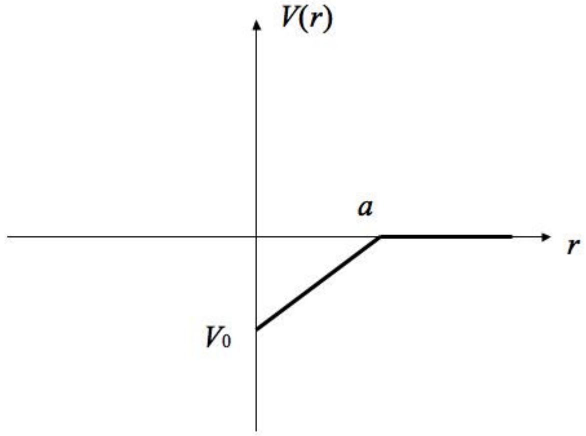
1. Consider the case of the following attractive potential given in the figure. Find the scattering amplitude in the Born approximation.



You are given that:

$$\int x\sin qx dx = \frac{\sin qx - qx\cos qx}{q^2},$$

$$\int x^2 \sin qx dx = \frac{-q^2x^2\cos qx + 2\cos qx + 2qx\sin(qx)}{q^3}$$

Solution: *From the figure we must derive the expression for the potential. The expression is given by*

$$V = \begin{cases} \frac{V_0}{a}r - V_0 & r \le a \\ 0 & r > a \end{cases}$$

The scaterring amplitude is given:

$$f_B(\theta) = -\frac{2m}{q\hbar^2}\int_0^a r\left(\frac{V_0}{a}r - V_0\right)\sin(qr)\,dr = -\frac{2mV_0}{q\hbar^2}\int_0^a r\left(\frac{r}{a} - 1\right)\sin(qr)\,dr =$$

$$-\frac{2mV_0}{q\hbar^2}\left\{\frac{1}{a}\int_0^a r^2\sin(qr)\,dr - \int_0^a r\sin(qr)\,dr\right\} =$$

$$-\frac{2mV_0}{q\hbar^2}\left\{\frac{1}{a}\left[\frac{-q^2r^2\cos qr + 2\cos qr + 2qr\sin(qr)}{q^3}\right]_0^a - \right.$$

$$\left.\left[\frac{\sin qr - qr\cos qr}{q^2}\right]_0^a\right\} = -\frac{2mV_0}{q\hbar^2}\left\{\frac{1}{a}\left[\frac{-q^2a^2\cos qa + 2\cos qa + 2qa\sin(qa) - 2}{q^3}\right] - \left[\frac{\sin qa - qa\cos qa}{q^2}\right]\right\}$$