King Saud University
College of Sciences
Mathematics Department

Academic Year (G) 2019–2020
Academic Year (H) 1441
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Model Answer of the first midterm exam ACTU–462 (25%)

October 7, 2019 (Fall 2019)

Problem 1. (6 marks)

1. **(2 marks)** For a 30–year fully discrete term life insurance with face amount 1000 on (35), you are given: (i) Premiums are calculated using the equivalence principle. (ii) Premiums are payable for 15 years. (iii) Mortality follows the Illustrative Life Table and $i = 0.06$. Calculate the annual net premium. (${}_{15}E_{35} = {}_{10}E_{35}\ {}_{5}E_{45}$ and ${}_{30}E_{35} = {}_{10}E_{35}\ {}_{20}E_{45}$)

2. **(2 marks)** For a special fully discrete 35–payment whole life insurance on (30): (i) The death benefit is 1 for the first 20 years and is 5 thereafter. (ii) The initial net premium paid during each of the first 20 years is one fifth of the net premium paid during each of the 15 subsequent years. (iii) Mortality follows the Illustrative Life Table, $i = 0.06$ $A_{30:\overline{20}|} = 0.32307$ and $\ddot{a}_{30:\overline{35}|} = 14.835$. Calculate the initial annual net premium.

3. **(2 marks)** For a special fully discrete 5–year term insurance on (20), you are given: (i) The death benefit is 2000 during the first 3 years and 1000 thereafter. In addition, in the event of death, all premiums are refunded without interest. (ii) Mortality is uniformly distributed with $\omega = 100$. Calculate the net premium P which are payable according to the following schedule for $i = 0.03$

Policy year	1	2	3	4 and later
Net premium	P	$2P$	$3P$	0

Solution:

1. By the equivalence principle, we know that the annual net premium is the quotient of the insurance over the premium annuity: that is

$$ {}_{15}P^{1}_{35:\overline{30}|} = \frac{A^{1}_{35:\overline{30}|}}{\ddot{a}_{35:\overline{15}|}} $$

The expected present value of the insurance for the unit benefit is $A^{1}_{35:\overline{30}|} = A_{35} - {}_{30}E_{35}A_{65}$. And ${}_{30}E_{35}$ can be calculated using the Illustrative Life Table as ${}_{10}E_{35}\ {}_{20}E_{45}$. Hence ${}_{30}E_{35} = 0.54318 \times 0.25634 = 0.13924$ and

$$ A^{1}_{35:\overline{30}|} = 0.12872 - (0.13924)(0.43980) = 0.067482. $$

The expected present value of the annuity is $\ddot{a}_{35:\overline{15}|} = \ddot{a}_{35} - {}_{15}E_{35}\ddot{a}_{50}$. Again ${}_{15}E_{35}$ can be calculated as ${}_{10}E_{35}\ {}_{5}E_{45}$. Thus ${}_{15}E_{35} = (0.54318)(0.72988) = 0.39646$ and

$$ \ddot{a}_{35:\overline{15}|} = 15.3926 - (0.39646)(13.2668) = 10.13284. $$

The annual net premium is

$$ 1000\ {}_{15}P^{1}_{35:\overline{30}|} = 1000\frac{0.067482}{10.13284} = \mathbf{6.6597}. $$

2. The value of the special insurance is

$$A^{1}_{30:\overline{20}|} + 5\left(A_{30} - A^{1}_{30:\overline{20}|}\right)$$

We look up $A_{30} = 0.10248$ in the Illustrative Life Table and calculate:

$$A^{1}_{30:\overline{20}|} = 0.32307 - {}_{20}E_{30} = 0.32307 - 0.29374 = 0.02933,$$

then the APV of the special insurance is $0.02933 + 5(0.10248 - 0.02933) = 0.39508$. We calculate the value of a premium annuity of 1 for 20 years, 5 for 15 years afterwards by using

$$\ddot{a}_{30:\overline{20}|} = \frac{1 - A_{30:\overline{20}|}}{d} = \frac{1 - 0.32307}{\frac{0.06}{1.06}} = 11.9591$$

The APV of the annuity is

$$\ddot{a}_{30:\overline{20}|} + 5\left(\ddot{a}_{30:\overline{35}|} - \ddot{a}_{30:\overline{20}|}\right) = 11.9591 + 5(14.835 - 11.9591) = 26.3386.$$

The **initial annual net premiums** are therefore $P = \frac{0.39508}{26.3386} = \mathbf{0.015}$.

