KING SAUD UNIVERSITY
College of Science
Department of Mathematics

M-106

Second Semester (1431/1432)

Solution Second Mid-Exam

Name:	Number:
Name of Teacher:	Group No:

Max Marks: 20 Time: 90 minutes

Marks:

Multiple Choice (1-10)	
Question # 11	
Question # 12	
Question # 13	
Question # 14	
Total	

Multiple Choice

Q.No:	1	2	3	4	5	6	7	8	9	10
$\{a, b, c, d\}$	d	b	d	c	b	c	c	a	b	c

Q. No: 1 If $2x = \sec\theta$ for $\frac{\pi}{2} < \theta < \pi$, then $\sqrt{4x^2 - 1}$ is equal to:

(a) $\tan\theta$ (b) $2\tan\theta$ (c) $\frac{1}{2}\tan\theta$ (d) $-\tan\theta$

Q. No: 2 The partial fraction decomposition of $\dfrac{3x}{(x^2-1)(x^2+1)}$ takes the form:

(a) $\frac{Ax+B}{x^2-1} + \frac{Cx+D}{x^2+1}$ (b) $\frac{A}{x-1} + \frac{B}{x+1} + \frac{Cx+D}{x^2+1}$ (c) $\frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{x^2+1}$
(d) $\frac{A}{x-1} + \frac{B}{x+1} + \frac{Cx+D}{x^2-1}$

Q. No: 3 To evaluate the integral $\displaystyle\int x^3\sqrt{2x^2+8}dx$, we use the substitution:

(a) $x = 2\sec\theta$ (b) $x = 2\sqrt{2}\tan\theta$ (c) $x = 2\sqrt{2}\sec\theta$ (d) $x = 2\tan\theta$

Q. No: 4 The value of the integral $\displaystyle\int_0^{\frac{\pi}{3}} \sec^2(x)\tan x \, dx$ is equal to:

(a) $\frac{\pi^2}{18}$ (b) $\frac{2}{3}$ (c) $\frac{3}{2}$ (d) $\frac{-\pi^2}{18}$

Q. No: 5 To evaluate the integral $\displaystyle\int \frac{1}{x^{\frac{2}{3}} + x^{\frac{4}{3}}}dx$, we use the substitution:

(a) $x = u^2$ (b) $x = u^3$ (c) $x = u^4$ (d) $x = u^{\frac{2}{3}}$

Q. No: 6 The value of the integral $\displaystyle\int x\sin x dx$ is equal to:

$(a)\sin x + x\cos x + c$ $(b) -\sin x + x\cos x + c$ $(c)\sin x - x\cos x + c$
$(d) -\sin x - x\cos x + c$

Q. No: 7 To evaluate the integral $\displaystyle\int \cos^3 x\sin^4 x dx$, we use the substitution:

(a) $u = \sec x$ (b) $u = \cos x$ (c) $u = \sin x$ (d) $u = \tan x$

Q. No: 8 The value of the integral $\displaystyle\int \frac{1}{x^2+2x+5}dx$ is equal to:

(a) $\frac{1}{2}\tan^{-1}\left(\frac{x+1}{2}\right) + c$ (b) $\tanh^{-1}\left(\frac{x+1}{2}\right) + c$ (c) $\tan^{-1}\left(\frac{x+1}{2}\right) + c$
(d) $\frac{1}{2}\tanh^{-1}\left(\frac{x+1}{2}\right) + c$

Q. No: 9 The improper integral $\int_0^1 \frac{1}{x^{\frac{3}{2}}} dx$

(a) converges to 3 (b) diverges (c) converges to $\frac{2}{3}$ (d) converges to $\frac{3}{2}$

Q. No: 10 The substitution $u = \tan\left(\frac{x}{2}\right)$ transforms the integral $\int \frac{1}{1+\cos x} dx$ into:

(a) $\int 2du$ (b) $\int udu$ (c) $\int du$ (d) $\int \frac{1}{1+u} du$

Full Questions

Question No: 11 **Evaluate** $\lim_{x\to\infty}\left(1+\frac{3}{x}\right)^{2x}$ [**2**]

Solution:

Let $y=\left(1+\frac{3}{x}\right)^{2x}$. Then $\ln y = 2x\ln\left(1+\frac{3}{x}\right) = \dfrac{\ln\left(1+\frac{3}{x}\right)}{\frac{1}{2x}}$.

