Exercise 1 *You are given that losses follow a Pareto distribution with* $\alpha = 3$ *and* $\theta = 1,000$*. The company implements a franchise deductible so that the* $E(Y^P)$ *with the franchise deductible is* 130% *of* $E(X)$ *without any deductible. Calculate the franchise deductible.*

Exercise 2 *Losses follow a uniform distribution over the range of* 0 *to* 1000*. Calculate the Loss Elimination Ratio if an ordinary deductible of* 200 *is applied.*

Exercise 3 *Under an unmodified geometric distribution,* $Var(N) = 20$*. Under a zero-modified geometric distribution,* $Var(N) = 20.25$*. The parameter* β *is the same for both distributions. Calculate* p_0^M*.*

Exercise 4 *An insurance policy pays claims up to a limit of* 2000*. A random sample of three claim payments is obtained as follows:* $300, 1000,$ *and* 2000*. The claims amounts are assumed to follow a Uniform distribution on* $(0, \omega)$*. Based on the data, we can surmise that the claim amounts could be grouped as follows:*

Claim Amounts	*Count*
$0 - 2000$	2
$2000+$	1

Calculate the maximum likelihood estimate for ω*.*

Exercise 5 *You are given the following data which is assumed to be drawn from a uniform distribution over the range* 0 *to* ω*:*

Claim amount	***Count***
$0 - 200$	6
$200 - 500$	8
$500 - 1000$	12
$1000 - 5000$	14
5000 *and over*	10

a) Determine the maximum likelihood estimate of ω*.*
b) If you knew that the 10 largest claims ranged from 5150 *and* 8000*, what would be the maximum likelihood estimate of* ω*.*

Exercise 6 *One hundred laptop computers are observed for a period of* 12 *months. Thirty laptops malfunction during the observation period, with the following distribution:*

Time Till Malfunction in Months	*Number of Malfunctions*
1	8
2	6
3	0
4	0
5	1
6	0
7	1
8	2
9	2
10	3
11	3
12	4

The remaining seventy laptops are still functioning at the end of 12 *months. The lifetime of a laptop is believed to follow an exponential survival function with mean of* θ.
Calculate the maximum likelihood estimate of θ.

Exercise 7 *A Life Insurance Company is completing a mortality study on a 4 year term insurance policy. Use a Kaplan-Meier product-limit estimator to approximate* $S_{25}(2.5)$.
The following Table 1 is available:

Table 1

Life	Date of Entry	Date of Exit	Reason for Exit
1	0	0.2	Lapse
2	0	0.3	Lapse
3	0	0.5	Lapse
4	0	0.5	Death
5	0	0.7	Lapse
6	0	1.0	Death
7	0	2.0	Lapse
8	0	2.5	Death
9	0	3.0	Lapse
10	0	3.5	Death
11	0	4.0	Expiry of Policy
12	0	4.0	Expiry of Policy
13	0	4.0	Expiry of Policy
14	0	4.0	Expiry of Policy
15	0	4.0	Expiry of Policy
16	0	4.0	Expiry of Policy
17	0	4.0	Expiry of Policy
18	0	4.0	Expiry of Policy
19	0.5	4.0	Expiry of Policy
20	0.7	1.0	Death
21	1.0	3.0	Death
22	1.0	4.0	Expiry of Policy
23	2.0	2.5	Death
24	2.0	2.5	Lapse
25	3.0	3.5	Death

loss HW 2

Ex 1 $X \sim \text{Pareto}(\alpha = 3, \theta = 1000)$.

