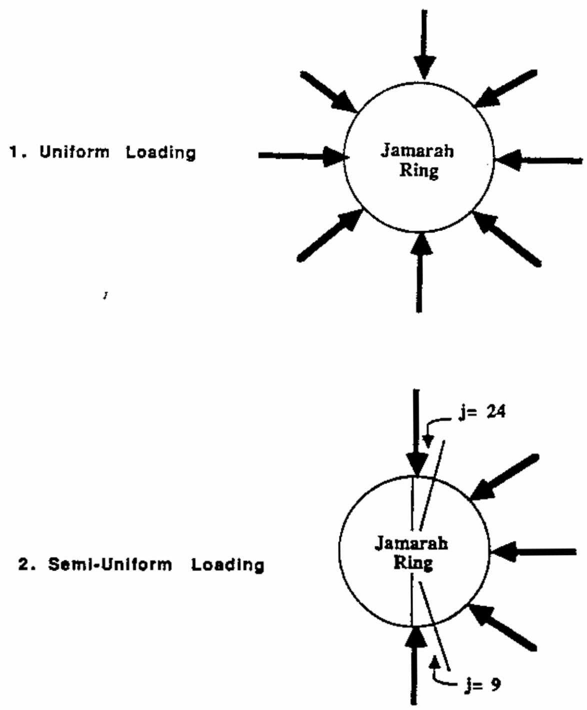

FIGURE 4 Spatial loading patterns of Jamarah ring.

An extensive investigation of the dynamic properties of the system is reported elsewhere (*31*). An illustration of the dynamic behavior of the system is given here. For this purpose, pilgrims are injected from one side only at a constant rate (of 70,000 pilgrims/hr, well in excess of the optimum level of 44,000 pilgrims/hr) over a given period (0.3 hr in this case, which is sufficient to illustrate concentration build-up), after which the inflow is shut off until the system clears. The model yields the values of the concentrations of the two classes of users in every cell at every time step, as well as a variety of associated summary measures. Figure 6 shows the dynamic concentration profile, for each class of facility users, prevailing at the rim of the Jamarah ring (i.e., for Ring $i = 2$), after the inflow has been shut off. Note that loading was taking place from one side only, i.e., for Sectors $j = 9, 10, \ldots, 24$. Figure 7 shows the corresponding dynamic concentration profile for a selected sector ($j = 13$). These figures correspond to an extremely congested situation, because the loading rate was well in excess of the system capacity. Class 2 users have been able to push their way out radially, away from the rim, which is dominated by Class 1 users, and into Rings 7 through 13.

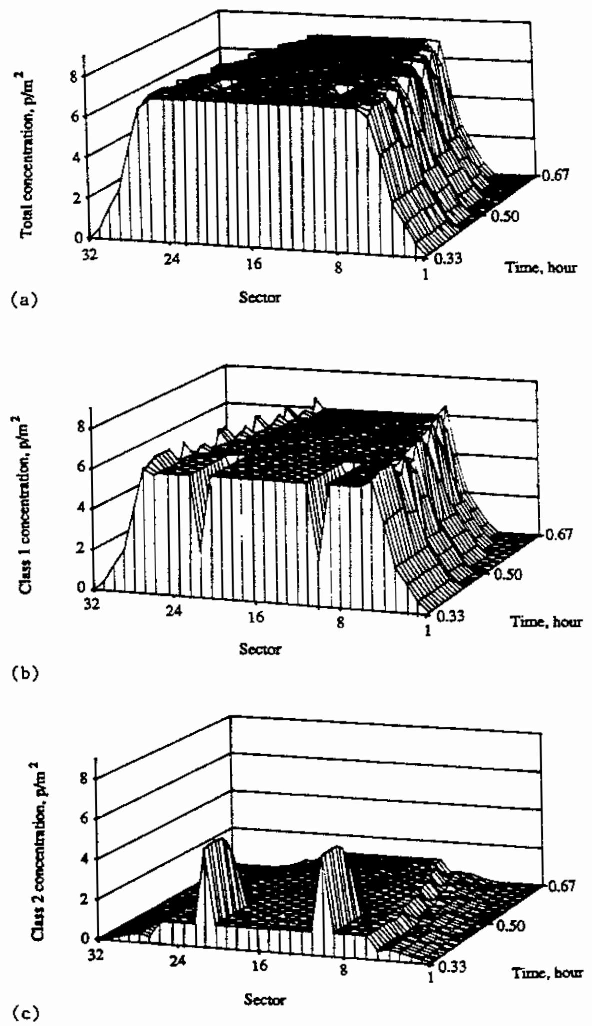

FIGURE 6 Concentration profile dynamics, at the rim of the Jamarah (Ring $i = 2$), with 70,000 pilgrims/hr loading rate from one side only, with loading shut off at time = 0.3 hr, for (a) both classes, (b) Class 1, and (c) Class 2 facility users.

