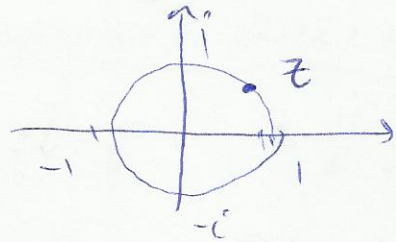


5.3

①

① Q $\sum_{j=0}^{\infty} z^j$ convergence at no points on its circle of convergence $|z|=1$ \ll diverges on the circle of convergence \gg

Ans: since $|z^j| = |z|^j = 1^j = 1 \geq 1$



\Rightarrow the geometric series is diverges

Q the series $\sum_{j=1}^{\infty} \frac{z^j}{j^2}$ convergence on the circle of convergence $|z|=1$

Ans:

Since $\left| \frac{z^j}{j^2} \right| = \frac{|z|^j}{j^2} = \frac{1}{j^2}$

and $\sum \frac{1}{j^2}$ is p-series and $p=2 > 1$

$\Rightarrow \sum \frac{1}{j^2}$ convergence

$\Rightarrow \sum \frac{z^j}{j^2}$ converges

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② Q $\sum_{j=0}^{\infty} j^3 z^j$ ($z_0=0$)

$\lim_{j \rightarrow \infty} \left| \frac{a_{j+1}}{a_j} \right| = \lim_{j \rightarrow \infty} \left| \frac{(j+1)^3}{j^3} \right| = 1 = L$

\Rightarrow the radius of convergence is $R = \frac{1}{L} = \frac{1}{1} = 1$

the circle of convergence is $|z-0| \leq 1$