King Saud University

Department of Mathematics

1st Semester 1437-1438H

MATH 151 (Discrete Mathematics)

2nd Midterm Exam

Duration: 90 Minutes

Name:

Sequence Number:

Teacher's Name:

Section:

Note:* *The exam consists of 5 pages.

Question	**Mark**
Question I	
Question II	
Question III	
Question IV	
Total	

Question Number	1	2	3	4	5
Answer					

<u>Question I</u>:

Choose the correct answer, then fill in the table above:

(1) For the equivalence relation on $\mathbb{Z}$ defined by $aRb \Leftrightarrow a \equiv b(mod3)$, $\mathbb{Z}$ can be partitioned as follows:

(a) $\{[0],[1],[2],[3]\}$

(b) $\{[1],[2],[3],[4]\}$

(c) $\{[0],[1],[2]\}$

(d) None of the previous

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(2) The set $\{(1,2),(2,4),(3,8),(4,16)\}$ represents the relation

(a) $xRy \Leftrightarrow x+2<y$

(b) $xRy \Leftrightarrow y=2^x$

(c) $xRy \Leftrightarrow x+y \text{ is odd}$

(d) None of the previous

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(3) Let $R_1=\{(a,b)\in \mathbb{R}^2 : a=b\}$ and $R_2=\{(a,b)\in \mathbb{R}^2 : a>b\}$, then $R_1 \cap R_2 =$

(a) ϕ (b) R_1 (c) R_2 (d) None of the previous

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(4) In the poset $(\mathbb{Z}, \geq)$, any two elements are comparable

(a) True (b) False (c) None of the previous

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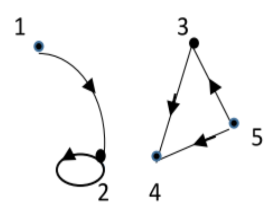
(5) The relation R, defined on $\{1,2,3,4,5\}$ whose graph is below, is



(a) reflexive (b) symmetric (c) transitive (d) None of the previous

Question II:

Let $\{a_n\}$ be a sequence defined inductively as

$$a_0 = 0, \qquad a_1 = 2, \qquad a_{n+1} = 4a_n - 3a_{n-1}, \ \ \forall n \geq 1.$$

Using **Strong Induction** prove that

$$a_n = 3^n - 1, \ \ \forall n \geq 0.$$

Question III:

A. Let R be a relation defined on $\mathbb{Z}$ by

$$aRb \Leftrightarrow 3 \ divides \ a^2 - b^2.$$

Then answer the following:

(i) Prove that R is an equivalence relation on $\mathbb{Z}$.

(ii) Find the equivalence class [4].

B. Find the transitive closure of the relation $R = \{(1,1), (1,3), (2,2), (3,1), (3,2)\}$ defined on the set $A = \{1,2,3\}$.

Question IV:

A. Let S be a nonempty set and R be the relation defined on $\mathcal{P}(S)$ by

$$ARB \Leftrightarrow A \subseteq B,\ \ \forall A, B \in \mathcal{P}(S).$$

Prove that $(\mathcal{P}(S), R)$ is a partially ordered set. Is it totally ordered? Justify your answer.

B. Draw the Hasse diagram for the relation R on the set $\{1,3,4,6,12,16,32\}$ given by:

$$aRb \Leftrightarrow a \text{ divides } b.$$

Good Luck☺