Answer

جامعة الملك سعود
King Saud University

Name:.....
ID:.....
Num of attendance:.....

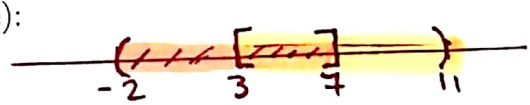
Time: 1h 30 min **Midterm 1 - Math 218- Semester II- 1443 H** Marks: 25

Question	1	2	3	4	5	6	7	8	9	10
Answer	B	D	A	B	C	D	A	B	D	A

1 × 10 = 10 Marks

I) Choose the correct answer (write it in the table above):

1) $(-2, 7] \cap [3, 11) =$



(a) $(-2, 11)$	(b) $[3, 7]$	(c) $(-2, 3] \cup [7, 11)$	(d) $[7, 11)$

2) $\dfrac{(x^{-2}y)^{-3}}{x^2y^{-4}} =$ $x^6 y^{-3} x^{-2} y^4 = x^4 y$

(a) $x^{-7}y$	(b) $x^{-1}y^{-7}$	(c) x	(d) x^4y

3) The domain of $\dfrac{1}{\sqrt{4-2x}}$ is $D = \{x \in \mathbb{R} / \ 4-2x > 0\} = \{x \in \mathbb{R} / \ 4 > 2x\} = \{x \in \mathbb{R} / \ 2 > x\}$

(a) $(-\infty, 2)$	(b) $(-\infty, 2]$	(c) $(2, \infty)$	(d) $[2, \infty)$

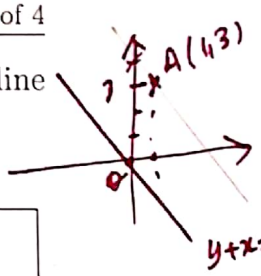
4) $\dfrac{1 + \dfrac{1}{x(x-2)}}{\dfrac{x-1}{x-2}}$ equals $\dfrac{(x-2)}{(x-1)}\left[1 + \dfrac{1}{x(x-2)}\right] = \dfrac{x-2}{x-1} + \dfrac{1}{x(x-1)} = \dfrac{x(x-2)+1}{x(x-1)} = \dfrac{x^2-2x+1}{x(x-1)}$

$= \dfrac{(x-1)^2}{x(x-1)} = \dfrac{x-1}{x}$

(a) $\dfrac{x}{x-1}$	(b) $\dfrac{x-1}{x}$	(c) 1	(d) x

Please go on to the next page...

5) The equation of the line passing through the point $A(1,3)$ and parallel to the line $y+x=0$ is



(a) $y=x+2$	(b) $y=-2x+5$	(c) $y+x-4=0$	(d) $2y-3x-3=0$

$y-3=-(x-1)$
$y-3=-x+1$
$y+x-4=0$

6) The center of the circle with equation $(x-5)^2+(y+2)^2=49$ is the point

$(x-x_0)^2+(y-y_0)^2=R^2 \rightarrow x_0=5 \quad R=7$
$y_0=-2$

(a) $C(-5,2)$	(b) $C(2,-5)$	(c) $C(-2,5)$	(d) $C(5,-2)$

7) If $z_1=3+i$ and $z_2=2-i$, then z_1z_2 equals

$z_1z_2=(3+i)(2-i)$
$=6-3i+2i-i^2=7-i$

(a) $7-i$	(b) $7+i$	(c) $5-i$	(d) $5+i$

8) The solution of equation $4-3x=-2(x+1)$ is

$4-3x=-2x-2$
$4+2=3x-2x \Leftrightarrow x=6$

(a) $x=3$	(b) $x=6$	(c) $x=-6$	(d) $x=2$

9) The solution of $|5-2x|<3$ is

$-3<5-2x<3$
$-8<-2x<-2 \Leftrightarrow 1<x<4$

(a) $x\in(-8,-2)$	(b) $x\in(-3,3)$	(c) $x\in(-4,-1)$	(d) $x\in(1,4)$

10) The remainder when $P(x)=x^3-2x^2+4$ is divided by $x+1$ is $P(-1)=1$

(a) $x=1$	(b) $x=2$	(c) $x=3$	(d) $x=7$

$x^3-2x^2+4=(x+1)(x^2-3x+3)+1$

II) A) Find all the solutions (real or complex) of the equations:

i) $x^2 - 4x + 5 = 0$

$a = 1$; $b = -4$; $c = 5$

[1Mark]

