Faculty of Engineering
Mechanical Engineering Department

# CALCULUS FOR ENGINEERS
# MATH 1110


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**Example 1**

# SINE & COSINE INTEGRALS

## Evaluate $\int \cos^3 x\, dx$

– Simply substituting $u = \cos x$ isn't helpful, since then $du = -\sin x\, dx$.

– In order to integrate powers of cosine, we would need an extra $\sin x$ factor.

– Similarly, a power of sine would require an extra $\cos x$ factor.

**Example 1**

# SINE & COSINE INTEGRALS

Thus, here we can separate one cosine factor and convert the remaining $\cos^2 x$ factor to an expression involving sine using the identity $\sin^2 x + \cos^2 x = 1$:

$$\cos^3 x = \cos^2 x \cdot \cos x = (1 - \sin^2 x) \cos x$$

**Example 1**

# SINE & COSINE INTEGRALS

We can then evaluate the integral by substituting *u* = sin *x*.

So, *du* = cos *x dx* and

$$\int \cos^3 x\,dx = \int \cos^2 x \cdot \cos x\,dx$$

$$= \int (1-\sin^2 x)\cos x\,dx$$

$$= \int (1-u^2)\,du = u - \tfrac{1}{3}u^3 + C$$

$$= \sin x - \tfrac{1}{3}\sin^3 x + C$$

# Combinations of sin, cos

- General form $$\int \sin^m x \cdot \cos^n x \, dx$$
- If either *n* or *m* is odd, use techniques as before
  - Split the odd power into an even power and power of one
  - Use Pythagorean identity
  - Specify *u* and *du,* substitute
  - Usually reduces to a polynomial
  - Integrate, un-substitute

**Example 2**

# SINE & COSINE INTEGRALS

Find $\int \sin^5 x \cos^2 x \, dx$

- We could convert $\cos^2 x$ to $1 - \sin^2 x$.
- However, we would be left with an expression in terms of $\sin x$ with no extra $\cos x$ factor.

**Example 2**

# SINE & COSINE INTEGRALS

$$\sin^5 x \cos^2 x = (\sin^2 x)^2 \cos^2 x \sin x$$
$$= (1 - \cos^2 x)^2 \cos^2 x \sin x$$

## Example 2

Substituting $u = \cos x$, we have $du = \sin x\, dx$.

So,

$$\int \sin^5 x \cos^2 x\, dx = \int (\sin^2 x)^2 \cos^2 x \sin x\, dx$$

$$= \int (1-\cos^2 x)^2 \cos^2 x \sin x\, dx = \int (1-u^2)^2 u^2 (-du)$$

$$= -\int (u^2 - 2u^4 + u^6) du = -\left(\frac{u^3}{3} - 2\frac{u^5}{5} + \frac{u^7}{7}\right) + C$$

$$= -\tfrac{1}{3}\cos^3 x + \tfrac{2}{5}\cos^5 x - \tfrac{1}{7}\cos^7 x + C$$

## EXAMPLE 4:

Find $\int cos^3x\, sin^3x\, dx.$

$$\int cos^2x\, sin^3x\, cos\, x\, dx =$$

$$\int (1 - sin^2x) sin^3x\, cos\, x dx$$

$$= \int (sin^3x - sin^5x) cos\, x\, dx\,.$$

Let $u = sin\, x \quad \Rightarrow \quad du = cos\, x\, dx\,.$

$$\int \left(u^3 - u^5\right) du = \frac{u^4}{4} - \frac{u^6}{6} = \frac{1}{4} sin^4x - \frac{1}{6} sin^6x + C$$

## EXAMPLE 5:

Find $\int \cos^{-4} x \sin^3 x \, dx$ .

$$\int \cos^{-4} x \sin^2 x \, (\sin x) dx =$$

$$\int \cos^{-4} x \left(1 - \cos^2 x\right) \sin x \, dx =$$

where $u = \cos x$, $du = -\sin x \, dx$ .

$$\int \cos^{-4} x \left(1 - \cos^2 x\right) \sin x \, dx =$$

$$-\int u^{-4} (1 - u^2)(-\sin x) dx =$$

$$-\int \left(u^{-4} - u^{-2}\right) du$$

$$-\int u^{-4} (1 - u^2)(-\sin x) dx =$$

$$-\int \left(u^{-4} - u^{-2}\right) du$$

$$= \int \left(u^{-2} - u^{-4}\right) du = \frac{u^{-1}}{-1} - \frac{u^{-3}}{-3} =$$

$$-\frac{1}{\cos x} + \frac{1}{3 \cos^3 x} + C.$$