phase angles. Compare the two sets of results and identify the buses at which the estimated angles are equal in the two cases. Does the presence of line ①-④ (but with no line measurement) affect the identification of those buses? Compare the identified buses with those identified in Prob. 15.10(*d*).

T

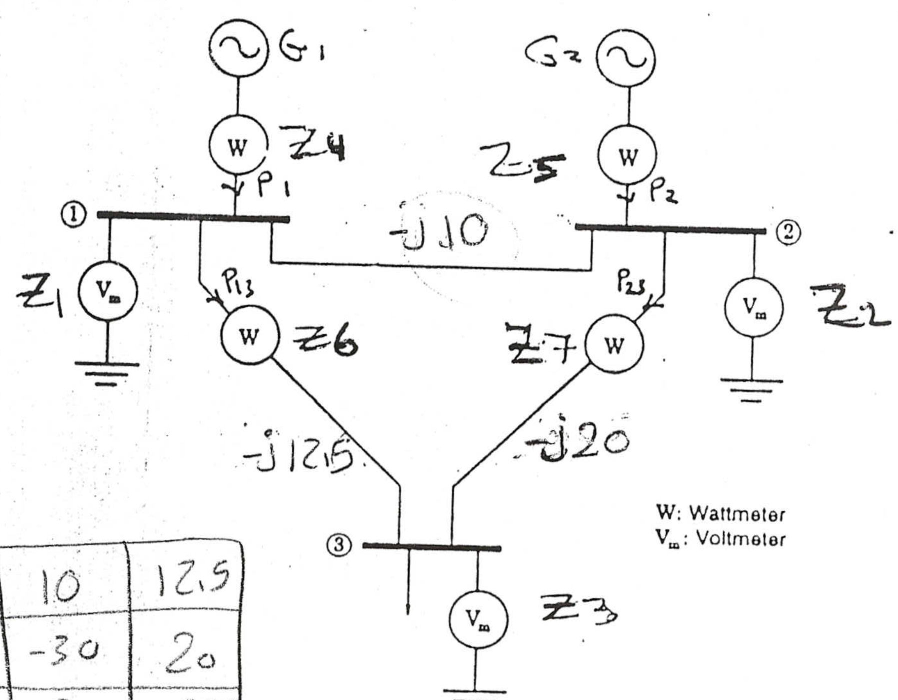
15.14. Three voltmeters and four wattmeters are installed on the three-bus system of Fig. 15.10, where per-unit reactances of the lines are $X_{12} = 0.1$, $X_{13} = 0.08$, and $X_{23} = 0.05$. The per-unit values of the three voltmeter measurements are $z_1 = |V_1| = 1.01$, $z_2 = |V_2| = 1.02$, and $z_3 = |V_3| = 0.98$. The readings of the two wattmeters measuring MW generation at buses ① and ② are $z_4 = 0.48$ per unit and $z_5 = 0.33$ per unit, respectively. The measurement of the wattmeter on line ①-③ at bus ① shows $z_6 = 0.41$ per unit and that of the wattmeter on line ②-③ at bus ② is $z_7 = 0.38$ per unit. The variances of the measurement errors are given in per unit as

$$\sigma_1^2 = \sigma_2^2 = \sigma_3^2 = (0.02)^2$$

$$\sigma_4^2 = \sigma_5^2 = \sigma_6^2 = \sigma_7^2 = (0.05)^2$$

(*a*) Use bus ① as reference to find expressions for the elements of the matrix

(1)



Ybus

-22.5	10	12.5
10	-30	20
12.5	20	-32.5

FIGURE 15.10
Three-bus network.

(1)

$\mathbf{H}_x^{(k)}$ and those of the measurement errors $e_i^{(k)}$ in terms of state variables as done in Example 15.5.

(*b*) Using the initial value of $1.0\angle 0°$ per unit for all bus voltages, find the values of the state variables that will be obtained at the end of the first iteration of the weighted least-squares state-estimation process.

T

15.15. Application of the weighted least-squares state estimation to the three-bus system with all the measurements described in Prob. 15.14 yields the following estimates of the states:

✓O

$|V_1| = 1.0109$ per unit

$|V_2| = 1.0187$ per unit $\qquad \delta_2 = -0.0101$ radians

$|V_3| = 0.9804$ per unti $\qquad \delta_3 = -0.0308$ radians

The sequence of diagonal elements in the covariance matrix $\mathbf{R}'$ is 0.8637×10^{-6}, 0.1882×10^{-5}, 0.2189×10^{-6}, 0.7591×10^{-3}, 0.8786×10^{-3}, 0.1812×10^{-2}, and 0.1532×10^{-2}. Find the estimates of the measurement errors $\hat{e}_i$ and the corresponding standardized errors.