$$E(Y^P) = \frac{E(X) - E(X \wedge d)}{P(X > d)} + d \qquad E(X) = \frac{\theta}{\alpha - 1} = 500$$

$$= \frac{\frac{\theta}{\alpha-1}\left(\frac{\theta}{d+\theta}\right)^{\alpha-1}}{\left(\frac{\theta}{d+\theta}\right)^{\alpha}} + d = \frac{\frac{\theta}{\alpha-1}}{\left(\frac{\theta}{d+\theta}\right)} + d = \frac{d+\theta}{\alpha-1} + d$$

$$= \frac{d+1000}{2} + d = \frac{3d+1000}{2} = 1.3\,(500)$$

$$\Leftrightarrow 3d + 1000 = 1.3 \times 1000 = 1300$$

$$\Leftrightarrow 3d = 300 \Rightarrow \boxed{d = 100}$$

Ex 2 $X \sim \mathcal{U}(0, w)$ $\quad f(x) = 1/w$, $w = 1000$

$$LER = \frac{E(X) - E(Y^L)}{E(X)} = \frac{E(X \wedge d)}{E(X)}, \quad d = 200$$

$$X \wedge d = \begin{cases} X & x \le d \\ d & x > d \end{cases}$$

$E(X) = w/2 = 500.$

$$E(X \wedge d) = \int_0^{200} \frac{x}{1000}\,dx + 200\left[\frac{1000-200}{1000}\right]$$

$$= \frac{1}{1000}\left[\frac{200^2}{2}\right] + 200\left[8/10\right] = 180$$

$$LER = \frac{180}{500} = \boxed{0.36}$$

Ex 3 $N \sim \text{Geom.}(\beta)$. $V(N) = 20$ $\quad p_0 = (1+\beta)^{-1}$

$N^{(M)}$ zero modif. $\Rightarrow$ $p_k^{(M)} = \frac{1 - p_0^{M}}{1 - p_0} p_k$.

$V(N) = \beta(1+\beta) = 20 \Rightarrow \beta = 4$.

$$E(N^{(M)}) = \sum_{1}^{\infty} k\, p_k^{(M)} = \frac{1-p_0^M}{1-p_0} E(N) = a\,E(N) = a\beta = 4a$$

$$E(N^{(M)2}) = \frac{1-p_0^M}{1-p_0} E(N^2) = a\,E(N^2)$$

$$V(N^{(M)}) = a\,\underbrace{E(N^2)}_{E(N)^2 + V(N)} - (4a)^2 = a\left[4^2 + 20\right] - 16a^2$$

$$= 36a - 16a^2 = 20.25$$

$$\Leftrightarrow 16a^2 - 36a + 20.25 = 0 \Leftrightarrow (4a - 4.5)^2 = 0$$

$$\Rightarrow a = \frac{4.5}{4} = \frac{1-p_0^M}{1-p_0} \Rightarrow p_0^M = 1 - a(1-p_0)$$

$$= 1 - \frac{4.5}{4}\left(1 - \frac{1}{5}\right) = \boxed{0.1}$$

Ex 4

$$L(w|x) = [F(2000)]^2 \times S(2000)$$
$$= \left(\frac{1}{w}\right)^2 \left(\frac{w-2000}{w}\right) = \frac{w-2000}{w^3}$$

$$\ell(w|x) = \log(w-2000) - 3\log(w).$$

$$\frac{d\ell}{dw} = \frac{1}{w-2000} - \frac{3}{w} = 0 \iff w = 3(w-2000)$$
$$\iff 2w = 3 \times 2000 \iff \hat{w} = 3000$$

$$\frac{d^2\ell}{dw^2} = -\frac{1}{(w-2000)^2} + \frac{3}{w^2} = \frac{1}{w^2}\left[3 - \left(\frac{w}{1000}\right)^2\right]$$
$$< 0 \text{ for } w = \hat{w}.$$

Ex 5
(a) $X \sim U[0,w)$, $w > 5000$. $f(w) = \frac{1}{w}$.

$$L(w|x) = \left(\frac{200}{w}\right)^6 \left(\frac{300}{w}\right)^8 \left(\frac{500}{w}\right)^{12} \left(\frac{4000}{w}\right)^{14} \left(\frac{w-5000}{w}\right)^{10}$$
$$= K w^{-50} (w-5000)^{10}$$

$$\ell(w|x) = C - 50\log(w) + 10\log(w-5000)$$

$$\frac{d\ell}{dw} = -\frac{50}{w} + \frac{10}{w-5000} = 0 \iff \frac{50}{w} = \frac{10}{w-5000}$$
$$\iff 50(w-5000) = 10w \iff 4w = 50 \times 5000$$
$$\iff w = \frac{5}{4} \times 5000 = 6{,}250$$