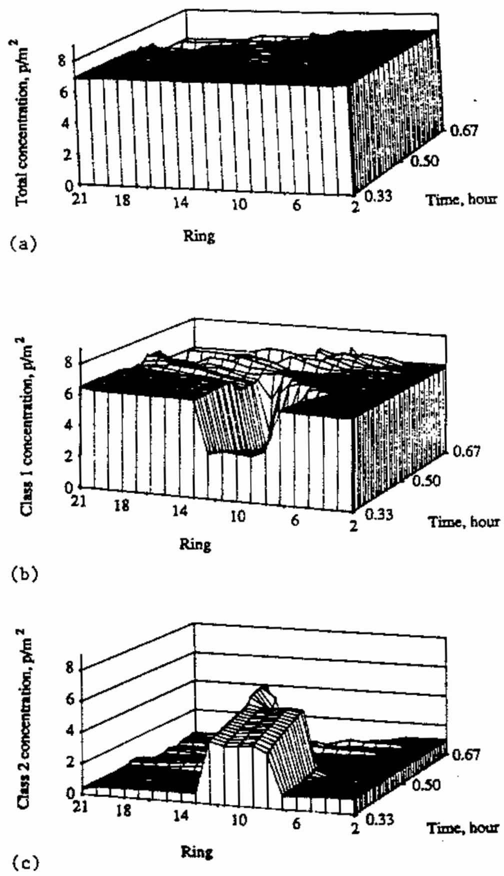

FIGURE 7 Concentration profile dynamics, at Sector $j = 13$, with 70,000 pilgrims/hr loading rate one side only, with loading shut off at time = 0.3 hr, for (a) both classes, (b) Class 1, and (c) Class 2 facility users.

Design and Control Strategies

So far, only a constant-rate temporal loading pattern was considered to examine the maximum throughput of the system. Another loading pattern was considered by pulsing pilgrims into the system: the system is loaded continuously for a given time, TPULSE, then shut off for an equal time before resuming loading again, and so on. Different runs were performed using different values of TPULSE ranging from 5 to 15 min. A similar strategy was successfully used for improving vehicular traffic flow in tunnels (*33*). However, for the case considered here, no improvement in system performance was observed, probably because of the fact that the case is not dealing with a continuous movement of individual users, because each pilgrim is supposed to completely stop to perform stoning.

In order to attain the maximum throughput of the Jamarat system under the current situation, the analysis presented earlier suggests that the input rate should be maintained at

about the optimum level. However, system stability considerations suggest that a safety buffer should be allowed for both types of spatial loading patterns considered. The actual rate at which pilgrims would be admitted to the system should be below the theoretical optimum by the corresponding buffer. Furthermore, on the basis of the analysis presented earlier, the all-directions spatial loading pattern should have a larger buffer than the one-side-only loading pattern because of impending system breakdown when the Jamarah is loaded from all directions at rates higher than the optimum.

In practice, pilgrims' flow into the system could be controlled by scheduling their arrival times through their official guides, who are responsible for providing them with accommodations, transportation, and logistical support. However, this may not work too well in part because of the fact that those guides do not have much control over their pilgrims' local movement within the Mina area, where the Jamarat is located. Therefore, use of some kind of direct control at the facility itself may be considered. One possibility would be to use flexible physical barriers at the entrances of the Jamarat bridge that could funnel the flow of pilgrims into the system so as to result in the established loading rate (e.g., using an inverted cone-shaped barrier). Extreme caution should be taken to avoid the use of fixed hard barriers for this purpose, because it might result in a dangerous pedestrian crush; one possibility is to use horses to form the boundaries of the funneling apparatus, with personnel riding on the horses to direct the crowd.

On the other hand, to improve the maximum throughput of the system, one may consider changing the design of the physical system or affecting the behavior of the crowd in one way or another to improve the efficiency of the system.