The discriminant $D = b^2 - 4ac = 16 - 4.1.5 = -4 = (2i)^2$

$$x_1 = \frac{-b - \sqrt{D}}{2a} = \frac{4 - 2i}{2} = 2 - i$$

$$x_2 = \frac{-b + \sqrt{D}}{2a} = 2 + i$$

ii) $\frac{x+4}{x-1} = \frac{3}{x+1} + \frac{15}{x^2-1}$. $D_E = \mathbb{R} \setminus \{\pm 1\}$

Let $x \in \mathbb{R} \setminus \{\pm 1\}$

[2Marks]

$$\frac{(x+4)(x+1)}{x^2-1} = \frac{3(x-1)}{x^2-1} + \frac{15}{x^2-1} \Leftrightarrow (x+4)(x+1) = 3(x-1)+15$$

$$\Leftrightarrow x^2 + 5x + 4 - 3x + 3 - 15 = 0$$

$$\Leftrightarrow x^2 + 2x - 8 = 0$$

$$\Leftrightarrow (x-2)(x+4) = 0$$

So $x = 2 \in D_E$ or $x = -4 \in D_E$

$S_{\mathbb{R}} = \{2, -4\}$

B) Solve the inequality $\frac{(2-x)(x+1)}{x-3} \leq 0$.

[2Marks]

x	-1		2		3	
$x+1$	$-$	0	$+$	$+$	$+$	
$2-x$	$+$		$+$	0	$-$	$-$
$x-3$	$-$		$-$		$-$	0 $+$
Q	$+$	0	$-$	0	$+$	$\|\|$ $-$

$$S = [-1, 2] \cup (3, +\infty)$$

C) Let $z = 1 + i\sqrt{3}$, compute $|z|$, $\bar{z}$ and z^2.

$z = x + iy$ with $x = 1$ & $y = \sqrt{3}$

[2 Marks]

- $|z| = \sqrt{x^2+y^2} = \sqrt{1+3} = 2$
- $\bar{z} = x - iy = 1 - i\sqrt{3}$
- $z^2 = (1+i\sqrt{3})^2 = 1^2 + 2i\sqrt{3} + (i\sqrt{3})^2 = -2 + 2\sqrt{3}\,i$

D) Find the coordinates of the center and the radius of the circle with equation $x^2 + y^2 - 2x - 4y + 1 = 0$

[3 Marks]

$(x^2-2x) + (y^2-4y) + 1 = 0$

$[(x-1)^2 - 1] + [(y-2)^2 - 2^2] + 1 = 0$

$(x-1)^2 + (y-2)^2 = 2^2$

It's an eq of a circle of center C(1, 2) and radius R

E) Use long division to find the quotient and the remainder when $P(x) = 2x^4 - 3x^2 + 5x - 2$ is divided by $x^2 + x + 1$.

[3 Marks]

- The quotient is $(2x^2 - 2x - 3)$
- The remainder is $(10x + 1)$

$P(x) = (x^2+x+1)(2x^2-2x-3) + (10x+1)$

$2x^4 - 3x^2 + 5x - 2 \;\big|\; x^2 + x + 1$

$-(2x^4 - 2x^3 - 2x^2) \;\big|\; 2x^2 - 2x - 3$

$-2x^3 - 5x^2 + 5x - 2$

$-(2x^3 - 2x^2 + 2x)$

$-3x^2 + 7x - 2$

$-(3x^2 + 3x + 3)$

$10x + 1$

F) Find a polynomial of degree 4 that has zeros (-2), 0, 1 and 2 and the coefficient of x^3 is (-8).

[2 Marks]

As (-2), 0, 1 & 2 are zeroes of P then

$P(x) = a\,x\,(x-1)(x-2)(x+2)$

$= a\,(x^2-x)(x^2-4)$

$= a\,[x^4 - 4x^2 - x^3 + 4x]$

$= a\,x^4 - a\,x^3 - 4a\,x^2 + 4a\,x$

As the coefficient of x^3 is (-8) then $a = 8$

We deduce that $\boxed{P(x) = 8x(x-1)(x^2-4)}$

End of exam