HW

15.16. Solve Prob. 15.14 when the two wattmeters installed on lines ①-③ and ②-③ are replaced with two varmeters and their readings are 0.08 and 0.24 per unit, respectively.

15.17. Suppose that real and reactive power flows are measured at both ends of each of the five lines in the four-bus system of Fig. 15.9 using ten wattmeters and ten varmeters. The voltage magnitude is measured at bus ② only, and bus injected powers are not measured at all.

(*a*) Determine the structure of $\mathbf{H}_x$ by writing the partial derivative form of its nonzero elements, as shown in Example 15.8. Assume that line-flow measurements are ordered in the following sequence: ①-②, ①-③, ②-③, ②-④, and ③-④ (and the same sequence also in reverse directions).

(*b*) Suppose that the elements of the $\mathbf{Y}_{bus}$ of the network are given by

$$Y_{ij} = G_{ij} + jB_{ij} = |Y_{ij}|\angle\theta_{ij}$$

and that the total charging susceptance of line ⓘ-ⓙ is B'_{ij}. Write out nonlinear functions which express the measured quantities P_{21} and Q_{21} in terms of state variables.

(*c*) In terms of state variables write out the expressions, similar to those given in Example 15.8, for the nonzero elements in the rows of the matrix $\mathbf{H}_x$ corresponding to measurements P_{21} and Q_{21}.

15.18. The method of Example 15.8 based on measurements of only line flows (plus a voltage measurement at one bus) is applied to the three-bus system of Fig. 15.10 using three wattmeters and three varmeters. The per-unit values of the

جامعة الملك سعود

Power Flow Equations for Any System:

$$P_i = V_i \sum_{j=1}^{N} V_j Y_{ij} \cos(\delta_i - \delta_j - \theta_{ij})$$

$$Q_i = V_i \sum_{j=1}^{N} V_j Y_{ij} \sin(\delta_i - \delta_j - \theta_{ij})$$

$$P_{ij} = -V_i^2 G_{ij} + Y_{ij} V_i V_j \cos(\delta_i - \delta_j - \theta_{ij})$$

$$Q_{ij} = V_i^2 B_{ij} + Y_{ij} V_i V_j \sin(\delta_i - \delta_j - \theta_{ij})$$

Where δ is angle of volt and $\bar{Y} = Y\angle\theta = G + jB$

But when Real Part of Z is neglected then:

$G = 0$ and $\theta = \pm 90$ $\quad \cos(\alpha - 90) = \sin\alpha$, $\sin(\alpha - 90) = -\cos\alpha$

Now, Power flow equations Can be defined as:

$$P_i = V_i \sum_{j=1}^{N} V_j B_{ij} \sin(\delta_i - \delta_j) \quad \text{------} \quad (1)$$

$$Q_i = -V_i \sum_{\substack{j=1 \\ j \neq i}}^{N} V_j B_{ij} \cos(\delta_i - \delta_j) + B_{ii} V_i^2 \quad (2)$$

$$P_{ij} = V_i V_j B_{ij} \sin(\delta_i - \delta_j) \quad \text{------} \quad (3)$$

$$Q_{ij} = V_i^2 B_{ij} - V_i V_j B_{ij} \cos(\delta_i - \delta_j) \quad \text{----} \quad (4)$$

$Y = 3 + j4$ $\quad$ $Y = 3 - j4$ $\quad$ $Y = j4$ $\quad$ $Y = -j4$

$B = 4$ $\quad$ $B = -4$ $\quad$ $B = 4$ $\quad$ $B = -4$

State X $\longrightarrow$ max .no of X = $2n-1$

measurement Z $\longrightarrow$ max .no of Z = $3n+4L$

*** 15.14 ***

$$Z = \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \\ Z_6 \\ Z_7 \end{bmatrix} = \begin{bmatrix} 1.01 \\ 1.02 \\ 0.98 \\ 0.48 \\ 0.33 \\ 0.41 \\ 0.38 \end{bmatrix} = \begin{bmatrix} |V_1| \\ |V_2| \\ |V_3| \\ p_1 \\ p_2 \\ p_{13} \\ p_{23} \end{bmatrix} \qquad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} \delta_2 \\ \delta_3 \\ |V_1| \\ |V_2| \\ |V_3| \end{bmatrix}$$

دائمًا نبدأ بـ δ

ref $\delta_1 = 0$
من السؤال

$h_1 = |V_1| = x_3$, $h_2 = |V_2| = x_4$, $h_3 = |V_3| = x_5$

$h_4 = P_1 = v_1\, v_2\, y_{12} \sin(\delta_1 - \delta_2) + v_1\, v_3\, y_{13} \sin(\delta_1 - \delta_3)$