From the design standpoint, one could change the size of the Jamarah ring itself. The circumference of a circle increases linearly with the radius, so in order to allow more pilgrims to simultaneously perform stoning at the Jamarah, one could increase the radius of the ring. The simulation model was used to explore the maximum throughput of a system when the ring has a 12-m radius, instead of the present 8 m. The maximum throughput was found to be about 116,000 pilgrims/hr when loading from all directions and 65,000 pilgrims/hr when loading from one side only. Thus, one way to increase the service rate of the system is to increase the size of the Jamarah ring itself. However, before considering the implementation of such a strategy, the effect of enlarging the size of the ring on the surrounding physical geometry, and movement within it, should be carefully evaluated. In addition, the height of the stoning monument should be increased accordingly to keep functioning as an orientation mark for the crowd.

Another possible physical design strategy is to change the shape of the Jamarah ring into an elliptical one. Using an ellipse instead of a circle with the same area would provide a larger circumference, thereby allowing more pilgrims to simultaneously perform stoning. For each spatial loading pattern, two simulation runs were performed using an ellipse with major and minor radii of 1.33 and 0.75 times the radius of the circular ring with equivalent area, respectively. It was found that the throughput would only improve slightly (3 to 5 percent) at the expense of increasing the prevailing average concentration level. However, such shape is expected to encourage more lateral movement of pilgrims and thus more uniform distribution of pilgrims around the ring, provided that the major axis of the ellipse is aligned along the centerline of the Jamarat bridge. One could also enlarge the stone monument, located at the center of the elliptical ring, to provide a wider target at the long sides of the ellipse, therefore encouraging further lateral spread of the pilgrims. These two strategies, which involve changes to the physical system, would only be applicable if they are in accordance with the religious rules governing the stoning process. It is the understanding of the authors that such changes are permissible, but opinions of the appropriate religious scholars should be obtained before implementing such strategies.

Even if the current physical layout of the system is kept unchanged, pilgrims approaching the Jamarah could be encouraged to perform stoning sooner than is currently done (i.e., having a larger fraction of Class 1 users switch from a given distance from the ring). This can be reflected in the model by a different value of the parameter γ in the relation $FR[r_i, K(i,j)] = 1.005\ (r_i)^{\gamma}[K(i,j)]^{2.341}$ (Equation 8). In the current model, $\gamma = -0.793$, but using values of -0.5, -0.3 and -0.1 instead yield maximum throughputs of 87,000, 98,000 and 110,000 pilgrims/hr, respectively, when loading from all directions; and 51,000, 59,000, and 65,000 pilgrims/hr, respectively, for the one-side only loading pattern. In practice this could be implemented through educating pilgrims to stone sooner, and possibly encouraging them to do so on the spot through a public address system. Also, one could elevate the position of the throwers by slightly inclining the surface towards the Jamarah ring. One might also consider encouraging pilgrims to stone faster (i.e., lower TRAJM). Theoretically such strategy could result in a dramatic increase in the system throughput. However, it would be extremely difficult to implement in practice because pilgrims who have made it through the crowd all the way to their stoning position would like to spend enough time to perform stoning successfully. This reluctance could be further appreciated when one realizes that the stoning time (average of 27 sec) is already small when compared to the time it takes a pilgrim to push through the crowd, which was observed to range from 4 to 10 min.

Finally, it was shown earlier that in the extreme case the throughput increased from 44,000 to 74,000 pilgrims/hr when the distribution of pilgrims changed from being semiuniform to being uniform around the Jamarah ring. This process suggests that encouraging more lateral movement of pilgrims would result in better overall system performance. Several simulation runs were performed by varying each of the parameters of the lateral movement relation (Equation 4). In these runs, values of 1,700, 1.0, and 0.1 persons/m^2 for the parameters α, β, and $\Delta K^{m}_{\min}$, respectively, were used instead of the previously reported values of 850, 0.005, and 1.0 persons/m^2, respectively. The change in the values of each of these three parameters resulted in higher system throughput, lower average concentration on the side where loading is taking place, and higher concentration on the other side of the ring. In addition, these changes resulted in better use of the space around the Jamarah and faster evacuation of the system after the input flow of pilgrims was shut off. In practice, this could be implemented through providing the pilgrims with real-time video information (e.g., through a giant elevated high-resolution screen indicating the concentration profiles around the ring) regarding the availability of vacant or less-congested positions at the other side of the Jamarah where no loading is taking place.

CONCLUSION

The model presented provides a useful framework for the simulation of crowd behavior and movement, and for the analysis of design features and control schemes for the effective management and operation of the Jamarat system. The need for such a modeling capability and for the development and application of sound scientific principles for crowd management problems was recently heightened by the tragic events in which over 1,400 people perished in the latest pilgrimage season in July 1990 (*34*).

