

Integral Calculus (M-106), Serie N: 4

Exercise 1:

Find $f'(x)$ if $f(x)$ is the given expression.

$$\begin{array}{lll} 1) x \ln x & 2) \frac{1}{\ln x} + \ln \frac{1}{x} & 3) \ln |5x^2 - 1|^3 \\ 4) \cos(\ln 2x) & 5) \ln |\csc x - \cot x| & 6) \ln \frac{\sqrt{x^2 + 1}}{(9x - 4)^2} \end{array}$$

Exercise 2:

a) Use implicit differentiation to find $y'(x)$.

$$1) x \ln y(x) - y(x) \ln x = 0 \quad 2) y^3(x) + x^2 \ln y(x) = 5x + 3$$

b) Use logarithmic differentiation to find $\frac{dy(x)}{dx}$.

$$1) y(x) = \sqrt{4x + 7}(x - 5)^3 \quad 2) y(x) = \frac{(x^2 + 3)^5}{\sqrt{x + 1}}$$

Exercise 3:

a) Find $f'(x)$ if $f(x)$ is the given expression.

$$\begin{array}{lll} 1) \frac{x}{e^{x^2}} & 2) e^{\sqrt{x}} + \sqrt{e^x} & 3) e^{\sin 5x} \\ 4) e^{-3x} \cos 3x & 5) \frac{e^x - e^{-x}}{e^x + e^{-x}} & 6) e^{-x} \tan^2 x \end{array}$$

b) Use implicit differentiation to find $y'(x)$.

$$1) e^x \cos y = x e^y \quad 2) x e^y + 2x - \ln(y + 1) = 3$$

Exercise 4:

Evaluate the integral.

1)

$$a) \int \frac{1}{4-5x} dx \quad b) \int_{-1}^0 \frac{1}{4-5x} dx$$

2)

$$a) \int \frac{3x}{x^2+4} dx \quad b) \int_1^2 \frac{3x}{x^2+4} dx$$

3)

$$a) \int x^2 e^{3x^3} dx \quad b) \int_1^2 x^2 e^{3x^3} dx$$

4)

$$a) \int (\csc x - 1)^2 dx \quad b) \int e^{2x} \sec e^{2x} dx \quad c) \int \frac{\cot \sqrt[3]{x}}{\sqrt[3]{x^2}} dx$$

Exercise 5:

Solve the differential equation subject to the given conditions.

1)

$$y'(x) = 4e^{2x} + 3e^{-2x}; \quad y = 4 \text{ if } x = 0$$

2)

$$y'(x) = 3e^{4x} - 8e^{-2x}; \quad y = -2 \text{ if } x = 0$$

Exercise 6:Find $f'(x)$ if $f(x)$ is given expression.

$$\begin{array}{lll} 1) (\cos 2x)^{2x} & 2) (10^x + 10^{-x})^{10} & 3) \log_5 \left| \frac{1-x^2}{2-5x^3} \right| \\ 4) (x\pi)^x & 5) (\log_2 |x| \pi)^x & 6) x^{\tan x} \end{array}$$

Exercise 7:

Evaluate the integral.

1)

$$\begin{array}{ll}
 a) \int 2^{3x-1} dx & aa) \int_{-1}^1 2^{3x-1} dx \\
 b) \int 6^{-6x} dx & bb) \int_1^2 6^{-6x} dx
 \end{array}$$

2)

$$\begin{array}{ll}
 a) \int x 3^{-x^2} dx & b) \int \frac{(2^x + 1)^2}{2^x} dx \\
 c) \int \frac{2^x}{2^x + 1} dx & d) \int 3^{\cos x} \sin x dx \\
 e) \int e^{\pi} dx & f) \int \frac{10^{\sqrt{x}}}{\sqrt{x}} dx
 \end{array}$$