Dr Bahen

Answer

King Saud University	Midterm Exam Math 218	*Department of Mathematics*
Name: ID:	Semester I (1443)	Time: 2H

Calculators are not allowed

Question	I(5 marks)	II (12 marks)	III (8 marks)	IV(5 marks)	Total
Grade					

Question	1	2	3	4	5	6	7	8	9	10
Answer	b	a	d	c	d	b	a	a	c	b

I) Choose the correct answer (write it in the table above):

1) $\frac{\sqrt{6}}{\sqrt{2}}$ equals

(a) $\sqrt{2}$	(b) $\sqrt{3}$	(c) $3\sqrt{2}$	(d) $\frac{1}{2}$

2) $(2x-3)^2$ equals

(a) $4x^2-12x+9$	(b) $4x^2-6x+9$	(c) $4x^2+12x+9$	(d) $2x^2+6x+9$

3) The distance between -5 and 4 on the real number line is

(a) 1	(b) 5	(c) 6	(d) 9

4) $(1-\sqrt{-1})(2+\sqrt{-1})$ equals

(a) $3+i$	(b) 2	(c) $3-i$	(d) $2-i$

5) The solutions of $|x+2|<4$ are

(a) $x\in(-4,-4)$	(b) $x\in(-\infty,4)$	(c) $x\in(-6,4)$	(d) $x\in(-6,2)$

6) The reminder when $P(x) = x^3 - 2x^2 + 4$ is divided by $x - 2$ is

(a) -8	(b) 4	(c) $x - 1$	(d) 8

7) The domain of the function $f(t) = \ln(4 - 2t)$ is

(a) $(-\infty, 2)$	(b) $[2, \infty)$	(c) $(2, \infty)$	(d) $(-\infty, 2]$

8) $\log_2 80 - \log_2 10$ equals

(a) 3	(b) $\sqrt{3}$	(c) $\log_2 70$	(d) 4

9) $\cos(\frac{7\pi}{6})$ equals

(a) $\frac{\sqrt{3}}{2}$	(b) $\frac{1}{2}$	(c) $-\frac{\sqrt{3}}{2}$	(d) $\frac{\sqrt{2}}{2}$

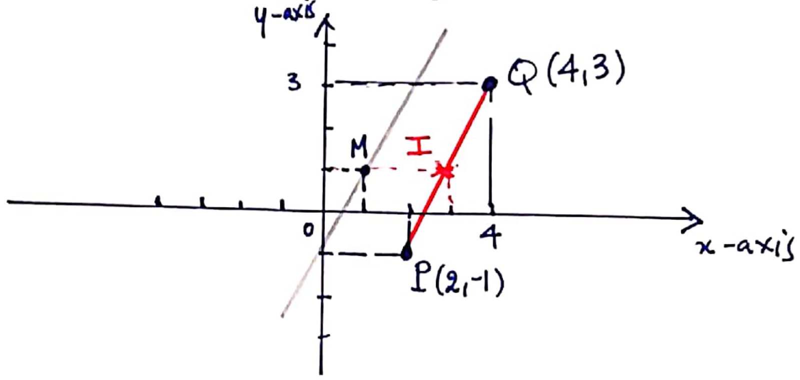
10) $\sin^{-1}(\sin \frac{13\pi}{6})$ equals

(a) $\frac{13\pi}{6}$	(b) $\frac{\pi}{6}$	(c) $\frac{7\pi}{6}$	(d) $\frac{\pi}{3}$

II) A) Let $P(2,-1)$ and $Q(4,3)$ be two points in the coordinate plane.

i) Plot P and Q in the coordinate plane.

1



ii) Find the distance between P and Q.

1

$$dist(P,Q) = \sqrt{(x_Q - x_P)^2 + (y_Q - y_P)^2}$$

$$= \sqrt{(4-2)^2 + (3-(-1))^2}$$

$$= \sqrt{4 + 16} = \sqrt{20} = 2\sqrt{5}.$$

iii) Find the midpoint of the segment PQ.

1

- let I the midpoint of [P,Q]

$$I\left(x_I = \frac{x_P + x_Q}{2}, \; y_I = \frac{y_P + y_Q}{2}\right)$$

$$I(3, 1)$$

iv) Find the slope of the line that contains P and Q.

1

- The slope of (PQ) is $m = \dfrac{y_Q - y_P}{x_Q - x_P} = \dfrac{3-(-1)}{4-2} = \dfrac{4}{2} = 2$

v) Find the equation of the line that passes through the point $M(1,1)$ and is parallel to PQ.

2

- The equation of the line is:

$$y - 1 = 2(x-1)$$

$$y - 1 = 2x - 2$$

$$\boxed{y = 2x - 1}$$

B) Let $f(x) = \dfrac{2\sqrt{x}}{x+5}$ and $g(x) = x^2 - 4x - 5$.

i) Find $f(0)$ and $f(4)$.

1

- $f(0) = 0$
- $f(4) = \dfrac{2 \cdot 2}{4+5} = \dfrac{4}{9}$

ii) Find the domain of f.

1

$D_f = \{ x \in \mathbb{R} \,/\, x \geq 0 \text{ and } x \neq -5 \}$

$D_f = [0, \infty)$

iii) Find all the solutions of equation $g(x) = 0$.

2

$$g(x) = 0 \iff \underbrace{x^2 - 4x}\, - 5 = 0$$
$$\underbrace{(x-2)^2 - 4} - 5 = 0$$
$$(x-2)^2 - 3^2 = 0$$
$$(x-2-3)(x-2+3) = 0$$
$$x = 5 \text{ or } x = -1$$

iv) Determine whether $g(x)$ has a maximum or a minimum and find this value.

