King Saud University
College of Sciences
Mathematics Department

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Solution of the final exam ACTU–462 Spring 2020 (20%)

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Problem 1. (5 marks)

1. For a special fully discrete whole life insurance on (40) you are given:
   (i) The benefit is 12000 if death occurs before age 55 and 6000 if death occurs afterwards.
   (ii) Expenses are

	Per premium	Per policy
First year	80%	120
Renewal	4%	10

   (iii) Mortality follows the Illustrative Life Table and $i = 0.06$.

   (a) Calculate the level gross premium.

   (b) Calculate the reserve at the end of the year 15.

2. Calculate the FPT reserve at the end of year 2 for a fully discrete 20–year endowment insurance of 1000 on (40) using ILT and $i = 0.06$.

3. You are given: (i) $1000\bar{A}_{38} = 235.68$ and (ii) $1000\bar{A}_{50} = 281.05$, (iii) Deaths are uniformly distributed between integral ages. Calculate the semi–continuous reserve at time 12 for a whole life insurance of 1000 on (38) when $i = 0.05$.

Solution:

1.

   (a) $\mathbf{APV(F.P.)}_0 = G\ddot{a}_{40} = 14.8166G,$

$$\begin{aligned}
\mathbf{APV(D.B.)}_0 &= 12000A^{1}_{40:\overline{15|}} + 6000\ {}_{15|}A_{40} = 12000\left(A_{40} - {}_{15|}A_{40}\right) + 6000\ {}_{15|}A_{40} \\
&= 12000A_{40} - 6000\ {}_{15|}A_{40} = 12000A_{40} - 6000\ {}_{15}E_{40}A_{55} \\
&= 12000A_{40} - 6000A_{55}\ {}_{5}E_{40}\ {}_{10}E_{45} \\
&= 12 \times 161.32 - 6 \times 305.14 \times 0.73529 \times 0.52652 = 1227.
\end{aligned}$$

   and

$$\begin{aligned}
\mathbf{APV(F.Exp.)}_0 &= 120 + 0.8G + (10 + 0.04)\,a_{40} \\
&= 120 + 0.8G + (10 + 0.04G)\,(\ddot{a}_{40} - 1) \\
&= 110 + 0.76G + (10 + 0.04G)\,\ddot{a}_{40} \\
&= 110 + 0.76G + (10 + 0.04G)\,(14.8166) \\
&= 1.3527G + 258.17
\end{aligned}$$

   Finally G is given as the solution of $1.3527G + 258.17 + 1227 = 14.8166G$, hence $G = \mathbf{110.31}$

(b) The gross reserve at time 15 is

$$
\begin{aligned}
{}_{15}V^g &= 6000A_{55} + (10 + 0.04G)\,\ddot{a}_{55} - G\ddot{a}_{55} \\
&= 6000A_{55} + (10 - 0.96G)\,\ddot{a}_{55} \\
&= 6 \times 305.14 + (10 - 0.96 \times 110.31) \times 12.2758 = \mathbf{653.620}.
\end{aligned}
$$

2. The FPT reserve at time 2 is the time 1 level net premium reserve for a 21–year endowment insurance on (39), that is

$$
{}_2V^{FPT}_{40:\overline{20}|} = {}_1V_{41:\overline{19}|} = 1 - \frac{\ddot{a}_{42:\overline{18}|}}{\ddot{a}_{41:\overline{19}|}}
$$

and

$$
\begin{aligned}
\ddot{a}_{42:\overline{18}|} &= \ddot{a}_{42} - {}_{18}E_{42}\ddot{a}_{60} = \ddot{a}_{42} - v^{18}\frac{\ell_{60}}{\ell_{42}}\ddot{a}_{60} \\
&= 14.551 - (1.06)^{-18}\frac{8188074}{9259571} \times 11.1454 = 11.098.
\end{aligned}
$$

We can backup $\ddot{a}_{41:\overline{19}|}$ using recursion on annuities.

$$
\ddot{a}_{40:\overline{20}|} = (\ddot{a}_{40} - {}_{20}E_{40}\ddot{a}_{60}) = 1 + v\ p_{40}\ddot{a}_{41:\overline{19}|}
$$

$$
\ddot{a}_{41:\overline{19}|} = \frac{\ddot{a}_{40} - {}_{20}E_{40}\ddot{a}_{60} - 1}{\frac{1-q_{40}}{1+i}} = \frac{14.8166 - 0.27414 \times 11.1454 - 1}{\frac{1-0.00278}{1.06}} = 11.439
$$