3. If death occurs in the first year, the premium refund is P; in the second year, it is $3P$; in the third year or later, the refund is $6P$. Therefore, the actuarial present value of future benefits is

$$\begin{aligned}
\mathrm{APV}(F.B.)_0 &= (2000+P)vq_{20} + (2000+3P)v^2 p_{20}\ q_{21} + (2000+6P)v^3\ {}_2p_{20}\ q_{22} \\
&\quad +(1000+6P)\left(v^4\ {}_{32}p_{20}\ q_{23} + v^5\ {}_4p_{20}\ q_{24}\right) \\
&= (2000+P)\frac{1}{1.03}\frac{1}{80} + (2000+3P)\frac{1}{1.03^2}\frac{1}{80} \\
&\quad +(2000+6P)\frac{1}{1.03^3}\frac{1}{80} + (1000+6P)\frac{1}{80}\left(\frac{1}{1.03^4} + \frac{1}{1.03^5}\right) \\
&= 0.24745P + 92.604.
\end{aligned}$$

The probabilities of survival are $p_{20} = \frac{79}{80}$, ${}_2p_{20} = \frac{78}{80}$. The actuarial present value of future premiums is

$$P + (2P)v\frac{79}{80} + (3P)v^2\frac{78}{80} = P\left(1 + \frac{2}{1.03}\frac{79}{80} + \frac{3}{1.03^2}\frac{78}{80}\right) = 5.6746P$$

By equating the annuity with the insurance, we get $0.24745P + 92.604 = 5.6746P \iff P = \mathbf{17.063}$.

<u>Problem 2.</u> (6 marks)

1. **(2 marks)** For a special decreasing 15–year term life insurance on a person age 30, you are given: $\mu_{30+t} = \frac{1}{70-t}$, $0 \le t < 70$, the benefit payment is $2,000$ for the first 10 years and $1,000$ for the last 5 years. The death benefit is payable at the end of the year of death and $v = 0.95$. Calculate the level annual net premium for this insurance.

2. **(2 marks)** For a fully discrete 10–year term insurance of 1000 on (60) , you are given: (i) Premiums are payable in all years. (ii) $i = 0.06$, (iii) $q_{60+k} = \frac{1}{40-k}$, $k = 0, 1, \ldots, 39$. The net premium is 25. Calculate the expected value of the net future loss.

3. **(2 marks)** Determine the annual net premium so that the expected value of the future loss will be -1.

Solution:

1. This can be treated as a sum of a 15–year level term insurance for 1000 and a 10–year level term insurance for 1000. Under uniform mortality, the expected present values of these are

$$A^{1}_{x:\overline{n}|} = \sum_{k=1}^{n} \frac{v^k}{\omega - x} = \sum_{k=1}^{n} \frac{v^k}{\omega - x} = \frac{a_{\overline{n}|}}{\omega - x} = \frac{1 - v^n}{i\,(\omega - x)}.$$

So the $\text{APV(F.B.)}_0 = 1000\left(A^{1}_{30:\overline{10}|} + A^{1}_{30:\overline{15}|}\right)$ and

$$A^{1}_{30:\overline{10}|} = \frac{1 - (0.95)^{10}}{0.052632 \times 70} = 0.10891 \quad \text{and} \quad A^{1}_{30:\overline{15}|} = \frac{1 - (0.95)^{15}}{0.052632 \times 70} = 0.14568$$

Therefore

$$\text{APV(F.B.)}_0 = 1000\left(A^{1}_{30:\overline{10}|} + A^{1}_{30:\overline{15}|}\right) = 1000(0.108914 + 0.145678) = 254.592.$$

And the actuarial present value of future premiums (or the annuity) is $Pa_* * 30{:}\overline{15}|$. But we know that

$$a_* * 30{:}\overline{15}| = \frac{1 - A_{30:\overline{15}|}}{d} = \frac{1 - \left(A^{1}_{30:\overline{15}|} + {}_{15}E_{30}\right)}{d} = \frac{1 - \left(0.14568 + (0.95)^{15}\,\frac{55}{70}\right)}{0.05} = 9.8061,$$

hence by the E.P. $P = \frac{1000\left(A^{1}_{30:\overline{10}|} + A^{1}_{30:\overline{15}|}\right)}{a_* * 30:\overline{15}|} = \frac{254.592}{9.8061} = \mathbf{25.963}$.