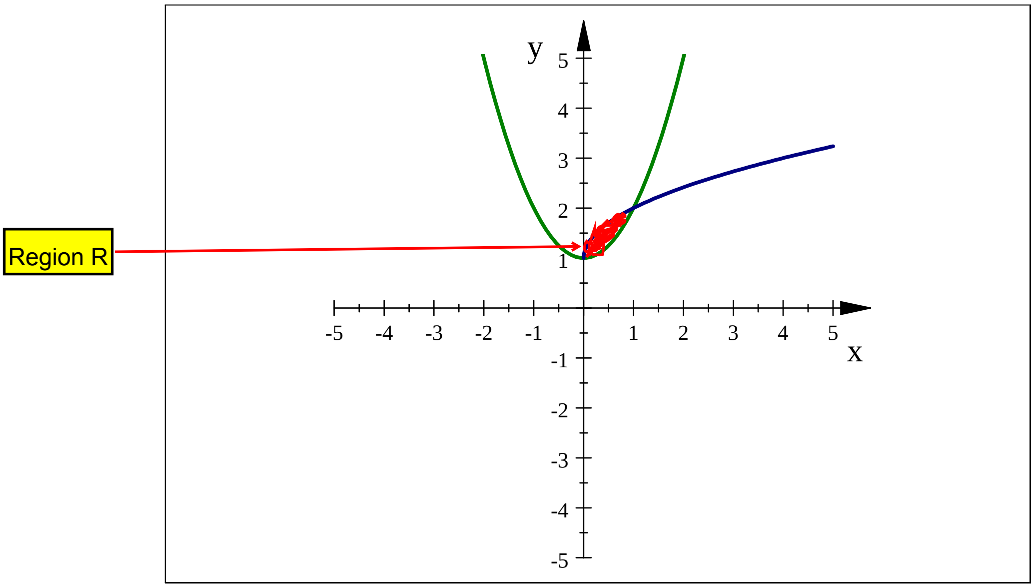
$$\lim_{x\to\infty}\ln y = \lim_{x\to\infty}\frac{\ln\left(1+\frac{3}{x}\right)}{\frac{1}{2x}} = \lim_{x\to\infty}\frac{6}{1+\frac{3}{x}} = 6. \quad (1)$$

So

$$\lim_{x\to\infty}\left(1+\frac{3}{x}\right)^{2x} = e^6 \quad (1)$$

Question No: 12 Let R be the region **bounded** by the graphs of $y=1+x^2$ and $y=1+\sqrt{x}$. **Sketch** the region R and **find** its area. [**2**]

Solution:



y=1+x^2 and y=1+$\sqrt{x}$ (1)

$$\text{Area}=\int_0^1 \left[(1+\sqrt{x})-(1+x^2)\right]dx = \int_0^1 \left(\sqrt{x}-x^2\right)dx = \tfrac{1}{3}. \quad (1)$$

Question No: 13 **Evaluate** $\displaystyle\int \frac{1}{x^2\sqrt{x^2-1}}dx$ [**3**]

Solution:

Let $x = \sec\theta$, $\quad 0 \le \theta < \frac{\pi}{2}$ or $\pi \le \theta < \frac{3\pi}{2}$. $(dx = \sec\theta\tan\theta d\theta)$ (0.5)

$\sqrt{x^2-1} = \sqrt{\sec^2\theta - 1} = \sqrt{\tan^2\theta} = \tan\theta. \qquad (\tan\theta \ge 0)$

So

$$\begin{aligned}
\int \frac{1}{x^2\sqrt{x^2-1}}dx &= \int \frac{1}{\sec^2\theta\tan\theta}\sec\theta\tan\theta d\theta \\
&= \int \cos\theta d\theta && (1) \\
&= \sin\theta + c && (0.5) \\
&= \frac{\sqrt{x^2-1}}{x} + c && (1)
\end{aligned}$$

Question No: 14 **Evaluate** $\displaystyle\int \frac{x-1}{(x+1)(x^2+1)}dx$ [**3**]

Solution:

$$\begin{aligned}
\frac{x-1}{(x+1)(x^2+1)} &= \tfrac{Ax+B}{x^2+1} + \tfrac{C}{x+1} && (0.5) \\
&= \tfrac{x}{x^2+1} - \tfrac{1}{x+1} && (0.5+0.5+0.5)
\end{aligned}$$

So

$$\begin{aligned}
\int \frac{x-1}{(x+1)(x^2+1)}dx &= \int \tfrac{x}{x^2+1}dx - \int \tfrac{1}{x+1}dx \\
&= \tfrac{1}{2}\ln|x^2+1| - \ln|x+1| + c && (0.5+0.5)
\end{aligned}$$