

Question # 3 (10 marks)

A synchronous machine with inertia constant $H = 5.0$ second is delivering a steady-state power of 2.0 pu to a 60 Hz power system representing an infinite bus at voltage 1 pu. The internal voltage of the machine is 1.1 pu and the transfer reactance between the machine and system is 0.22 pu. A three-phase fault occurred at a point in the system. The power-angle equation during fault is $P_e^{\text{During}} = 3.0 \sin(\delta)$ pu. The fault was cleared after 0.325 seconds returning the system to its original status.

- Write the power-angle equation before fault (substitute all known quantities). Then calculate the initial rotor angle.
- Write the swing equation for the machine during the fault (substitute all known quantities).
- Taking a time interval of 0.05 seconds, perform a step-by-step solution procedure to compute the swing curve of the machine for a period of 0.5 seconds after fault occurrence.
- If the electrical power output during fault was reduced to zero, what would be the value of rotor angle after 0.2 seconds.
- Comparing the results from c) and d) above, which case is worse from the stability point of view

Question # 5 (10 marks)

The set of measurements fed to the state estimator for a given power system, with 3 states, are as follows:
 $z_1 = 0.92, z_2 = 1.02, z_3 = 0.605, z_4 = 0.598, z_5 = 0.305$.

The variances of the measurement errors are:

$$\sigma_1^2 = \sigma_2^2 = (0.01)^2, \sigma_3^2 = \sigma_5^2 = (0.02)^2, \sigma_4^2 = (0.015)^2$$

The state estimates at the end of a state estimation stage are as follows:

$$\hat{x}_1 = -0.1762, \hat{x}_2 = 0.9578, \hat{x}_3 = 0.9843$$

The system model $h(x)$ representing measurements as functions of states has the following values:

$$h_1 = 0.9578, h_2 = 0.9843, h_3 = 0.3240, h_4 = 0.6610, h_5 = -0.0430$$

The coefficient matrix of the system model at the solution point is given by

$$H_x = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -0.6610 & -3.8761 & 4.4303 \\ -3.7125 & 0.6901 & 0.6716 \\ -0.6610 & 3.7863 & -3.7719 \end{bmatrix}$$

Use the following table for $\chi^2_{k,\alpha}$ distribution to determine whether there are any bad data in the above five measurements, and which measurement would be rejected if any. Use a value of $\alpha = 0.01$.

| α | | | | | α | | | | |
|----------|-------|-------|-------|-------|----------|-------|-------|-------|-------|
| k | 0.05 | 0.025 | 0.01 | 0.005 | k | 0.05 | 0.025 | 0.01 | 0.005 |
| 1 | 3.84 | 5.02 | 6.64 | 7.88 | 6 | 12.59 | 14.45 | 16.81 | 18.55 |
| 2 | 5.99 | 7.38 | 9.21 | 10.60 | 7 | 14.07 | 16.01 | 18.48 | 20.28 |
| 3 | 7.82 | 9.35 | 11.35 | 12.84 | 8 | 15.51 | 17.54 | 20.09 | 21.96 |
| 4 | 9.49 | 11.14 | 13.28 | 14.86 | 9 | 16.92 | 19.02 | 21.67 | 23.59 |
| 5 | 11.07 | 12.83 | 15.09 | 16.75 | 10 | 18.31 | 20.48 | 23.21 | 25.19 |

Q5

$$e = Z - h = \begin{bmatrix} 0.92 \\ 1.02 \\ 0.602 \\ 0.598 \\ 0.305 \end{bmatrix} - \begin{bmatrix} 0.9578 \\ 0.9843 \\ 0.324 \\ 0.661 \\ -0.043 \end{bmatrix} = \begin{bmatrix} -0.0378 \\ 0.0357 \\ 0.281 \\ -0.063 \\ 0.348 \end{bmatrix}$$

$$f = \sum_{i=1}^5 \frac{e_i^2}{\sigma_i^2} = \sum_{i=1}^5 e_i^2 w_i$$

$$f = (-0.0378)^2(10000) + (0.0357)^2(10000) \\ + (0.281)^2(2500) + (-0.063)^2(4444.44) \\ + (0.348)^2(2500) = 544.83$$

$$w_i = \begin{bmatrix} 10000 & 0 & 0 & 0 & 0 \\ 0 & 10000 & 0 & 0 & 0 \\ 0 & 0 & 2500 & 0 & 0 \\ 0 & 0 & 0 & 4444.44 & 0 \\ 0 & 0 & 0 & 0 & 2500 \end{bmatrix}$$

$$\chi^2_{K, \alpha}$$

$$K = \#Z - \#X = 5 - 3 = 2$$

$$\alpha = 0.01$$

$$\chi^2_{2, 0.01} = 9.21$$

$$f > \chi^2_{2, 0.01}$$

\therefore There is at least one bad data

$$\text{Standardized error} = \frac{\hat{e}_i}{\sqrt{R_{ii}}}$$

$$R' = [I - H(H^T W H)^{-1} H^T W] W^{-1}$$

$$R' = \begin{bmatrix} 0.0000438 & & & & \\ & 0.0000502 & & & \\ & & 0.0001877 & & \\ & & & 0.0000077 & \\ & & & & 0.0002223 \end{bmatrix}$$

$$\text{Standardized error} = \frac{\hat{e}_i}{\sqrt{R_{ii}}} = \begin{bmatrix} -5.711 \\ 5.038 \\ 20.51 \\ -22.703 \\ 23.34 \end{bmatrix} \rightarrow \begin{matrix} \text{Badest} \\ \text{measurement} \end{matrix}$$

$\therefore z_s$ rejected