Department of Statistics & Operations Research

College of Science, King Saud University

STAT 324, Test II

Semester I, 1433 – 1434 H

Student Name:			
Student Number:		Section Number:	
Teacher Name:		Attendance Number	

- Mobile Telephones are not allowed in the classrooms.
- Time allowed is 90 minutes
- Answer all questions.
- Choose the nearest number to your answer.
- WARNING: Do not copy answers from your neighbors. They have different questions forms.
- For each question, put the code in capital letter of the correct answer, in the following table, beneath the question number:

1	2	3	4	5	6	7	8	9	10
D	A	E	E	A	C	C	D	E	B

11	12	13	14	15	16	17	18	19	20
D	A	E	D	D	E	A	B	A	E

21	22	23	24	25	26	27	28	29	30
A	C	E	D	B	D	A	B	B	E

⏩ Suppose that the percentage of females in a certain population is 50 %. If three people are selected randomly from this population, then:

1) The probability that at most two females are selected is:

(A) 0.001	(B) 0.900	(C) 0.008	(D) 0.875	(E) 0.810

2) The probability that no females are selected is:

(A) 0.125	(B) 0.896	(C) 0.784	(D) 0.652	(E) 0.247

3) The mean number of females is:

(A) 3	(B) 1	(C) 1.75	(D) 1.25	(E) 1.50

4) The standard deviation of the number of females is:

(A) 0.50	(B) 0.87	(C) 2.75	(D) 3.25	(E) 0.75

⏩ Suppose that a family has 5 children, 3 of them are boys. A sample of 2 children is selected randomly without replacement. Then:

5) The probability that at most one boy are selected is:

(A) 0.7	(B) 0.1	(C) 0.6	(D) 0.9	(E) 0.3

6) The probability that no boys are selected is:

(A) 0.0	(B) 0.9	(C) 0.1	(D) 0.7	(E) 0.3

7) On average, the number of boys in the sample is:

(A) 2	(B) 3	(C) 1.2	(D) 1	(E) 1.5

8) The variance of the number of boys is:

(A) 0.60	(B) 0	(C) 1	(D) 0.36	(E) 0.67

⏩ Let X represents the outcome when a balanced die is tossed.

9) The expected value of the random variable X is:

(A) 2	(B) 1.5	(C) 1	(D) 1.2	(E) 3.5

10) The standard deviation value of the random variable X is:

(A) 2.92	(B) 1.71	(C) 1	(D) 0	(E) 2.67

11) P(X=1 or 3) is:

(A) 0.125	(B) 0	(C) 1	(D) 0.333	(E) 0.667

⏩ Suppose that the number of telephone calls received per day follows a Poisson distribution with an average of 4 calls per day.

12) The probability that two calls will be received in a given day is:

(A) 0.147	(B) 0	(C) 0.333	(D) 0.125	(E) 0.667

13) The expected number of telephone calls received in a given week is:

(A) 4	(B) 8	(C) 32	(D) 1	(E) 28

14) The probability that at least two calls will be received in a period of 12 hours is:

(A) 0.294	(B) 0.194	(C) 0.094	(D) 0.594	(E) 0.394

15) The standard deviation of the number of telephone calls received in 3 days is:

(A) 3.125	(B) 0	(C) 12	(D) 3.464	(E) 3.667

⏩ Suppose that the random variable X has a uniform distribution on the interval (2,5) then:

16) P(X < 3) is:

(A) 1	(B) 0.275	(C) 3	(D) 0.225	(E) 0.333

17)The mean of X is:

(A) 3.500	(B) 0.896	(C) 0.350	(D) 3.655	(E) 3.247

18) The variance of X is :

(A) 3	(B) 0.75	(C) 1.75	(D) 1.50	(E) 1

random variable with a probability density function of the form:

$$f(x) = \begin{cases} 0.5\ e^{-0.5x} & ; x \geq 0 \\ 0 & ; elsewhere \end{cases}$$

19) The probability that the system will need between 4 and 7 hours to be repaired is:

(A) 0.105	(B) - 0.5	(C) 5	(D) 0.502	(E) 1

20) The mean time of repairing such system is:

(A) 0.5 hour	(B) 5 hours	(C) 0.2 hour	(D) any time	(E) 2 hours

21) The probability that the system will need more than 4 hours to be repaired is:

(A) 0.1353	(B) 0.5	(C) 0.2086	(D) - 0.5	(E) 0.8647

22) The variance of repair time of such system is:

(A) 2	(B) 0.25	(C) 4	(D) 0.5	(E) 2.5

▸▸ In a photographic process, the developing time of prints can be considered as a random variable having normal distribution with a mean of 16.28 seconds, and a standard deviation of 0.12 seconds. Then the probability that it will take:

23) Anywhere from 16 to 16.5 seconds to develop one of the prints is:

(A) 1	(B) 0.0432	(C) 0.1052	(D) 0.5	(E) 0.9565

24) At least 16.2 seconds to develop one of the prints is:

(A) 0.2389	(B) 0.2514	(C) 0.3974	(D) 0.7486	(E) 0.3156

25) At most 16.35 seconds to develop one of the prints is:

(A) 0.2810	(B) 0.7190	(C) 0.4167	(D) 0.5833	(E) 0.1353

⏩ In a certain city, the average temperature (X) during month of August is nicely modeled by a normal distribution with mean of 45 degrees centigrade and variance 25, then:

26) The percentage of months of August that are expected to have temperatures more than 40 in this city is:

(A) 15.87 %	(B) 5%	(C) 16.60%	(D) 84.13%	(E) 17.36%

27) $P(X > 45)$ is:

(A) 0.5	(B) 0.4129	(C) 0.3632	(D) 0.25	(E) 0

28) The value of k such that $P(30 < X < k) = 0.84$ is:

(A) 35	(B) 50	(C) 45	(D) 40	(E) 55

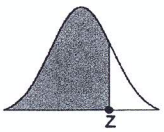
29) The probability that in any August month the temperature is at least 35 is:

(A) 0.0228	(B) 0.9772	(C) 0.2846	(D) 0.7190	(E) 0.9830

30) The probability that in any August month the temperature is between 45 and 50 is:

(A) -0.3413	(B) 0.1885	(C) 0.8413	(D) -0.8413	(E) 0.3413

Areas under the Standard Normal Curve



Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Q1 Binomial distribution

$n = 3$, $p = 0.5 \longrightarrow q = 0.5$

X -- no. of females in the sample

$f(x) = {}^{3}C_{x} (0.5)^{x} (0.5)^{3-x}$

① $P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$

$= {}^{3}C_{0} (0.5)^{3} + {}^{3}C_{1} (0.5)^{1} (0.5)^{2} + {}^{3}C_{2} (0.5)^{2} (0.5)^{1}$

$= 0.875$

or $P(X \leq 2) = 1 - P(X > 2) = 1 - P(X=3)$

$= 1 - {}^{3}C_{3} (0.5)^{3} = 0.875$

② $P(X=0) = {}^{3}C_{0} (0.5)^{3} = 0.125$

③ $\mu_X = n \cdot p = 3 * 0.5 = 1.5$

④ $\sigma_X^2 = n \cdot p \cdot q = 3 * 0.5 * 0.5 = 0.75$

Q2 Hypergeometric distribution

X -- no. of boys in sample

$N = 5$, $n = 2$, $k = 3$

② → | 3 B | 2 G | → 5

(5) $P(X \leq 1) = P(X=0) + P(X=1) = \dfrac{\binom{3}{0}\binom{2}{2} + \binom{3}{1}\binom{2}{1}}{\binom{5}{2}}$

$= 0.7$

(6) $P(X=0) = \dfrac{\binom{3}{0}\binom{2}{2}}{\binom{5}{2}} = 0.1$

(7) $\mu_X = n \cdot \frac{K}{N} = 2 * \frac{3}{5} = 1.2$

(8) $\sigma_X^2 = n \cdot \frac{K}{N}\left(1 - \frac{K}{N}\right)\frac{N-n}{N-1} = 0.36$

Q3 X discrete r.v.

x	1	2	3	4	5	6
f(x)	1/6	1/6	1/6	1/6	1/6	1/6

X is has a discrete uniform distribution, K=6

$f(x) = \frac{1}{6}$; $x = 1, 2, 3, 4, 5, 6$

$= 0$ elsewhere.

(9) $\mu_X = \frac{\sum x_i}{K} = \frac{1+2+3+4+5+6}{6} = 3.5$

(10) $\sigma_X^2 = \frac{\sum (x_i - \bar{x})^2}{K} = \frac{((-2.5)^2 + (-1.5)^2 + (-0.5)^2 + (0.5)^2 + (1.5)^2 + (2.5)^2)}{6}$

$= 2.92$

$\sigma_X = \sqrt{2.92} = 1.71$

(11) $P(X=1 \text{ or } 3) = P(X=1) + P(X=3) = \frac{2}{6} = 0.333$

Q4 $X \sim$ Poisson (μ) $f(x) = e^{-\mu}\frac{\mu^x}{x!}$

$\lambda = 4$ calls/day

(12) $P(X=2) = e^{-4}\frac{4^2}{2!}$ $\equiv \mu = \lambda t = 4 * 1 = 4$

$= 0.147$

(13) $\mu_X = \lambda * t = 4 * 7 = 28$ calls / week

(14) $\mu_X = \lambda * t = 4 * \frac{1}{2} = 2$ calls / 12 hours

$$P(X \geq 2) = 1 - P(X < 2)$$
$$= 1 - [P(X=0) + P(X=1)]$$
$$= 1 - \left[e^{-2} \cdot \frac{2^0}{0!} + e^{-2} \frac{2}{1!} \right]$$
$$= 1 - 3e^{-2} = 0.594$$