$= 10\, x_3\, x_4 \sin(-x_1) + 12.5\, x_3\, x_5 \sin(-x_2)$

$h_5 = P_2 = v_2\, v_1\, y_{21} \sin(\delta_2 - \delta_1) + v_2\, v_3\, y_{23} \sin(\delta_2 - \delta_3)$

$= 10\, x_4\, x_3\, (x_1) + 20\, x_4\, x_5 \sin(x_1 - x_2)$

$h_6 = P_{13} = v_1\, v_3\, y_{13} \sin(\delta_1 - \delta_3) = 12.5\, x_3\, x_5 \sin(-x_2)$

$h_7 = P_{23} = v_2\, v_3\, y_{23} \sin(\delta_2 - \delta_3) = 20\, x_4\, x_5 \sin(x_1 - x_2)$

H_x = Jacobian matrix $= \left[\dfrac{\delta h}{\delta x}\right]_{Z \times X}$

$$\begin{bmatrix}
\left[\frac{\delta h_1}{\delta x_1}\right] & \left[\frac{\delta h_1}{\delta x_2}\right] & \left[\frac{\delta h_1}{\delta x_3}\right] & \left[\frac{\delta h_1}{\delta x_4}\right] & \left[\frac{\delta h_1}{\delta x_5}\right] \\
\left[\frac{\delta h_2}{\delta x_1}\right] & \left[\frac{\delta h_2}{\delta x_2}\right] & \left[\frac{\delta h_2}{\delta x_3}\right] & \left[\frac{\delta h_2}{\delta x_4}\right] & \left[\frac{\delta h_2}{\delta x_5}\right] \\
\left[\frac{\delta h_3}{\delta x_1}\right] & \left[\frac{\delta h_3}{\delta x_2}\right] & \left[\frac{\delta h_3}{\delta x_3}\right] & \left[\frac{\delta h_3}{\delta x_4}\right] & \left[\frac{\delta h_3}{\delta x_5}\right] \\
\left[\frac{\delta h_4}{\delta x_1}\right] & \left[\frac{\delta h_4}{\delta x_2}\right] & \left[\frac{\delta h_4}{\delta x_3}\right] & \left[\frac{\delta h_4}{\delta x_4}\right] & \left[\frac{\delta h_4}{\delta x_5}\right] \\
\left[\frac{\delta h_5}{\delta x_1}\right] & \left[\frac{\delta h_5}{\delta x_2}\right] & \left[\frac{\delta h_5}{\delta x_3}\right] & \left[\frac{\delta h_5}{\delta x_4}\right] & \left[\frac{\delta h_5}{\delta x_5}\right] \\
\left[\frac{\delta h_6}{\delta x_1}\right] & \left[\frac{\delta h_6}{\delta x_2}\right] & \left[\frac{\delta h_6}{\delta x_3}\right] & \left[\frac{\delta h_6}{\delta x_4}\right] & \left[\frac{\delta h_6}{\delta x_5}\right] \\
\left[\frac{\delta h_7}{\delta x_1}\right] & \left[\frac{\delta h_7}{\delta x_2}\right] & \left[\frac{\delta h_7}{\delta x_3}\right] & \left[\frac{\delta h_7}{\delta x_4}\right] & \left[\frac{\delta h_7}{\delta x_5}\right]
\end{bmatrix}$$

$Z * X$

$Hx^{(k)} =$

h_1	0	0	1	0	0
h_2	0	0	0	1	0
h_3	0	0	0	0	1
h_4	$-10x_3x_4$ $\cos(-x_1)$	$-12.5x_3x_5$ $\cos(-x_2)$	$10x_4\sin(-x_1)$ $+12.5x_5\sin(-x_2)$	$10x_3\sin(-x_1)$	$12.5x_3\sin(-x_2)$
h_5	$10x_4x_3\cos(x_1)$ $+20x_4x_5$ $\cos(x_1-x_2)$	$-20x_4x_5$ $\cos(x_1-x_2)$	$10x_4\sin(x_1)$	$10x_3\sin(-x_1)$ $+20x_5\sin(x_1-x_2)$	$20x_4$ $\sin(x_1-x_2)$
h_6	0	$-12.5x_3x_5$ $\cos(-x_2)$	$12.5x_5$ $\sin(-x_2)$	0	$12.5x_3$ $\sin(-x_2)$
h_7	$20x_4x_5$ $\cos(x_1-x_2)$	$-20x_4x_5$ $\cos(x_1-x_2)$	0	$20x_5$ $\sin(x_1-x_2)$	$20x_4$ $\sin(x_1-x_2)$

