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| KSU logo tiff.tif | **King Saud University** |
| **College of Sciences** |
| **Department of Mathematics** |
| **373 Math** |
| **Final Exam** |
| **First semester 1433-1434** |

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**Question 1:**

1. Let be a function from a space into a space . Prove that is continuous if and only if for every , .
2. Let,… be topological spaces, and let be a function from a space into a product space , given by . Prove that is continuous if and only if are continuous.
3. Let and be functions. Prove that if is open and is continuous onto function, then is open.

**Question 2:**

1. Let be a metric space. Prove that the set is closed, where and . (This set is called the closed ball with center and radius
2. Let be the usual topology on , . Prove that is a metrizable.

**Question 3:**

1. Prove that in a Hausdorff space any convergent sequence has a unique limit. Give an example to show the converse of the statement does not hold.
2. Let and be sequences in the spaces and , respectively. Prove that the sequence converges to if and only if converges to and converges to .

**Question 4:**

1. Define a compact space.
2. Prove that with Co-finite topology is compact, but with usual topology is not compact.
3. Prove that any closed set of a compact space is compact.

**Question 5:**

1. Prove that if is a continuous function from a compact space into , then attains its maximal and its minimal.
2. Prove that if is a continuous bijection function from a compact space onto a Hausdorff space , then is a homeomorphism.

**Question 6:**

1. Let be a metrizable space. Prove that is limit point compact space if and only if is sequentially compact.
2. If is not a metrizable space, then prove that the statement in I is not true.

**Bonus:**

Let be a metric on . Prove that for any

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