Then the net premium reserves,

$$
{}_2V^{\text{FPT}} = 1000\left(1 - \frac{11.098}{11.439}\right) = \mathbf{29.81}
$$

3. We have $A_{38} = 0.23568\frac{\ln(1.05)}{0.05} = 0.22998$ and $A_{40} = 0.28105\frac{\ln(1.05)}{0.05} = 0.27425$

$$
\begin{aligned}
{}_{12}V &= \frac{0.05}{\ln(1.05)}1000\left(\frac{A_{40} - A_{38}}{1 - A_{38}}\right) \\
&= \frac{0.05}{\ln(1.05)}1000\left(\frac{0.27425 - 0.22998}{1 - 0.22998}\right) = \mathbf{58.918}.
\end{aligned}
$$

Problem 2. (5 marks)

1. For a double decrement model you are given the following

$$
q^{(2)}_{25} = 0.125,\ \ {}_{1|}q^{(1)}_{25} = 0.252 \ \text{ and } q^{(1)}_{26} = 0.335
$$

Calculate $q^{(1)}_{25}$.

2. Consider a double–decrement model: (1) if the cause of death is accident and (2) if the cause of death is due to other means. Given $q^{(\tau)}_x = \frac{x}{200}$ and $q^{(2)}_x = 2q^{(1)}_x$, calculate the probability that an individual age 20 will die from accident within 3 years.

3. Assuming constant forces of decrement in each year of age, you are given:

$$
q'^{(1)}_{40} = 0.15,\ \ q'^{(2)}_{41} = 0.25 \ \text{ and } \mu^{(2)}_{40} = 0.3,\ \mu^{(1)}_{41} = 0.2
$$

Calculate ${}_{1|}q^{(2)}_{40}$ for a double–decrement table.

Solution:

1. We know that

$$q_{25}^{(1)} = q_{25}^{(\tau)} - q_{25}^{(2)} = 1 - p_{25}^{(\tau)} - q_{25}^{(2)} = 1 - 0.125 - p_{25}^{(\tau)} = 0.875 - p_{25}^{(\tau)}$$

Moreover ${}_{1|}q_{25}^{(1)} = p_{25}^{(\tau)}\; q_{26}^{(1)}$ which implies that $p_{25}^{(\tau)} = \frac{{}_{1|}q_{25}^{(1)}}{q_{26}^{(1)}} = \frac{0.252}{0.335} = 0.75224$, thus $q_{25}^{(1)} = 0.875 - 0.75224 = \mathbf{0.12276}$.

2. The probability of dying from accident within 3 years is the sum of the probabilities of dying in each year, which are

$$q_{20}^{(1)} + p_{20}^{(\tau)}\; q_{21}^{(1)} + \;{}_2p_{20}^{(\tau)}\; q_{22}^{(1)}.$$

We know $q_x^{(1)} = q_x^{(\tau)} - q_x^{(2)} = q_x^{(\tau)} - 2q_x^{(1)}$, hence $q_x^{(1)} = \frac{1}{3}q_x^{(\tau)}$, thus $q_{20}^{(1)} = \frac{1}{3}\frac{20}{200} = \frac{1}{30}$, $q_{21}^{(1)} = \frac{1}{3}\frac{21}{200} = \frac{7}{200}$ and $q_{22}^{(1)} = \frac{1}{3}\frac{22}{200} = \frac{11}{300}$. Moreover $p_{20}^{(\tau)} = 1 - \frac{20}{200} = 0.9$ and $p_{21}^{(\tau)} = 1 - \frac{21}{200} = 0.895$ then

$${}_2p_{20}^{(\tau)} = \; p_{20}^{(\tau)}\; p_{21}^{(\tau)} = 0.9 \times 0.895 = 0.8055.$$

Finally $\frac{1}{30} + 0.9 \times \frac{7}{200} + 0.8055 \times \frac{11}{300} = \mathbf{0.094368}$.

