



Faculty of Engineering
Mechanical Engineering Department

CALCULUS FOR ENGINEERS

MATH 1110

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$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta = 1 - 2 \sin^2 \theta = 2 \cos^2 \theta - 1$$

$$\tan 2\theta = \frac{2 \frac{\tan \theta}{1 - \tan^2 \theta}}$$

Find $\int \cos^4 x \, dx$.

Use $\cos^2 x = \frac{1 + \cos 2x}{2}$.

$$\int \cos^4 x \, dx = \int \left(\frac{1 + \cos 2x}{2} \right)^2 dx =$$

$$\int \frac{1 + 2 \cos 2x + \cos^2 2x}{4} dx =$$

$$\int \left(\frac{1}{4} + \frac{\cos 2x}{2} + \frac{\cos^2 2x}{4} \right) dx =$$

$$\frac{x}{4} + \frac{\sin 2x}{4} + \frac{1}{4} \int \frac{1 + \cos 4x}{2} dx =$$

$$\frac{x}{4} + \frac{\sin 2x}{4} + \frac{1}{8} x + \frac{1}{32} \sin 4x =$$

$$\frac{3x}{8} + \frac{\sin 2x}{4} + \frac{\sin 4x}{32} + C.$$

EXAMPLE 7:

Find $\int \sec^4 x \tan^{10} x \, dx$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

We will let $u = \tan x$, $du = \sec^2 x \, dx$, and use the identity

$$1 + \tan^2 x = \sec^2 x.$$

$$\tan^2 \theta + 1 = \sec^2 \theta.$$

Peel off a factor of $\sec^2 x$.

$$\int \sec^4 x \tan^{10} x \, dx = \int \sec^2 x \tan^{10} x \sec^2 x \, dx =$$

$$\int (1 + \tan^2 x) \tan^{10} x \sec^2 x \, dx =$$

$$\int (\tan^{10} x + \tan^{12} x) \sec^2 x \, dx =$$

$$\int (u^{10} + u^{12}) \, du =$$

$$\frac{\tan^{11} x}{11} + \frac{\tan^{13} x}{13} + C.$$

EXAMPLE 8:

Find $\int \sec^3 x \tan^3 x \, dx$.

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

Technique: Peel off a ***sec x tan x***.

$$\tan^2 \theta + 1 = \sec^2 \theta$$

Then let

$$u = \sec x, \quad du = \sec x \tan x \, dx,$$

and use the identity $1 + \tan^2 x = \sec^2 x$.

$$\int \sec^3 x \tan^3 x \, dx =$$

$$\int \sec^2 x \tan^2 x (\mathbf{\sec x \tan x}) \, dx$$

$$u = \sec x, \quad du = \sec x \tan x \, dx$$

$$\tan^2 x = \sec^2 x - 1 = u^2 - 1$$

$$\begin{aligned} \int \sec^2 x \sec x \tan^2 x \tan x \, dx &= \\ \int \sec^2 x (\sec^2 x - 1) \sec x \tan x \, dx &= \\ = \int (\sec^4 x - \sec^2 x) (\sec x \tan x) \, dx &= \\ \int (u^4 - u^2) \, du & \\ \text{(where } u = \sec x, \, du = \sec x \tan x \, dx) & \\ = \frac{u^5}{5} - \frac{u^3}{3} = \frac{\sec^5 x}{5} - \frac{\sec^3 x}{3} + C. & \end{aligned}$$

EXAMPLE 9:

$$\text{Find } \int \sec^6 x \tan^5 x \, dx.$$

Now we have a choice.

We can keep one $\sec^2 x$, use the identity $\sec^2 x = 1 + \tan^2 x$, and substitute $u = \tan x$.

$$\begin{aligned} \int \sec^2 x \sec^4 x \tan^5 x \, dx &= \\ \int (1 + \tan^2 x)^2 \tan^5 x \sec^2 x \, dx &= \\ \int (1 + u^2)^2 u^5 \, du &= \int (1 + 2u^2 + u^4) u^5 \, du = \\ \int (u^5 + 2u^7 + u^9) \, du &= \frac{u^6}{6} + \frac{2u^8}{8} + \frac{u^{10}}{10} = \\ \frac{\tan^6 x}{6} + \frac{\tan^8 x}{4} + \frac{\tan^{10} x}{10} + C. & \end{aligned}$$

EXAMPLE 10:

Find $\int \sec^6 x \tan^5 x dx$ a 2nd way.

We can peel off a $\sec x \tan x$ and make the substitution $u = \sec x$, $du = \sec x \tan x dx$.

$$\begin{aligned} \int \sec^6 x \tan^5 x dx &= \\ \int \sec^5 x \tan^4 x (\sec x \tan x dx) &= \\ \int \sec^5 x (\sec^2 x - 1)^2 (\sec x \tan x dx) &= \\ \int u^5 (u^2 - 1)^2 du = \int u^5 (u^4 - 2u^2 + 1) du &= \\ \int (u^9 - 2u^7 + u^5) du = & \\ \frac{\sec^{10} x}{10} - \frac{\sec^8 x}{4} + \frac{\sec^6 x}{6} + C. & \end{aligned}$$

EXAMPLE 11:

Find $\int \csc^4 x \cot^6 x \, dx$.

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

Let $u = \cot x$, $du = -\csc^2 x \, dx$,
 $1 + \cot^2 x = \csc^2 x$.

$$\begin{aligned} \int \cot^6 x \csc^2 x \csc^2 x \, dx &= \\ &= - \int u^6 (1 + u^2) \, du = \\ &= - \int (u^6 + u^8) \, du = -\frac{u^7}{7} - \frac{u^9}{9} = \\ &= -\frac{\cot^7 x}{7} - \frac{\cot^9 x}{9} + C. \end{aligned}$$

EXAMPLE 12:

Find $\int \tan^2 x \, dx$.

Use $\tan^2 x = \sec^2 x - 1$.

$$\int \tan^2 x \, dx = \int (\sec^2 x - 1) \, dx =$$

$$\int \sec^2 x \, dx - \int 1 \, dx =$$

$$\tan x - x + C.$$