



(6) since f has a zero of order m

$$\Rightarrow f(z) = (z - z_0)^m h(z), \quad g(z_0) \neq 0$$

$$\Rightarrow f'(z) = m(z - z_0)^{m-1} h(z) + (z - z_0)^m h'(z)$$

$$\Rightarrow g(z) = \frac{f'}{f} = \frac{m(z - z_0)^{m-1} h(z) + (z - z_0)^m h'(z)}{(z - z_0)^m h(z)}$$

$$= \frac{m h + (z - z_0) h'(z)}{(z - z_0) h(z)}$$

$$= \frac{k(z)}{z - z_0}, \quad k(z) = \frac{m h + (z - z_0) h'(z)}{h(z)}$$

$$k(z) = \frac{m h}{h(z)} \neq 0 \quad m \neq 0$$

k is analytic at z_0

$$\text{Res}(g, z_0) = \lim_{z \rightarrow z_0} (z - z_0) g(z)$$

$$= \lim_{z \rightarrow z_0} k(z)$$

$$= k(z_0)$$

$$= m$$

Sim
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