3. We have

$${}_{1|}q_{40}^{(2)} = p_{40}^{(\tau)}\; q_{41}^{(2)} = \; p_{40}^{\prime(1)}\; p_{40}^{\prime(2)}\; q_{41}^{(2)} = (1 - 0.15)\, e^{-0.3}\; q_{41}^{(2)}.$$

Now, we need to find $q_{41}^{(2)}$. We have

$$\begin{aligned} q_{41}^{(2)} &= \int_0^1 {}_sp_{41}^{(\tau)}\; \mu_{41+s}^{(2)}\; ds = \int_0^1 \; {}_sp_{41}^{\prime(1)}\; {}_sp_{41}^{\prime(2)}\; \mu_{41+s}^{(2)}\; ds \\ &= \int_0^1 e^{-0.2s} \left(p_{41}^{\prime(2)}\right)^s \; \mu_{41+s}^{(2)}\; ds = \int_0^1 e^{-0.2s}\left(1 - q_{41}^{\prime(2)}\right)^s \; \mu_{41+s}^{(2)}\; ds \\ &= \int_0^1 e^{-0.2s}\,(0.75)^s\,(-\ln(0.75))\;\; ds = 0.22767. \end{aligned}$$

Finally ${}_{1|}q_{40}^{(2)} = 0.85e^{-0.3}0.22767 = \mathbf{0.14336}$.

<u>Problem 3.</u> (5 marks)

1. For a 3–year fully discrete **term** insurance of 5000 on (50), subject to a double decrement model: Decrement (d) is death. Decrement (w) is withdrawal.

x	$\ell_x^{(\tau)}$	$d_x^{(w)}$	$d_x^{(d)}$
50	3000	50	30
51	–	30	40
52	–	–	50

Assume that a withdrawal benefit of 1000 is possible in the second year. Calculate the level annual premium for this insurance for $i = 0.04$.

2. For a fully continuous whole life insurance of 12000 on (x) is subject to CFM and CFI that is $\mu_x = \mu$ for all x, and $\delta = 0.05$. The net single premium for is 4000. Suppose that we change the mortality assumption to a double decrement model with $\mu_x^{(1)} = \mu$ and $\mu_x^{(2)}$ such that $\mu_x^{(2)} = 2\mu_x^{(1)}$. Assume that the benefit amount and force of interest are unchanged. Determine the net single premium for the double decrement model.

3. An insurance company uses a double model to calculate premiums and reserves for a special 10–year term insurance policy. Let (1) be death due to accident and (2) be death due to other causes and suppose
$$\mu_x^{(1)} = 0.005 + 0.0004e^{0.08x} \text{ and } \mu_x^{(2)} = 0.0001$$
The basic death benefit is 10,000, but the death benefit becomes 15,000 if death occurs as a result of an accident. The death benefit is payable immediately on death. Premiums are payable continuously throughout the term. Ignore expenses. Using a force of interest of 5% per year and ignoring expenses, for a policy issued to a life aged 25. The level premium is 161.13.

   (a) Write down the Thiele's differential equation satisfied by the benefit reserve.

   (b) Use Euler's method with $h = 1$ and a backward recursion to find for the reserve at the end of year 7.

Solution:

1. We have

$$\begin{aligned}
\mathbf{APV(F.P.)}_0 &= P\left(1 + v\ p_{50}^{(\tau)} + v^2\ {}_2p_{50}^{(\tau)}\right) = P\left(1 + v\ \frac{\ell_{51}^{(\tau)}}{\ell_{50}^{(\tau)}} + v^2\ \frac{\ell_{52}^{(\tau)}}{\ell_{50}^{(\tau)}}\right) \\
&= P\left(1 + v\ \frac{\ell_{50}^{(\tau)} - d_{50}^{(\tau)}}{\ell_{50}^{(\tau)}} + v^2\ \frac{\ell_{51}^{(\tau)} - d_{51}^{(\tau)}}{\ell_{50}^{(\tau)}}\right) \\
&= P\left(1 + v\ \frac{\ell_{50}^{(\tau)} - d_{50}^{(\tau)}}{\ell_{50}^{(\tau)}} + v^2\ \frac{\ell_{50}^{(\tau)} - d_{50}^{(\tau)} - d_{51}^{(\tau)}}{\ell_{50}^{(\tau)}}\right) \\
&= P\left(1 + \frac{1}{1.04}\frac{3000-80}{3000} + \frac{1}{(1.04)^2}\frac{3000-80-70}{3000}\right) = 2.8142P.
\end{aligned}$$

and

$$\begin{aligned}
\mathbf{APV(F.B.)}_0 &= 5000A_{50:\overline{3|}}^{1\,(d)} + 1000v^2\ {}_{1|}q_{50}^{(w)} \\
&= 5000\left(v\ q_{50}^{(d)} + v^2\ {}_{1|}q_{50}^{(d)} + v^3\ {}_{2|}q_{50}^{(d)}\right) + 1000\ v^2\ {}_{1|}q_{50}^{(w)} \\
&= \frac{5}{3}\left(v\ d_{50}^{(d)} + v^2\ d_{51}^{(d)} + v^3\ q_{52}^{(d)}\right) + \frac{1}{3}\ v^2\ d_{51}^{(w)} \\
&= \frac{5}{3}\left(\frac{30}{1.06} + \frac{40}{(1.06)^2} + \frac{50}{(1.06)^3}\right) + \frac{1}{3}\frac{30}{(1.04)^2} \\
&= 183.80 + 9.2456 = 193.0456
\end{aligned}$$

Thus $P = \frac{193.0456}{2.8142} = \mathbf{68.5970}$.

