**PHYS 502**
**2nd Midterm Exam**
**Wednesday 8th May 2013**

**Instructor: Dr. V. Lempesis**

**Student Name: …….**

**Student ID Number……..**

**Student Grade: ……./20**

***Please answer question 1 and one question from 2 and 3***

**1.** Use the following relations for the Bessel functions: $\frac{d}{dx}\left\{x^n J_n(x)\right\} = x^n J_{n-1}(x)$, $\frac{d}{dx}\left\{x^{-n} J_n(x)\right\} = -x^{-n} J_{n+1}(x)$ and prove the following relations:

a) $\frac{dJ_n(x)}{dx} = \frac{1}{2}\left\{J_{n-1}(x) - J_{n+1}(x)\right\}$.

(2 marks)

b) $J_{n-1}(x) + J_{n+1}(x) = \frac{2n}{x} J_n(x)$

(2 marks)

**2.** A) Apply the method of separation of variables for the wave motion of a string of length $L$ with both ends fixed. Find the general solution for the motion of the string if the string is given an initial displacement $u(x,0) = f(x)$ and zero initial velocity.

(4 marks)

B) Find the specific solution if $f(x) = \begin{cases} x & 0 < x \le L/2 \\ -x + L & L/2 \le x < L \end{cases}$.

(4 marks)

**3.** A) Apply the method of separation of variables for temperature distribution in a beam length $L$, which has a coefficient of thermal conductivity $\sigma$ and an initial temperature $u(x,0) = f(x)$. Both ends of the beam remain at zero temperature.

(4 marks)

B) Find the specific general solution if $f(x) = x$, $L = 2$ and $\sigma = 3$.

(4 marks)

$$\int x\sin(kx)dx = \frac{\sin kx}{k^2} - \frac{x\cos kx}{k}, \quad \int x\cos(kx)dx = \frac{x\sin kx}{k} + \frac{\cos kx}{k^2}$$

$$\int \sin^2(kx)dx = \frac{x}{2} - \frac{1}{4k}\sin 2kx$$

$$\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2}, \quad \frac{\partial u}{\partial t} = \sigma\frac{\partial^2 u}{\partial x^2}$$