

Mathematics of Finance II: Derivative securities

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Second term 2015–2016

Chapter 2 : Options

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- 1 Introduction and motivations
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Introduction

We have seen that forward and futures contract are binding contract, a forward/futures contract obligates the buyer (the holder of the long position) to pay the forward price at expiration, even if the value of the underlying asset at expiration is less than the forward price.

Because losses are possible with a forward/futures contract, it is natural to wonder: Could there be a contract where the buyer has the right to walk away from the deal?

The answer is yes; a **call option** is a contract where the buyer has the right to buy, but not the obligation to buy.

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Introduction: Learning objectives

- Understand how call and put options are used and how they are priced
- Examine the instruments traded on the options market
- Understand how options can be used for either risk management or for speculative purposes

Definitions

A financial option contract gives its owner the right (but not the obligation) to purchase or sell an asset at a fixed price at some future date.

An option is a financial instrument like a stock or bond, an option is a derivative security.

It is also a non-binding contract with strictly defined terms and properties.

Here is an example illustrating how a call option works at expiration.

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Example

Suppose that the call buyer agrees to pay \$1020 for the S&P index in 6 months but is not obligated to do so. (The buyer has purchased a call option.)

If in 6 months the S&P price is \$1100, the buyer will pay \$1020 and receive the index. This leads to a payoff of \$80 per unit of the index.

If the S&P price is \$900, the buyer walks away, hence his payoff is 0.

Remark From the seller's point of view the buyer is in control of the option, deciding when to buy the index by paying \$1020. Thus, the rights of the option buyer are obligations for the option seller.

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If in 6 months the S&P price is \$1100, the seller will receive \$1020 and give up an index worth more, for a loss of \$80 per unit of the index.

If the S&P price is less than \$1020, the buyer will not buy, so the seller has no obligation.

Thus, at expiration, the seller will have a payoff that is zero (if the S&R price is less than \$1020) or negative (if the S&R price is greater than \$1020).

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Different type of options

Two distinct kinds of option contracts exist: **call options and put options**.

A **call option** gives the owner the right to **buy** the asset;
a **put option** gives the owner the right to **sell** the asset.

Because an option is a contract between two parties, for every owner of a financial option, there is also an option writer, the person who takes the other side of the contract.

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Initial payment is necessary

Because the buyer can decide whether to buy, the seller cannot make money at expiration. This situation suggests that the seller must get something to enter into the contract in the first place.

At the time the buyer and seller agree to the contract, the buyer must pay the seller an initial price, the **premium**.

This initial payment compensates the seller for being at a disadvantage at expiration.

Contrast this with a forward/futures contract, for which the initial premium is **zero**

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Examples

The most commonly encountered option contracts are options on shares of stock.

A stock option gives the holder the option to buy or sell a share of stock on or before a given date for a given price.

For example, a call option on 3M Corporation stock might give the holder the right to purchase a share of 3M for \$75 per share at any time up to, for example, July 19, 2016.

Similarly, a put option on 3M stock might give the holder the right to sell a share of 3M stock for \$50 per share at any time up to, say, June 16, 2016.

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Option Terminology

Here are some key terms used to describe options:

Strike price: The strike price, or exercise price, of a call option is what the buyer pays for the asset.

In the example above, the strike price was \$1020. The strike price can be set at any value.

Exercise: The exercise of a call option is the act of paying the strike price to receive the asset. In the above example, the buyer decided after 6 months whether to exercise the option—that is, whether to pay \$1020 (the strike price) to receive the S&P index.

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Option Terminology

Expiration: The expiration of the option is the date by which the option must either be exercised or it becomes worthless. The option in previous example had an expiration of 6 months.

Exercise style: The exercise style of the option governs the time at which exercise can occur.

In the above example, exercise could occur only at expiration. Such an option is said to be a **European-style option**.

If the buyer has the right to exercise at any time during the life of the option, it is an **American-style option**.

If the buyer can only exercise during specified periods, but not for the entire life of the option, the option is a **Bermudan-style option**.

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Option Terminology

The terms “European” and “American”, have nothing to do with geography. European, American, and Bermudan options are bought and sold worldwide.

An European call option gives the owner of the call the right, but not the obligation, to buy the underlying asset on the expiration date by paying the strike price.

The option described in two examples above is a 6-month European-style S&P call with a strike price of \$1020.

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Payoff and Profit for a purchased call option

We can graph call options as we did forward contracts.

The buyer is not obligated to buy the index, and hence will only exercise the option if the payoff is greater than zero.

The algebraic expression for the payoff to a purchased call is therefore

$$\text{Purchased call payoff} = \max(\text{spot price at expiration} - \text{strike price}; 0)$$

The expression $\max(a, b)$ means take the greater of the two values a and b .

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Example

Consider a call option on the S&P index with 6 months to expiration and a strike price of \$1000.

Suppose the index in 6 months is \$1100. Clearly it is worthwhile to pay the \$1000 strike price to acquire the index worth \$1100.

Using the equation (1), the call payoff is

$$\max(\$1100 - \$1000, 0) = \$100$$

If the index is \$900 at expiration, it is not worthwhile paying the \$1000 strike price to buy the index worth \$900. The payoff is then

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See the link for more information:

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Profit for a purchased call option

Notice that the payoff does not take account of the initial cost of acquiring the position.

For a purchased option, the premium is paid at the time the option is acquired. In computing profit at expiration, suppose we defer the premium payment; then by the time of expiration we accrue 6 months' interest on the premium.

The option profit is computed as

$$\begin{aligned} \text{Purchased call profit} = & \max(\text{spot price at expiration} - \text{strike price}, 0) \\ & - \text{future value of option premium} \end{aligned} \quad (2)$$

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Example illustrating the computation of the profit

Use the same option as before, and suppose that the risk-free rate is 2% over 6 months.

Assume that the index spot price is \$1000 and that the premium for this call is \$93.81.

Hence, the future value of the call premium is $\$93.81 \times 1.02 = \95.68 . If the S&P index price at expiration is \$1100, the owner will exercise the option. Using equation (2), the call profit is $\max(\$1100 - \$1000, 0) - \$95.68 = \4.32

If the index is \$900 at expiration, the owner does not exercise the option. It is not worthwhile paying the \$1000 strike price to buy the index worth \$900.

The Profit is then $\max(\$900 - \$1000, 0) - \$95.68 = -\95.68 reflecting the loss of the premium.

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