2. We know that

$$E\left[{}_0L\right] = 1000A^{1}_{60:\overline{10}|} - 25\ddot{a}_{60:\overline{10}|}$$

Under De Moivre's law the actuarial present value of the term insurance is

$$1000A^{1}_{60:\overline{10}|} = 1000\frac{a_{\overline{10}|}}{40} = 1000\frac{1 - (1.06)^{-10}}{40 \times 0.06} = 184.002.$$

For the expected present value of the premiums, we need $\ddot{a}_{60:\overline{10}|}$. For uniform mortality it is easier to first calculate a 10–year endowment to get a 10–year temporary life annuity: that is

$$\ddot{a}_{60:\overline{10}|} = \frac{1 - A_{60:\overline{10}|}}{d} = \frac{1 - \left(A^{1}_{60:\overline{10}|} + {}_{10}E_{60}\right)}{d} = \frac{1 - \left(0.184002 + (1.06)^{-10}\,\frac{3}{4}\right)}{0.06 \times (1.06)^{-1}} = 7.01723.$$

Therefore $E\left[{}_0L\right] = 184.002 - 25 \times 7.01723 = \mathbf{8.5713}$.

3. We want the premium P such that

$$E\left[{}_0L\right] = 184.002 - P(7.01723) = -1 \text{ that is } P = \frac{184.002 + 1}{7.01723} = \mathbf{26.364}.$$

<u>Problem 3.</u> (6 marks)

1. **(2 marks)** For a special whole life insurance on (35), you are given: (i) The annual net premium is payable at the beginning of each year. (ii) The death benefit is equal to 1000 plus the return of all net premiums paid in the past without interest. (iii) The death benefit is paid at the end of the year of death. (iv) $A_{35} = 0.42898$, (v) $(IA)_{35} = 6.16761$. Calculate the annual net premium for this insurance for $i = 0.05$.

2. **(2 marks)** An insured, age 25, purchases a 10–year continuous payment, continuous whole life insurance policy with a benefit of 1. You are given that the insured is subject to a constant force of mortality equal to 0.025 and a constant force of interest equal to 0.075. Determine the net annual premium for this policy.

3. **(2 marks)** For a fully continuous whole life insurance of 1000 on (45), the force of interest is 0.04 and the probability of survival is ${}_tp_{45} = \frac{3}{5}e^{-0.01t} + \frac{2}{5}e^{-0.03t}$. Calculate the net premium for this policy.

Solution:

1. Let P be the annual net premium. The actuarial present value of the death benefit is $1000A_{35} + P(IA)_{35} = 428.98 + 6.16761P$. The actuarial present value of the premiums is

$$P\ddot{a}_{35} = P\left(\frac{1 - A_{35}}{d}\right) = P\left(\frac{(1 - 0.42898)(1.05)}{0.05}\right) = 11.99142P$$

Equating death benefits and premiums. $428.98 + 6.16761P = 11.99142P$, $P = \frac{428.98}{5.82381} = \mathbf{73.6597}$.

2. We know that under CFM

$$P = \frac{\bar{A}_{25}}{\bar{a}_{25:\overline{10}|}} = \frac{\frac{\mu}{\mu+\delta}}{\frac{1-e^{-10(\mu+\delta)}}{\mu+\delta}} = \frac{\frac{0.025}{0.025+0.075}}{\frac{1-e^{-10(0.025+0.075)}}{0.025+0.075}} = \frac{0.25}{6.3212} = \mathbf{0.039549}.$$

3. The net premium is given by $1000\overline{P}_{45}$, where

$$\overline{P}_{45} = \frac{\bar{A}_{45}}{\bar{a}_{45}} = \frac{(1 - \delta\bar{a}_{45})}{\bar{a}_{45}}.$$

and

$$\begin{aligned}\bar{a}_{45} &= \int_0^\infty e^{-\delta t}\ {}_tp_{45}dt = \int_0^\infty e^{-0.04t}\left(\frac{3}{5}e^{-0.01t} + \frac{2}{5}e^{-0.03t}\right)dt \\ &= \frac{3}{5}\int_0^\infty e^{-0.05t}dt + \frac{2}{5}\int_0^\infty e^{-0.07t}dt = \frac{3}{5}\frac{100}{5} + \frac{2}{5}\frac{100}{7} = \frac{124}{7} = 17.714\end{aligned}$$

so the net premium is $1000\overline{P}_{45} = \frac{1000(1-0.04\times17.714)}{17.714} = \mathbf{16.453}$.