2. The net single premium for the double decrement model is the APV of future premiums

$$12000\bar{A}_x^{(\tau)} = 12000\frac{\mu^{(\tau)}}{\mu^{(\tau)} + \delta} = 12000\frac{3\mu}{3\mu + \delta} = 12000\frac{3\mu}{3\mu + 0.05}.$$

We need just to find μ which can be backed from $4000 = 12000\frac{\mu}{\mu+0.05}$, hence $\mu = 0.025$ thus

$$12000\bar{A}_x^{(\tau)} = 12000\frac{3 \times 0.025}{3 \times 0.025 + 0.05} = \mathbf{7200}.$$

3. We have $\mu_x^{(1)} = 0.005 + 0.0004e^{0.08x}$ and $\mu_x^{(2)} = 0.0001$

$$\int_0^s \mu_{25+u}^{(\tau)}\, du = 10^{-4}\int_0^s \left(51 + 4e^{0.08(25+u)}\right) du = 10^{-4}\left(51s + \frac{4e^2}{0.08}\left(e^{0.08s} - 1\right)\right)$$

$$ {}_s p_{30}^{(\tau)} = \exp\left(-10^{-4}\left(51s + \frac{4e^2}{0.08}\left(e^{0.08s} - 1\right)\right)\right).$$

$$\begin{aligned}
\mathbf{APV(FP)}_0 &= P\int_0^{10} e^{-\delta s}\ {}_s p_{25}^{(\tau)} ds = \int_0^{10} e^{-0.05s}\exp\left(-0.0001\left(51s + \frac{4e^2}{0.08}\left(e^{0.08s} - 1\right)\right)\right) ds \\
&= 7.5547
\end{aligned}$$

and

$$\begin{aligned}
\mathbf{APV(FB)}_0 &= 1000\int_0^{10} e^{-\delta s}\ {}_s p_{25}^{(\tau)}\left(15\mu_{25+s}^{(1)} + 10\mu_{25+s}^{(2)}\right) ds \\
&= \int_0^{10} e^{-0.05s}\exp\left(-0.0001\left(51s + \frac{4e^2}{0.08}\left(e^{0.08s} - 1\right)\right)\right)\left(15\left(5 + 0.4e^{0.08(25+s)}\right) + 1\right) ds \\
&= 1067.4.
\end{aligned}$$

So the premium is $P = \frac{1067.4}{7.5547} = 141.2895$ a correction is taken $P = \mathbf{161.13.}$

(a) The Thiele's differential equation satisfied by the benefit reserve is

$$\begin{aligned}
\frac{d\ {}_tV}{dt} &= P + \left(\delta + \mu_{25+t}^{(\tau)}\right) {}_tV - b_t^{(1)}\mu_{25+t}^{(1)} - b_t^{(2)}\mu_{25+t}^{(2)} \\
&= 161.13 + \left(0.05 + 0.0051 + 0.0004e^{0.08(25+t)}\right) {}_tV - 15\left(5 + 0.4e^{0.08(25+t)}\right) - 1 \\
&= 161.13 + \left(0.0551 + 0.0004e^{0.08(25+t)}\right) {}_tV - 15\left(5 + 0.4e^{0.08(25+t)}\right) - 1 \\
&= 85.13 + \left(0.0551 + 0.0004e^{0.08(25+t)}\right) {}_tV - 6e^{0.08(25+t)}.
\end{aligned}$$