(b) $x = 8000$ is necessarily is the support of the uniform$[0,w]$ which is $(0,w]$. Hence $w \geq 8000$.
The MLE $\hat{w}$ is the solution of the program

$$(1)\begin{cases} \max L(w|x) \\ w \geq 8000 \end{cases}$$

$$L(w|x) = \left(\frac{200}{w}\right)^6 \left(\frac{300}{w}\right)^8 \left(\frac{500}{w}\right)^{12} \left(\frac{4000}{w}\right)^{14} \left(\frac{8000-5000}{w}\right)^{10}$$
$$= \frac{K}{w^{50}}$$ which is a decreasing function of w

The solution of the program (1) is then $\hat{w} = 8000$.

Ex 6

$$f(x) = \frac{1}{\beta} e^{-x/\beta} \qquad S(x) = e^{-x/\beta}$$

$$L(\beta|x) = \left[\prod \left(\frac{1}{\beta} e^{-x_i/\beta}\right)^{n_i}\right]\left(e^{-12/\beta}\right)^{70}$$

$$= \left(\frac{1}{\beta} e^{-1/\beta}\right)^{8} \left(\frac{1}{\beta} e^{-2/\beta}\right)^{6} \left(\frac{1}{\beta} e^{-5/\beta}\right)^{1}$$

$$\left(\frac{1}{\beta} e^{-7/\beta}\right)^{1} \left(\frac{1}{\beta} e^{-8/\beta}\right)^{2} \left(\frac{1}{\beta} e^{-9/\beta}\right)^{2}$$

$$\left(\frac{1}{\beta} e^{-10/\beta}\right)^{3} \left(\frac{1}{\beta} e^{-11/\beta}\right)^{3} \left(\frac{1}{\beta} e^{-12/\beta}\right)^{4} \left(e^{-12/\beta}\right)^{70}$$

$$= \beta^{-\sum n_i} e^{-[\sum n_i x_i + 70 \times 12]/\beta}$$

$$\ell(\beta|x) = -(\sum n_i)\log\beta - \frac{\sum n_i x_i + 70 \times 12}{\beta}$$

$$\frac{d\ell}{d\beta} = -\frac{\sum n_i}{\beta} + \frac{\sum n_i x_i + 70 \times 12}{\beta^2} = 0$$

$$\Leftrightarrow \hat{\beta} = \frac{\sum n_i x_i + 70 \times 12}{\sum n_i} = 33.9$$

$$\frac{d^2\ell}{d\beta^2} = \frac{\sum n_i}{\beta^2} - \frac{2}{\beta^3}\left[\sum n_i x_i + 70 \times 12\right]$$

$$= \frac{1}{\beta^3}\left[(\sum n_i)\beta - 2(\sum n_i x_i + 70 \times 12)\right]$$

$$\frac{d^2\ell}{d\beta^2}(\beta = \hat{\beta}) = \frac{1}{\hat{\beta}^3}\left[\underbrace{(\sum n_i)\hat{\beta} - 2(\sum n_i)\hat{\beta}}_{<0}\right] < 0$$

Ex 7

d	x	u
0	—	0.2
0	—	0.3
0	—	0.5
1	0.5	—
1	—	0.7
1	1	—
1	—	2
1	2.5	—
1	—	3
1	3.5	—
1	—	4
1	—	4
1	—	4
1	—	4
1	—	4
1	—	4
1	—	4
1	—	4
1	1	—
1	3	—
1	—	4
1	2.5	—
1	—	2.5
0	3.5	—

j	y_j	s_j	r_j
1	0.5	1	$8+15-7=16$
2	1	2	$7+13-5=15$
3	2.5	2	$5+12-1=16$
4	3	1	"
5	3.5	2	"

$$S_{25}(2.5) = \prod_{j=1}^{3} \left(1 - \frac{s_j}{r_j}\right)$$

$$= \left(1 - 1/16\right)\left(1 - 2/15\right)\left(1 - 2/15\right)$$

$$= 0.7109$$