

STAT324 HW based on chapter 8

Question No. 1

The average life of an industrial machine is 6 years, with a standard deviation of 1 year. If a random sample of 4 of such machines is selected and assumes that the life of such machines follows approximately a normal distribution, then:

- (a) Find the expected value of the samples mean (\bar{X}).
- (b) Find the variance of the samples mean (\bar{X}).
- (c) Find the $P(\bar{X} < 5.5)$
- (d) If $P(\bar{X} > a) = 0.1492$, find the numerical value of a .

Question No. 2

A random sample of size 25 is taken from a normal population having a mean of 80 and a standard deviation of 5. A second independent random sample of size 36 is taken from a different normal population having a mean of 75 and a standard deviation of 3, then,

- (a) Find $P(\bar{X}_1 - \bar{X}_2 < 2)$.
- (b) Find $P(1.5 < \bar{X}_1 - \bar{X}_2 < 2)$.

Question No. 3

A certain machine makes electrical resistors that have an average resistance of 100 (ohms) and a standard deviation of 36 (ohms). If a random sample of size 36 resistors are drawn from the product of this machine, then:

- (a) Find the probability that the average resistance of the 36 resistors will be less than 91(ohms)
- (b) Find the probability that the average resistance of the 36 resistors will be between 95 and 105 (ohms).

Question No. 4

Suppose that you take a random sample of size $n=75$ from a binomial population with parameter $p=0.18$ (proportion of successes). Let $\hat{p} = X/n$ be the sample proportion of successes, where X is the number of successes in the sample.

- (a) Find the mean of \hat{p} .
- (b) Find the standard error (standard deviation) of \hat{p} .
- (c) Find the probability that the sample proportion \hat{p} is less than 0.2.

Question No. 5

Suppose that 30% of the male students and 18% of the female students in a certain university smoke cigarettes. A random sample of 7 male students is taken. Another random sample of 12 female students is independently taken from this university. Let \hat{p}_1 and \hat{p}_2 be the proportions of smokers in the two samples, respectively.

- (a) Find the mean of $\hat{p}_1 - \hat{p}_2$.
- (b) Find the variance of $\hat{p}_1 - \hat{p}_2$.
- (c) Find $P(0.10 < \hat{p}_1 - \hat{p}_2 < 0.20)$.