(b) The backward recursion scheme of Euler's method is

$$\frac{{}_{10-kh}V - {}_{10-(k+1)h}V}{h} = 85.13 + \left(0.0551 + 0.0004e^{0.08(35-kh)}\right) {}_{10-kh}V - 6e^{0.08(35-kh)}$$

or equivalently

$${}_{10-(k+1)h}V - {}_{10-kh}V = -h\left(85.13 + \left(0.0551 + 0.0004e^{0.08(35-kh)}\right) {}_{10-kh}V - 6e^{0.08(35-kh)}\right)$$

with terminal value ${}_{10}V = 0$. Now we start with $k = 0$ and compute the reserves recursively, using $h = 1$:

we get

$$\begin{aligned}
{}_9V &= {}_{10}V - \left(85.13 + \left(0.0551 + 0.0004e^{0.08(35)}\right) {}_{10}V - 6e^{0.08(35)}\right) \\
&= 0 - \left(85.13 + \left(0.0551 + 0.0004e^{0.08(35)}\right)0 - 6e^{0.08(35)}\right) = \mathbf{98.668} \\
{}_8V &= {}_9V - \left(85.13 + \left(0.0551 + 0.0004e^{0.08(34)}\right) {}_9V - 6e^{0.08(34)}\right) \\
&= 98.668 - \left(85.13 + \left(0.0551 + 0.0004e^{0.08(34)}\right)(98.668) - 6e^{0.08(34)}\right) = \mathbf{98.584} \\
{}_7V &= {}_8V - \left(85.13 + \left(0.0551 + 0.0004e^{0.08(33)}\right) {}_8V - 6e^{0.08(33)}\right) \\
&= 98.584 - \left(85.13 + \left(0.0551 + 0.0004e^{0.08(33)}\right)(98.584) - 6e^{0.08(33)}\right) = \mathbf{91.549}
\end{aligned}$$

Problem 4. (5 marks)

1. Consider a permanent disability model of the following form:

$$\begin{array}{ccccc} \boxed{\text{(0) Healthy}} & & \xrightarrow{\mu_x^{01}} & & \boxed{\text{(1) Disabled}} \\ & \mu_x^{02} \searrow & & \swarrow \mu_x^{12} & \\ & & \boxed{\text{(2) Dead}} & & \end{array}$$

and

$$\mu_x^{01} = \frac{0.75}{100-x}\ \mu_x^{02} = \frac{0.25}{100-x};\ \ \mu_x^{12} = \frac{1}{100-x} \text{ for, } x < 100;$$

This leads to

$${}_tp_x^{00} = {}_tp_x^{11} = 1 - \frac{t}{100-x}, \text{ and } {}_tp_x^{01} = 0.75\left(1 - \frac{t}{100-x}\right)\ln\left(\frac{100-x}{100-x-t}\right)$$

Observe that ${}_tp_{40}^{02} = 1 - {}_tp_{40}^{00} - {}_tp_{40}^{01}$ for $t < 60$.

A 3–year insurance on (40) pays an annuity of 6000 per year at the beginning of each of the second and the third years if the insured is disabled and $12,000$ at the end of the year of death if death occurs within 3 years. Premiums are payable at the beginning of each year **if the insured is healthy at that time** and are determined by the equivalence principle.
Calculate the annual premium for $i = 0.03$.

2. Consider again a permanent disability model with three states: healthy (state 0), disabled (state 1), and dead (state 2). The forces of transition are, for all x: $\mu_x^{01} = 0.10$, $\mu_x^{02} = 0.05$ and $\mu_x^{12} = 0.25$ and interest $\delta = 0.05$.
A whole life insurance on (40) pays a benefit of 4000 **at the time** of transition to state 2. Continuous premiums of 400 per year are payable for 10 years while the insured is healthy. Calculate the reserve at time 2 if the insured is healthy.

Solution:

1. We know $\mathbf{APV(F.P.)}_0 = P$ times APV of an annuity–due of 1 per year in state 0 that is

$$\mathbf{APV(F.P.)}_0 = P\left(1 + \frac{p_{40}^{00}}{1.03} + \frac{{}_2p_{40}^{00}}{(1.03)^2}\right) = P\left(1 + \frac{1}{1.03}\frac{59}{60} + \frac{1}{(1.03)^2}\frac{58}{60}\right) = 2.8659P$$

and

$$\begin{aligned} \mathbf{APV(F.B.)}_0 &= \mathbf{APV(Disability\ Benifits)}_0 + \mathbf{APV(Death\ Benifits)}_0 \\ &= 6000\left(\frac{p_{40}^{01}}{1.03} + \frac{{}_2p_{40}^{01}}{(1.03)^2}\right) + 12000\left(\frac{p_{40}^{02}}{1.03} + \frac{{}_2p_{40}^{02}}{(1.03)^2} + \frac{{}_3p_{40}^{02}}{(1.03)^3}\right) \end{aligned}$$

