King Saud University **Academic Year (G) 2019–2020**
College of Sciences **Academic Year (H) 1441**
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Solution of Quiz 2 November 20, 2019 ACTU-462 from 4:45 to 6:15 PM

Quiz 2

1. You are given: (i) The future lifetimes of (40) and (50) are **independent**. (ii) The survival function for (40) is based on a constant force of mortality, $\mu = 0.05$. (iii) The survival function for (50) follows De Moivre's law with $\omega = 110$.
   Calculate the probability that (50) dies within 10 years and dies before (40).
   (Hint $_t q_{\frac{1}{x:y}} = \int_0^t \mathrm{P}\left(T_y > T_x \mid T_x = u\right) f_x(u)\ \mathrm{d}u = \int_0^t \mathrm{P}\left(T_y > u \mid T_x = u\right) f_x(u)\ \mathrm{d}u$)

Solution:

1. We need to compute $_{10}q_{\frac{1}{50:40}}$ which is given by definition and independence by

$$_{10}q_{\frac{1}{50:40}} = \int_0^{10} \mathrm{P}\left(T_{40} > u \mid T_{50} = u\right) f_{50}(u)\ \mathrm{d}u = \int_0^{10} {}_u p_{40}\ f_{50}(u)\ \mathrm{d}u$$

where

$$f_{50}(t) = \frac{1}{110 - 50} = \frac{1}{60} \text{ for } 0 < t < 60$$

Therefore,

$$_{10}q_{\frac{1}{50:40}} = \frac{1}{60} \int_0^{10} e^{-0.05t}\ \mathrm{d}t = \frac{1}{60} \frac{1 - e^{-0.5}}{0.05} = \mathbf{0.131156}.$$