<u>**Problem 4.**</u> **(6 marks)**

1. **(2 marks)** For a fully continuous 20–year endowment insurance of 1000 on (60) such that: $\mu_{60+t} = \frac{1}{40-t}$ for $0 \leq t < 40$, $\delta = 0.06$ and the annual premium is 36. Calculate the probability that the future loss is less than 0.

2. You are given the following life table for $i = 0.05$

x	45	63	64	65	77	78
ℓ_x	9,164,070	7,833,904	7,683,980	7,533,963	4,828,285	4,530,476

Deaths are uniformly distributed over each year of age.

(a) **(1 mark)** Consider a fully continuous 20–year endowment insurance of 250,000 on (45). Calculate the 50^{th} and 15^{th} percentile premium.

(b) **(1 mark)** Consider a fully discrete 20–year endowment insurance of 1000 on (45). Calculate the 50^{th} and 15^{th} percentile premium

3. **(2 marks)** For a life age (45) whose mortality follows $\mu_x = 0.015$ for all ages x with $\delta = 0.035$. **Find** the 20^{th}–percentile premium for a 12–year payment whole life insurance of $450,000$.

<u>**Solution**</u>:

1. If death occurs before year 20, the future loss is

$$_0L = 1000e^{-0.06T_{60}} - 36\left(\frac{1-e^{-0.06T_{60}}}{0.06}\right) = 1600e^{-0.06T_{60}} - 600$$

Then the probability that the future loss is less than 0 is

$$\begin{aligned} P(_0L &< 0) = P\left(1600e^{-0.06T_{60}} < 600\right) = P\left(8e^{-0.06T_{60}} < 3\right) = P\left(\frac{8}{3} < e^{0.06T_{60}}\right) \\ &= P\left(\ln(\frac{8}{3}) < 0.06T_{60}\right) = P\left(T_{60} > \frac{1}{0.06}\ln(\frac{8}{3})\right) = P\left(T_{60} > 16.347\right) \\ &= e^{-\int_0^{16.347}\frac{1}{40-u}du} = e^{[\ln(40-u)]_0^{16.347}} = 1 - \frac{16.347}{40} = \mathbf{0.59133}. \end{aligned}$$

2. a. For $\alpha = 0.5$ first find an integer k such that

$$_kq_{45} = 1 - \frac{\ell_{45+k}}{\ell_{45}} \leq 0.5 < {}_{k+1}q_{45} = 1 - \frac{\ell_{45+k+1}}{\ell_{45}}.$$

such integer k should satisfy the following inequalities

$$0.5\ell_{45} \leq \ell_{45+k} \quad \text{and} \quad \ell_{45+k+1} < 0.5\ell_{45}$$

From the Illustrative Life Table, with we have $0.5\ell_{45} = 0.5 \times 9164070 = 4582000$, hence we find $\ell_{45+32} = \ell_{77} = 4,828,285$ and $\ell_{45+33} = \ell_{78} = 4,530,476$ therefore

$$_{32}q_{45} = 1 - \frac{4828285}{9164070} = 0.47313 < 0.5 < {}_{33}q_{45} = 1 - \frac{4530476}{9164070} = 0.50563$$

therefore $32 < t_{0.5} < 33$, and $t_{0.5} > 20$, therefore the 50^{th}–percentile premium

$$P_{0.5} = \frac{250000}{\bar{s}_{\overline{20|}}} = \frac{250000 \times \ln(1.05)}{(1.05)^{20} - 1} = \mathbf{7377.7}.$$

For $\alpha = 0.15$, we know that $_{18}q_{45} < F_{45}(t) = 0.15 < {}_{19}q_{45}$ therefore $18 < t < 19$, hence

$$\begin{aligned} 0.85 &= S_{45}(t) = {}_tp_{45} = {}_{18+t-18}p_{45} = {}_{18}p_{45}\;{}_{t-18}p_{63} = \frac{\ell_{63}}{\ell_{45}}(1 - {}_{t-18}q_{63}) \\ &= \frac{\ell_{63}}{\ell_{45}}(1-(t-18)\,q_{63}) = \frac{\ell_{63}}{\ell_{45}}\left(1-(t-18)\left(1-\frac{\ell_{64}}{\ell_{63}}\right)\right). \end{aligned}$$