The model consists of three fundamental processes: radial movement, lateral movement, and stoning process characterization (switching mechanism). Mathematical representations were developed for each of these processes, and the parameters of the principal relations were calibrated using actual observations. However, these observations were somewhat limited, and may not have covered a sufficiently wide spectrum of conditions that might be encountered in the system. These limitations affect the three processes of interest to varying degrees. For instance, there is little confidence at the present time in the parameter values of the lateral movement model. More extensive data collection is necessary for a more definitive model of lateral movement, especially because the limited data available seemed to suggest the existence of possible directionality reflecting users' preferred movement directions. On the other hand, the speed-concentration model developed here for bidirectional crowd movement may be robust. In particular, the conclusion that the impedance of facility users going in their own direction is about twice that of those going in the opposite direction appears to be rather well supported by the available data. In addition, it corresponds to the authors' own experience in other heavily crowded conditions. Nevertheless, it is not clear that such a conclusion can be extrapolated to all situations regardless of the relative mix of the two user classes. Relations and parameter values for the third process, namely the stoning process, are also based on a sufficient number and range of observations to justify a reasonable degree of confidence in the results.

While motivated and presented for the Jamarat system, the modeling approach developed here is general and should essentially be applicable to a variety of heavy crowding situations. Of course, the application of any particular situation would require customization in terms of defining a spatial discretization scheme and identifying the principal classes of facility users. Of the relations developed for the Jamarat system, the bidirectional speed-concentration model is likely to be the most directly applicable in other situations, though some adjustment in parameter values would likely be necessary to reflect different cultural norms. It is hoped that additional effort will be directed at crowd phenomena in a variety of settings in order to build a body of scientific knowledge and engineering procedures to enhance the quality of the human experience in such settings and improve the safety and efficiency of the associated operations.

REFERENCES

1. J. J. Fruin. Crowd Disasters—A System Evaluation of Causes and Countermeasures. *Proc., Experts' Workshop on Crowd Ingress to Places of Assembly*, F. T. Ventre et al. (eds.), National Bureau of Standards, Gaithersburg, Md, 1981.
2. *Masarat Al-Mushat fi Al Mashair Al-Muqaddasah (Pedestrian Footways in the Holy Rites)*. Hajj Research Center, Umm Al Qura University, Makkah, Saudi Arabia, 1982.
3. *Al-Jamarat: Dirasat Alwada' Alrahin Wa Was'ail Tatweeruh (Jamarat: Study of Current Conditions and Means of Improvement)*. Hajj Research Center, Umm Al Qura University, Makkah, Saudi Arabia, 1984.
4. B. Hankin and R. Wright. Passenger Flow in Subways. *Operational Research Quarterly*, Vol. 9, No. 2, 1958, pp. 81–88.
5. D. Oeding. Verkehrsbelastung und Dimensionierung von Gehwegen und anderen Anlagen des Fussgangerverkehrs (Traffic Loads and Dimensions of Walkways and other Pedestrian Circulation Facilities). *Strassenbau und Strassenverkehrstechnik*, Vol. 22, Bonn, West Germany, 1963.
6. V. M. Predtechnskii. Dvizhenie Lyndkikh Potokovi Operadeknie Razmerove Kommunikatsionnykh Pomeshch Enii. Glov v Uchenbnike, Arkhitekura Grahdanskikh i Promyshlennykh Zdanii (Pedestrian Traffic Flow and the Determination of Passage Dimensions in Buildings. Chapter: Architecture of Public and Industrial Buildings), Stroiizdat, Moscow, 1966.
7. S. J. Older. Movement of Pedestrians on Footways in Shopping Streets. *Traffic Engineering and Control*, Vol. 10, No. 4, 1968, pp. 160–163.
8. F. Navin and R. Wheeler. Pedestrian Flow Characteristics. *Traffic Engineering*, Vol. 39, No. 9, 1969, pp. 30–33, 36.
9. J. J. Fruin. *Pedestrian Planning and Design*. Metropolitan Association of Urban Designers and Environmental Planners, New York, 1971.
10. A. Ceder. An Algorithm to Assign Pedestrian Groups Dispersing at Public Gatherings Based on Pedestrian-Traffic Modelling. *Applied Mathematical Modelling*, Vol. 3, April, 1979, pp. 116–124.
11. V. M. Predtechnskii and A. I. Milinski. *Planning for Foot Traffic Flow in Buildings*. (Translated from Russian, Stroiizdat Publishers, Moscow, 1969.) Amerind Publishing Co., New Delhi, India, 1978.
12. O. E. Klapp. *Currents of Unrest*. Holt, Rhinehart, and Winston, 1972.
13. J. L. Freedman. *Crowding and Behavior*. W. H. Freeman and Company, San Francisco, Calif., 1975.
14. M. Wolf. The Behavior of Pedestrians on 42nd Street, New York City. City University of New York, 1970.
15. J. Wolpert and D. Zillmann. The Sequential Expansion of a decision Model in a Spatial Context. *Environment and Planning*, Vol. 1, 1969, pp. 91–104.
16. I. B. Stilitz. The Role of Static Pedestrian Groups in Crowded Spaces. *Ergonomics*, Vol. 12, No. 6, 1969, pp. 821–839.
17. I. B. Stilitz. Pedestrian Congestion, RIBA Publication 61–72. *Architectural Psychology*, D. V. Canter (ed.), London, 1970.
18. A. E. Baer. *A Simulation Model of Multi-Directional Pedestrian Movement Within Physically Bounded Environments*. Report 47, Institute of Physical Planning, Carnegie-Mellon University, Pittsburgh, Pa., 1974.
19. W. Boles. Planning Pedestrian Environment: A Computer Simulation Model. *Proc., 4th National Seminar on Planning, Design, and Implementation of Bicycle and Pedestrian Facilities*, 1975.
20. K. Hirai and K. Tarui. A Simulation of the Behavior of a Crowd in Panic. *Proc., 1975 International Conference on Cybernetics and Society*, San Francisco, Calif., 1975, pp. 409–411.
21. F. H. Harlow and D. L. Sandoval. Human Collective Dynamics: The Mathematical Modeling of Mobs. Report LA-1076S-MS. *Mathematical Modeling of Biological Ensembles*, Los Alamos National Laboratory, Los Alamos, N. Mex., 1986, pp. 31–49.
22. P. Endean. *Pedestrian Movement In and Out of the Holy Areas of Arafat, Muzdalifah, and Mina during the 1395 (1975) Hajj*. Hajj Research Center, Umm Al Qura University, Makkah, Saudi Arabia, 1977.
23. P. Endean. *The Pedestrian Movement During the Nafrah (Departure from Arafat to Muzdalifah)*. Hajj Research Center, Umm Al Qura University, Makkah, Saudi Arabia, 1977.
24. I. J. Gibson. *An Analysis of Tawaf, Sa'ee, and Umra for the 1396 (1976) Hajj*. Hajj Research Center, Umm Al Qura University, Makkah, Saudi Arabia, 1977.
25. E. Haug, B. Gawenal, and M. A. B. Rasch. *Optimization of the*