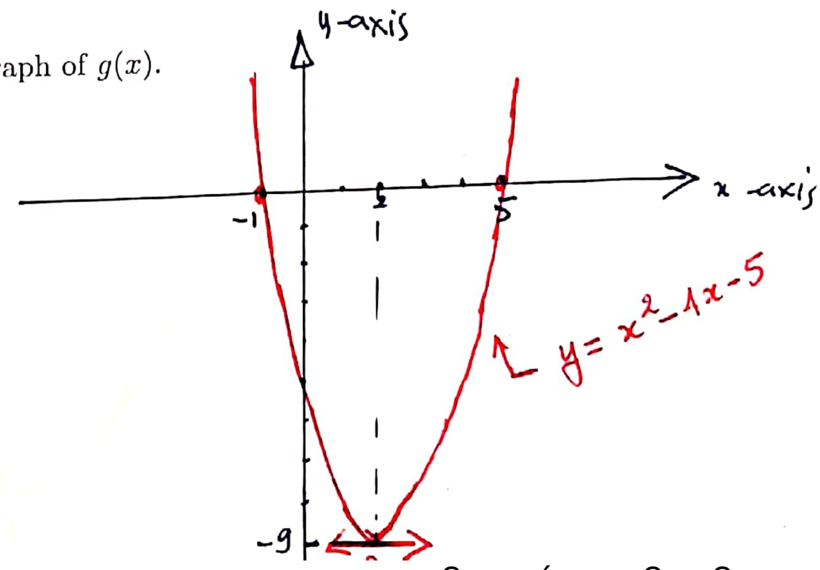
1

$g(x) = x^2 - 4x - 5 = (x-2)^2 - 9$

g has a minimum at $P(2, -9)$

v) Sketch the graph of $g(x)$.

1



III) A) Let $P(x) = 2x^3 - x^2 - 2x + 1$.

i) List all possible rational zeros of $P(x)$.

$x = \frac{p}{q}$ p is a factor of 1
q is " " of 2

So $x \in \{\pm 1 ; \pm 1/2\}$

$P(1) = 2 - 1 - 2 + 1 = 0$ So $x = 1$ is a zero of P.
$P(-1) = -2 - 1 + 2 + 1 = 0$ So $x = -1$ is a zero of P.
$P(1/2) = 2/8 - 1/4 - 2 \cdot 1/2 + 1 = 0$ So $x = 1/2$ is a zero of P.

ii) Find the complete factorization of $P(x)$.

From i) we deduce that

$$P(x) = 2\left(x - \frac{1}{2}\right)(x-1)(x+1)$$

3

iii) Find all zeros of $P(x)$.

$$P(x) = 0 \iff x \in \left\{-1, \frac{1}{2}, 1\right\}$$

1

iv) Use long division to find the quotient and the reminder when $P(x)$ is divided by $x^2 - x + 1$.

$$\boxed{\frac{P(x)}{x^2-x+1} = Q(x) + \frac{R(x)}{x^2-x+1}}$$ with $Q(x)$ is the quotient
$R(x)$ is the remainder

3

$$\begin{array}{r|l} 2x^3 - x^2 - 2x + 1 & x^2 - x + 1 \\ \cline{2-2} -2x^3 + 2x^2 - 2x & 2x+1 \\ \hline x^2 - 4x + 1 & \\ -x^2 + x - 1 & \\ \hline -3x & \end{array}$$

So $Q(x) = 2x+1$ (quotient)
$R(x) = -3x$

$$\frac{2x^3 - x^2 - 2x + 1}{x^2 - x + 1} = (2x+1) - \frac{3x}{x^2-x+1}$$

for every $x \in \mathbb{R}$

B) Find a fourth-degree polynomial with integer coefficients that has zeros 1 and -1, with -1 a zero of multiplicity 3.

$$P(x) = a\,(x+1)^3(x-1)$$ with a is an integer nonzero.

1

IV) A) Solve the following equations:

i) $\ln(2x+1) - \ln 5 = \ln(x-4)$.

$D_E = \left\{ x \in \mathbb{R} \,/\, \begin{matrix} 2x+1>0 \\ x-4>0 \end{matrix} \right\} = \{ x \in \mathbb{R} \,/\, x>4 \} = (4,\infty)$

let $x \in DE$

$\ln(2x+1) - \ln 5 = \ln\left(\frac{2x+1}{5}\right) = \ln(x-4)$

So $\frac{2x+1}{5} = x-4$

2

$2x+1 = 5x-20$

$21 = 3x$ So $x = 7 \in D_E$

unique solution $\{x=7\}$.

ii) $3^{2x} - 3^x - 2 = 0$ (Hint: denote $y = 3^x$).

⚠ $3^{2x} = (3^x)^2$

if $y = 3^x > 0$, the equation $3^{2x} - 3^x - 2 = 0$ becomes

$y^2 - y - 2 = 0$

$(y+1)(y-2) = 0$

2

$y = -1$ ~~y~~ or $y = 2$ because $y>0$

So $3^x = 2 \Leftrightarrow x = \log_3(2) = \frac{\ln 2}{\ln 3}$

B) If $\cos t = \frac{4}{5}$ and if the terminal point determined by t is in quadrant I, find $\tan t \cot t + \sin t$.

- As t in quadrant I then $t \in [0, \pi/2]$

Also $\cot t = \frac{1}{\tan t}$ then

1

$\tan t \cdot \cot t + \sin t = 1 + \sin t$

We have $\cos t = 4/5$ then $\sin t = \sqrt{1-\cos^2 t} = \sqrt{1-\frac{16}{25}} = \frac{3}{5}$

So $\tan t \cot t + \sin t = 1 + 3/5 = \frac{8}{5}$.