Hence

$$0.85 = \frac{7833904}{9164070}\left(1-(t-18)\left(1-\frac{7683980}{7833904}\right)\right).$$

and this gives $t_{0.15} = \mathbf{18.296455}$, therefore the 15^{th}–percentile premium

$$P_{0.15} = \frac{250000}{\bar{s}_{\overline{t_{0.15}|}}} = \frac{250000 \times \ln(1.05)}{(1.05)^{18.296455} - 1} = \mathbf{8460.6} \text{ since } t_{0.15} < 20.$$

b. For a fully discrete 20–year endowment insurance of 1000 on (45) we need to find an integer k such that

$${}_kq_{45} < 0.15 < {}_{k+1}q_{45} \quad \text{and} \quad {}_kq_{45} < 0.5 < {}_{k+1}q_{45}.$$

For $\alpha = 0.5$, from the previous question we know that ${}_{20}q_{45} < 0.5$, then the 50^{th}–percentile premium is

$$P_{0.5} = \frac{1000}{\ddot{s}_{\overline{20|}}} = \frac{1000}{(1.05)^{20} - 1}\frac{0.05}{1.05} = \mathbf{28.802}.$$

For $\alpha = 0.15$ we have ${}_{19}q_{45} > 0.15$, therefore 15^{th}–percentile premium is

$$P_{0.15} = \frac{1000}{\ddot{s}_{\overline{19|}}} = \frac{1000}{(1.05)^{19} - 1}\frac{0.05}{1.05} = \mathbf{31.186}.$$

3. The distribution of T_{45} is exponential with parameter 0.015. We know that the c.d.f. $F_{45}(t) = 1 - e^{-0.015t}$. Solving $F_{45}(t_{0.2}) = 0.20 = 1 - e^{-0.015t_{0.2}}$, we get $t_{0.20} = 14.876 > 12$. Then the 20^{th} –percentile premium for a whole life insurance of $450,000$ on (45) is given by since

$$P_{0.2} = \frac{450000e^{-0.035\times 14.876}}{\frac{1-e^{-0.035\times 12}}{0.035}} = \mathbf{27285}.$$

<u>Problem 5.</u> (6 marks)

1. **(2 marks)** For a fully discrete $100,000 whole life policy issued at age 30:
   (i) The annual net premium is $2,738.
   (ii) The net premium reserve at the end of 9 years is $8,931.
   (iii) The net premium reserve at the end of 10 years is $10,059.
   (iv) $\ell_x = \begin{cases} 112 - \frac{14}{10}x & \text{for } 20 < x < 80 \\ 0 & \text{for } x \geq 80. \end{cases}$ Calculate the effective interest rate i.

2. **(2 marks)** An annual premium 10–year payment whole life insurance policy with $10,000 face amount is issued to a life age 65. The net premium reserve at time 10 is $7,200; the net premium reserve at time 11 is $7,500. The effective annual rate of interest is 6%. Calculate the probability that a life age 75 will die within one year.

3. **(2 marks)** For a fully discrete whole life insurance of 1 on (55), you are given

$$\mu_{55+t} = \begin{cases} 0.01 & \text{for } t < 10 \\ 0.02 & \text{for } t \geq 10. \end{cases} \quad \text{and } i = 4\%.$$

   Calculate ${}_2V$ and ${}_{2.5}V$.

<u>Solution</u>:

1. By the recursion formula for net premium reserve we have

$$({}_9V + P)(1+i) = 10^5 q_{39} + {}_{10}V\, p_{39} \iff i = \frac{10^5(1 - p_{39}) + {}_{10}V\, p_{39}}{{}_9V + P} - 1$$

hence

$$i=\frac{10^5\left(1-\frac{\ell_{40}}{\ell_{39}}\right)+\ {}_{10}V\,\frac{\ell_{40}}{\ell_{39}}}{{}_9V+P}-1=\frac{10^5\left(1-\frac{112-1.4\times 40}{112-1.4\times 39}\right)+10059\frac{112-1.4\times 40}{112-1.4\times 39}}{8931+2738}-1=\mathbf{0.05002}.$$

Remark that $\frac{112-1.4\times 40}{112-1.4\times 39}=\frac{56}{57.4}=\frac{40}{41}$.

2. The required probability is $q_{75}=P\left(T_{75}\le 1\right)$ again by recursion

$$({}_{10}V+P_{10})(1+i)=10^4q_{75}+\ {}_{11}V\,p_{75}=\left(10^4-\ {}_{11}V\right)q_{75}+\ {}_{11}V$$

hence $q_{75}=\frac{({}_{10}V+P_{10})(1+i)-\ {}_{11}V}{10^4-\ {}_{11}V}=\frac{(7200)(1.06)-7500}{10^4-7500}=\mathbf{0.0528}$ (since $P_{10}=0$).