So we first calculate the required probabilities

$$p_{40}^{01} = 0.75\left(\frac{59}{60}\right)\ln\left(\frac{60}{59}\right) = 0.012395,\ \ {}_2p_{40}^{01} = 0.75\left(\frac{58}{60}\right)\ln\left(\frac{60}{58}\right) = 0.024579$$

and observe that

$$\begin{aligned} p_{40}^{02} &= 1 - p_{40}^{00} - p_{40}^{01} = 1 - \frac{59}{60} - 0.012395 = 0.0042717 \\ {}_2p_{40}^{02} &= 1 - {}_2p_{40}^{00} - {}_2p_{40}^{01} = 1 - \frac{58}{60} - 0.024579 = 0.0087543 \\ {}_3p_{40}^{02} &= 1 - {}_3p_{40}^{00} - {}_3p_{40}^{01} = 1 - \frac{57}{60} - 0.75\left(\frac{57}{60}\right)\ln\left(\frac{60}{57}\right) = 0.013454. \end{aligned}$$

Hence

$$\begin{aligned}\mathbf{APV}(\mathbf{F}.\mathbf{B}.)_0 &= 6000\left(\frac{0.012395}{1.03}+\frac{0.024579}{(1.03)^2}\right)\\ &\quad +12000\left(\frac{0.0042717}{1.03}+\frac{0.0087543}{(1.03)^2}+\frac{0.013454}{(1.03)^3}\right)\\ &= 6000\times 0.035202+12000\times 0.024711=507.7440\end{aligned}$$

Finally the net level premium is $P=\frac{507.7440}{2.8659}=\mathbf{177.16738}$.

2. We know that

$$\begin{aligned}{}_2V &= \mathbf{APV}(\mathbf{F}.\mathbf{B}.)_2-\mathbf{APV}(\mathbf{F}.\mathbf{P}.)_2\\ &= 4000\int_0^{\infty} e^{-0.05s}\sum_{k=0}^{1} {}_sp_{42}^{0k}\ \mu_{42+s}^{k2}ds-400\int_0^8 e^{-0.05s}\ {}_sp_{42}^{(00)}ds\end{aligned}$$

We first calclulate $\int_0^8 e^{-0.05s}\ {}_sp_{42}^{(00)}ds$ and $\int_0^{\infty} e^{-0.05s}\ \sum_{k=0}^{1}\ {}_sp_{42}^{0k}\ \mu_{42+s}^{k2}ds$.
The APV of a continuous annuity of 1 per year for 8 years at time 2 in the healthy state is:

$$\int_0^8 e^{-0.05s}\ {}_sp_{42}^{(00)}ds=\int_0^8 e^{-s(0.05+0.15)}ds=\frac{1-e^{-8\times 0.2}}{0.2}=3.9905.$$

Thus $\mathbf{APV}(\mathbf{F}.\mathbf{P}.)_2=400\times 3.9905=1596.20$. And

$$\int_0^{\infty} e^{-0.05s}\ {}_sp_{42}^{00}\ \mu_{42+s}^{02}\ ds=\int_0^{\infty} e^{-0.05s}0.05e^{-0.15s}ds=0.05\int_0^{\infty} e^{-0.2s}ds=\frac{0.05}{0.2}=0.25$$

But

$$\begin{aligned}{}_sp_{42}^{01} &= \int_0^s {}_tp_{42}^{00}\ \mu_{42+t}^{01}\ {}_{s-t}p_{42+t}^{11}\ dt=\int_0^s e^{-0.15t}0.1e^{-0.25(s-t)}dt\\ &= 0.1e^{-0.25s}\int_0^s e^{0.1t}dt=e^{-0.25s}\left(e^{0.1s}-1\right)=e^{-0.15s}-e^{-0.25s}\end{aligned}$$

thus

$$\begin{aligned}0.25\int_0^{\infty} e^{-0.05s}\left(e^{-0.15s}-e^{-0.25s}\right)ds &= 0.25\int_0^{\infty}\left(e^{-0.2s}-e^{-0.3s}\right)ds\\ &= 0.25\left(\frac{1}{0.2}-\frac{1}{0.3}\right)=0.41667.\end{aligned}$$

Therefore

$$\int_0^{\infty} e^{-0.05s}\sum_{k=0}^{1} {}_sp_{42}^{0k}\ \mu_{42+s}^{k2}\ ds=0.25+0.41667=0.66667$$

hus $\mathbf{APV}(\mathbf{F}.\mathbf{B}.)_2=4000\times 0.66667=2666.68$, finally ${}_2V=2666.68-1596.20=\mathbf{1070.48}$.