(15) $\mu_X = \lambda * t = 4 * 3 = 12$ calls / 3 days

$\sigma_X^2 = \lambda t = 12 \Rightarrow \sigma_X = \sqrt{12} = 3.464$

Q5 $X \sim U(2, 5)$

$a = 2, b = 5 \Rightarrow f(x) = \begin{cases} \frac{1}{3} & 2 < x < 5 \\ 0 & \text{elsewhere} \end{cases}$

$\frac{1}{b-a} = \frac{1}{5-2} = \frac{1}{3}$

(16) $P(X < 3) = \int_2^3 \frac{1}{3} dx = \frac{1}{3}(3-2) = \frac{1}{3} = 0.333$

(17) $\mu_X = \frac{a+b}{2} = \frac{2+5}{2} = 3.5$

(18) $\sigma_X^2 = \frac{(b-a)^2}{12} = \frac{(5-2)^2}{12} = 0.75$

Q6 $f(x) = \begin{cases} 0.5 e^{-0.5x} & x \geq 0 \\ 0 & \text{elsewhere} \end{cases}$

(19) $X \sim \exp(\beta) \Rightarrow \beta = 2$

$$P(4 \leq X \leq 7) = \int_4^7 0.5 e^{-0.5x} dx$$
$$= -e^{-0.5x} \Big|_4^7 = -e^{-3.5} + e^{-2} = 0.10$$
$$-0.0302 + 0.1353$$

$\mu_X = \beta = 2$

(21) $P(X > 4) = \int_4^{\infty} 0.5\, e^{-0.5x}\, dx$

$= -e^{-0.5x}\Big|_4^{\infty} = 0 + e^{-2} = 0.1353$

(22) $\sigma_X^2 = \beta^2 = 4$

Q7 $X \sim N(\mu, \sigma)$ $\mu = 16.28$, $\sigma = 0.12$

(23) $P(16 \leq X \leq 16.5)$

$= P(X \leq 16.5) - P(X \leq 16)$

$= P\left(Z \leq \frac{16.5 - 16.28}{0.12}\right) - P\left(Z \leq \frac{16 - 16.28}{0.12}\right)$

$= P(Z \leq 1.83) - P(Z \leq -2.33)$

$= 0.9664 - 0.0099 = 0.9565$

(24) $P(X \geq 16.2) = 1 - P(X < 16.2)$

$= 1 - P\left(Z < \frac{16.2 - 16.28}{0.12}\right)$

$= 1 - P(Z < -0.67)$

$= 1 - 0.2514 = 0.7486$

(25) $P(X \leq 16.35) = P\left(Z < \frac{16.35 - 16.28}{0.12}\right)$

$= P(Z < 0.58)$

$= 0.7190$

Q8 $X \sim N(\mu, \sigma)$

$\mu = 45$, $\sigma^2 = 25$, $\sigma = 5$

(26) $P(X > 40) = 1 - P(X < 40)$

$= 1 - P(Z < \frac{40-45}{5})$

$= 1 - P(Z < -1)$

$= 1 - 0.1587 = 0.8413$

percentage = prob. * 100 = 84.13%.

(27) $P(X > 45) = P(X > \mu) = 0.5$

$P(Z > 0)$

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(28) $P(30 < X < k) = 0.84$

$P(X < k) - P(X < 30) = 0.84$

$P(Z < \frac{k-45}{5}) - P(Z < \frac{30-45}{5}) = 0.84$

$P(Z < \frac{k-45}{5}) - P(Z < -3) = 0.84$

$P(Z < \frac{k-45}{5}) = 0.0013 + 0.84 = 0.8413$

$\frac{k-45}{5} = 1 \Rightarrow k = 50^\circ c$

(29) $P(X \geq 35) = 1 - P(X < 35)$

$= 1 - P\left(Z < \frac{35-45}{5}\right)$

$= 1 - P(Z < -2)$

$= 1 - 0.0228 = 0.9772$

(30) $P(45 < X < 50) = P(X < 50) - P(X < 45)$

$= P\left(Z < \frac{50-45}{5}\right) - P\left(Z < \frac{45-45}{5}\right)$

$= P(Z < 1) - P(Z < 0)$

$= 0.8413 - 0.5$

$= 0.3413$