$$e^{(k)} = z - h \Rightarrow \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ e_6 \\ e_7 \end{bmatrix} = \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \\ Z_6 \\ Z_7 \end{bmatrix} - \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \\ h_7 \end{bmatrix} = \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \\ Z_6 \\ Z_7 \end{bmatrix} - \begin{bmatrix} x_3 \\ x_4 \\ x_5 \\ 10x_3x_4\sin(-x_1)+12.5\,x_3x_5\sin(-x_2) \\ 10x_3x_4\sin(-x_1)+12.5\,x_3x_5\sin(x_1-x_2) \\ 12.5x_3x_5\sin(x_1-x_2) \\ 20x_4x_5\sin(x_1-x_2) \end{bmatrix}$$

using initial Value ($1\angle 0$) for all bus Voltage

(b) $V = X_3, X_4$ & $X_5 = 1$ من السؤال تعني عوض عن كل

$\delta = X_1$ & $X_2 = 0$

$H_x^{(0)} =$

0	0	1	0	0
0	0	0	1	0
0	0	0	0	1
– 10	– 12.5	0	0	0
30	– 20	0	0	0
0	– 12.5	0	0	0
20	– 20	0	0	0

$$h^{(0)} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad e^{(0)} = z - h^{(0)} = \begin{bmatrix} 1.01 \\ 1.02 \\ 0.98 \\ 0.48 \\ 0.33 \\ 0.41 \\ 0.38 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1.01 \\ 1.02 \\ -0.02 \\ 0.48 \\ 0.33 \\ 0.41 \\ 0.38 \end{bmatrix}$$

$$x^{(1)} = x^{(0)} + (H_x^{T(0)}\ R^{-1}\ H_x^{T(0)})^{-1}\ H_x^{T(0)} R^{-1}\ e^{(0)} \quad \ldots$$

$$R^{-1} = \frac{1}{\sigma_i^2} = \begin{bmatrix} 2500 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2500 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2500 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 400 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 400 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 400 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 400 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} -0.0101413 \\ -0.0307403 \\ 0.01 \\ 0.02 \\ -0.02 \end{bmatrix} = \begin{bmatrix} -0.0101413 \\ 0.0307403 \\ 1.01 \\ 1.02 \\ 0.98 \end{bmatrix}$$

radian

P.u

*** 15.15 ***

Now if finally we obtained the following estimates of the states.

$x_3 = |v_1| = 1.0109$ pu $\quad x_4 = |v_2| = 1.0187$ pu $\quad x_5 = |v_3| = 0.9804$ pu

and $\quad x_1 = \delta_2 = -\,0.0101$ Radian $\quad x_2 = \delta_2 = -\,0.0308$ Radian

Diagonal of R\` $= 0.8637 \times 10^{-6}$, 0.1882×10^{-5}, 0.2189×10^{-6},

0.7591×10^{-3}, 0.1812×10^{-2}, 0.1532×10^{-2}.

$$Z = \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \\ Z_6 \\ Z_7 \end{bmatrix} = \begin{bmatrix} 1.01 \\ 1.02 \\ 0.98 \\ 0.48 \\ 0.33 \\ 0.41 \\ 0.38 \end{bmatrix} \qquad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -0.0101 \\ -0.0308 \\ 1.0109 \\ 1.0187 \\ 0.9804 \end{bmatrix}$$

$$h = \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \\ h_7 \end{bmatrix} = \begin{bmatrix} 1.0109 \\ 1.0187 \\ 0.9804 \\ 0.4855 \\ 0.30948 \\ 0.3815 \\ 0.41347 \end{bmatrix}$$

* The estimates of the measurement errors $\hat{e}_i$.

$$\hat{e} = z - h = \begin{bmatrix} -0.0009 \\ 0.0013 \\ -0.0004 \\ -0.0055 \\ 0.02022 \\ 0.0285 \\ -0.03347 \end{bmatrix}$$

* Check for bad data.

$X^2_{k,\alpha}$ $\quad$ $k = \#\text{ of } z - \#\text{ of } x = 7 - 5 = 2$

$\boxed{X^2_{2,0.01} = 9.21}$ $\quad$ $\alpha = 0.01$

$$f = \sum e_i^2 / \alpha_i^2 = \sum e_i^2 \times w_i = (-0.009)^2 (2500) + (0.0013)^2 (2500)$$

$$+ (-0.0004)^2 (2500) + (-0.0055)^2 (400) + (0.02022)^2 (400)$$

$$+ (0.0285)^2 (400) + (-0.03347)^2 (400) = 0.9552$$

$f < X^2_{2,0.01}$ there is no bad data

* Standardized errors

$$\text{Standardized error} = \frac{\hat{e}_i}{\sqrt{R'_{ii}}} = \begin{bmatrix} -0.96848 \\ 0.9476 \\ -0.8549 \\ -0.1946 \\ 0.06747 \\ 0.6695 \\ -0.8551 \end{bmatrix}$$