3. By prospective approach we have ${}_2V=\mathrm{APV}(F.B.)_2-\mathrm{APV}(F.P.)_2=A_{57}-P_{55}\ddot{a}_{57}$ where

$$P_{55}=\frac{A_{55}}{\ddot{a}_{55}}=\frac{1-d\ddot{a}_{55}}{\ddot{a}_{65}}=\frac{1}{\ddot{a}_{55}}-d.$$

Therefore

$${}_2V=A_{57}-\left(\frac{1}{\ddot{a}_{55}}-d\right)\ddot{a}_{57}=1-d\ddot{a}_{57}-\left(\frac{1}{\ddot{a}_{55}}-d\right)\ddot{a}_{67}=1-\frac{\ddot{a}_{57}}{\ddot{a}_{55}}.$$

Now,

$$\begin{aligned}
\ddot{a}_{55} &= \ddot{a}_{55:\overline{10}|}+{}_{10|}\ddot{a}_{65}=\ddot{a}_{55:\overline{10}|}+{}_{10}E_{55}\ddot{a}_{65}\\
&= \sum_{k=0}^{9}v^k\ {}_kp_{55}+v^{10}\ {}_{10}p_{55}\sum_{k=0}^{\infty}v^k\ {}_kp_{55}=\sum_{k=0}^{9}\frac{e^{-0.01k}}{(1.04)^k}+\frac{e^{-0.01\times 10}}{(1.04)^{10}}\sum_{k=0}^{\infty}\frac{e^{-0.02k}}{(1.04)^k}\\
&= \sum_{k=0}^{9}\left(\frac{e^{-0.01}}{1.04}\right)^k+\frac{e^{-0.01\times 10}}{(1.04)^{10}}\sum_{k=0}^{\infty}\left(\frac{e^{-0.02}}{1.04}\right)^k=8.0935+10.631=18.725
\end{aligned}$$

and

$$\begin{aligned}
\ddot{a}_{57} &= \ddot{a}_{57:\overline{8}|}+{}_{8|}\ddot{a}_{67}=\ddot{a}_{57:\overline{8}|}+{}_8E_{57}\ddot{a}_{65}\\
&= \sum_{k=0}^{7}v^k\ {}_kp_{57}+v^8\ {}_8p_{57}\sum_{k=0}^{\infty}v^k\ {}_kp_{65}=\sum_{k=0}^{7}\frac{e^{-0.01k}}{(1.04)^k}+\frac{e^{-0.01\times 8}}{(1.04)^8}\sum_{k=0}^{\infty}\frac{e^{-0.02k}}{(1.04)^k}\\
&= \sum_{k=0}^{7}\left(\frac{e^{-0.01}}{1.04}\right)^k+\frac{e^{-0.01\times 8}}{(1.04)^8}\sum_{k=0}^{\infty}\left(\frac{e^{-0.02}}{1.04}\right)^k=6.776\,9+11.73=18.507
\end{aligned}$$

Finally ${}_2V=1-\frac{18.507}{18.725}=\mathbf{0.011642}$.
By recursion formula we have $({}_2V+P_2)(1+i)^{0.5}=v^{0.5}\ {}_{0.5}q_{57}+\ {}_{2.5}V\ {}_{0.5}p_{57}$, then

$$\begin{aligned}
{}_{2.5}V &= \frac{({}_2V+P_2)(1+i)^{0.5}-v^{0.5}\ {}_{0.5}q_{57}}{{}_{0.5}p_{57}}=\frac{({}_2V+\frac{1}{\ddot{a}_{55}}-d)(1+i)^{0.5}-v^{0.5}\left(1-\ {}_{0.5}p_{57}\right)}{{}_{0.5}p_{57}}\\
&= \frac{({}_2V+\frac{1}{\ddot{a}_{55}}-d)(1+i)^{0.5}-v^{0.5}\left(1-p_{57}^{0.5}\right)}{p_{57}^{0.5}}\\
&= \frac{(0.011642+\frac{1}{18.725}-\frac{0.04}{1.04})(1.04)^{0.5}-(1.04)^{-0.5}\left(1-e^{-0.01\times 0.5}\right)}{e^{-0.01\times 0.5}}=\mathbf{0.022332}.
\end{aligned}$$