King Saud University
Department of Mathematics

106
Final Exam, January 2017

NAME:

Group Number/Instructor's Name:

ID:

Question	Grade
I	
II	
III	
IV	
Total	

Question	1	2	3	4	5	6	7	8
Answer								

I) Choose the correct answer (write it on the table above) [8 *marks*]:

1) If $\ln(e^{4x}) - \ln(e) = 3$, then x equals

(A) 0	(B) 1	(C) 2	(D) None

2) If $y = \sinh^{-1}(\tanh x)$, then $\dfrac{dy}{dx}$ equals

(A) $\dfrac{\operatorname{sech}^2 x}{1+\tanh^2 x}$	(B) $\dfrac{\operatorname{sech}^2 x}{\sqrt{1+\tanh^2 x}}$	(C) $\dfrac{\operatorname{sech}^2 x}{\sqrt{\tanh^2 x - 1}}$	(D) None

3) If $f(x) = \sinh(x) + \cosh(x)$, then $f(0)$ equals

(A) 0	(B) 1	(C) e	(D) None

4) The graph of $r = \sin 3\theta$ is

(A) a circle	(B) a rose with 3 leaves	(C) a rose with 6 leaves	(D) None

5) The partial fraction decomposition of $\dfrac{x^2-5}{x^4+x^3+x^2}$ is

(A) $\dfrac{A}{x^2} + \dfrac{B}{x^2+x+1}$	(B) $\dfrac{Ax+B}{x^2} + \dfrac{Cx+D}{x^2+x+1}$	(C) $\dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{Cx+D}{x^2+x+1}$	(D) None

6) The rectangular coordinates (x, y) corresponding to the polar coordinates $(r, \theta) = (3, \pi)$ are

(A) $(-3, 0)$	(B) $(3, 0)$	(C) $(0, -3)$	(D) None

7) If $f(x) = 10^x$, then $f'(x)$ equals

(A) 10^x	(B) $10^x \ln 10$	(C) $\dfrac{10^x}{\ln 10}$	(D) None

8) $\dfrac{d}{dx}\displaystyle\int_0^1 \frac{x^3+x^2}{5x^{3/2}}dx$ equals

(A) 1	(B) 2	(C) 0	(D) None

II) Evaluate the following integrals [20 *marks*]:

1) $\int x \cosh x dx$

2) $\int \sin^4 x \cos^3 x dx$

3) $\int \frac{x^3 + 2x}{x^2 - x - 2} dx$

4) $\int \frac{e^{2x}}{\sqrt[3]{1 + e^x}} dx$

5) $\int_0^3 \sqrt{3x}(\sqrt{x}+\sqrt{3})dx$

6) $\int \sqrt{x}\ln x dx$

7) $\int \frac{x^2}{(1-9x^2)^{3/2}}dx$

8) $\int \frac{1}{x^2+2x+5}dx$

III) A) Determine whether the improper integrals converge and, if so, find their value [5 *marks*]:

1) $\int_{-8}^{1} \frac{1}{\sqrt[3]{x}} dx$

2) $\int_{4}^{\infty} \frac{1}{x-4} dx.$

B) Sketch the graph of the function $f(x) = e^x$ and compute the area of the region that lies under the graph of $y = e^x$, over the x-axis and to the left of $x = 0$ [3 *marks*].

IV) A) Sketch the graph of the curve with polar equation $r = 3 - 3\sin\theta$, for $0 \leq \theta \leq 2\pi$ [3 *marks*].

B) Find the area of the region enclosed by the graph of $r = 3 - 3\sin\theta$, for $0 \leq \theta \leq 2\pi$ [3 *marks*].

C) Find a polar equation that has the same graph as $xy = 4$ [2 *marks*].

Formulas

- $\int \frac{1}{\sqrt{a^2+u^2}} du = \sinh^{-1} \frac{u}{a} + c,\ a > 0;$
- $\int \frac{1}{a^2-u^2} du = \frac{1}{a} \tanh^{-1} \frac{u}{a} + c,\ |u| < a;$
- $\int \frac{1}{\sqrt{a^2-u^2}} du = \sin^{-1} \frac{u}{a} + c,\ a > 0;$
- $\int \frac{1}{a^2+u^2} du = \frac{1}{a} \tan^{-1} \frac{u}{a} + c,\ a > 0.$

Scrap paper