Tawaf Capacity of the Mataf Area. Islamic Agency for Technology, 1987.
26. *Dirasat Harakat Al-Hujjaj fi Al-Masa'a (Study of Pilgrims' Movement During Performing Sa'ee)*. Hajj Research Center, Umm Al Qura University, Makkah, Saudi Arabia, 1983.
27. *Dirasa'at Al-Masjid Al-Haram: Dirasat Al-Harakah fi Al-Mataf (The Sacred Mosque Studies: A Study of the Movement in the Tawaf Area)*. Hajj Research Center, Umm Al Qura University, Makkah, Saudi Arabia, 1987.
28. *Counting Pilgrims on the Jamarat Bridge During 1984 Pilgrimage*. Hajj Research Center, Umm Al Qura University, Makkah, Saudi Arabia, 1985.
29. A. H. Al-Rabeh, and S. Z. Selim. An Analysis of Congestion in the Jamarat Area. *Proc., 9th National Computer Conference*, Riyadh, Saudi Arabia, 1986.
30. S. A. H. AlGadhi, and H. S. Mahmassani. Modelling Crowd Behavior and Movement: Application to Makkah Pilgrimage. *Proc., 11th International Symposium on Transportation and Traffic Theory*, M. Koshi (ed.), Yokohama, Japan, July, 1990, pp. 59–78.
31. S. A. H. AlGadhi. *Characterization of Crowd Behavior and Movement*. Ph.D. dissertation, Department of Civil Engineering, The University of Texas, Austin, 1990.
32. D. C. Gazis, R. Herman, and G. H. Weiss. Density Oscillations Between Lanes of a Multilane Highway. *Operation Research*, Vol. 10, 1962, pp. 658–667.
33. L. C. Edie, and R. S. Foote. Effect of Shock Waves on Tunnel Traffic Flow. *Proc. HRB*, 1960, p. 39.
34. The Washington Post, July 3, 1990.

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