MMSE Detector for Narrowband Interference Suppression in DS Spread Spectrum

Leslie A. Rusch
Department of Electrical Engineering, Université Laval

Abstract – This research addresses narrowband interference suppression in a direct sequence spread spectrum (DSSS) overlay system. We assume a digital signal for the narrowband interferer and use multiuser detection theory to find the receiver that minimizes the mean square error in the spread spectrum bit estimate. This detector is of interest as it can be implemented adaptively without explicit knowledge of the narrowband interferer's signal waveform, timing and energy. We treat only the synchronous case.

Summary

In [1] Rusch and Poor first used multiuser detection theory to find a receiver that incorporates narrowband interference suppression in the estimation of the spread spectrum bit. Instead of the conventional matched filter detector, another linear detector was proposed based on the decorrelating detector [2]. To calculate the settings for this detector we require knowledge of the timing of the binary narrowband interference signal, the bit waveform, and the power. While estimation of these parameters should be no more difficult for the spread spectrum detector than for the intended receiver, it nonetheless complicates the receiver structure.

In [3] Honig, *et al.* develop a general expression for the receiver which minimizes the mean square error (MMSE) between the outputs and the data. This linear detector is of interest as an adaptive version can be implemented which does not require knowledge of the narrowband signal characteristics. We take the general expression for the MMSE receiver and apply it to the narrowband suppression problem, arriving at a closed-form solution for the MMSE detector and its probability of error.

A Virtual CDMA System

As described in [1], we assume the interferer is a binary signal with m narrowband bits in one SS bit. These m bits are interpreted as m mutually orthogonal code division multiple access (CDMA) signals, each with a "signature waveform" that is zero outside a particular bit interval. The bit waveform of the narrowband signal during the i^{th} narrowband bit interval is $s_i(t)$, which again is interpreted to be the i^{th} CDMA code. The spread spectrum bit waveform is $s_m(t)$. All bit waveforms are normalized to have unit power. The power of the narrowband signal is w_1, and the power of the SS bit is w_2. The cross correlation between the SS signal and the i^{th} narrowband bit is ρ_i.

$$\rho_i = \int_0^T s_i(t) s_m(t) dt$$

The vector $\underline{\rho}$ is formed from these m cross correlations. The cross correlation matrix is

$$[R]_{i,j} = \int_0^T s_i(t) s_j(t) dt \tag{1}$$

and the matrix of user powers is

$$W = diag(w_1, w_1, \cdots, w_1, w_2) \tag{2}$$

MMSE Detector and Probability of Error

Per ref [3], the multiuser detector which minimizes the MSE is determined by the inverse of the matrix $R+\sigma^2 W^{-1}$. For our system this inverse can be found explicitly as a function of the power and cross correlation vector $\underline{\rho}$.

$$\begin{aligned}\left[R+\sigma^2 W^{-1}\right]^{-1} &= \begin{bmatrix} I_m\left(1+\sigma^2/w_1\right) & \underline{\rho} \\ \underline{\rho}^T & 1+\sigma^2/w_2 \end{bmatrix}^{-1} \\ &== \frac{1}{1+\sigma^2/w_1}\begin{bmatrix} I_m + \dfrac{\underline{\rho}\underline{\rho}^T}{\left(1+\sigma^2/w_1\right)\left(1+\sigma^2/w_2\right)-\underline{\rho}^T\underline{\rho}} & \dfrac{-\underline{\rho}\cdot\left(1+\sigma^2/w_1\right)}{\left(1+\sigma^2/w_1\right)\left(1+\sigma^2/w_2\right)-\underline{\rho}^T\underline{\rho}} \\ \dfrac{-\underline{\rho}^T\cdot\left(1+\sigma^2/w_1\right)}{\left(1+\sigma^2/w_1\right)\left(1+\sigma^2/w_2\right)-\underline{\rho}^T\underline{\rho}} & \dfrac{\left(1+\sigma^2/w_1\right)^2}{\left(1+\sigma^2/w_1\right)\left(1+\sigma^2/w_2\right)-\underline{\rho}^T\underline{\rho}} \end{bmatrix}\end{aligned} \tag{3}$$

where σ^2 is the additive white Gaussian noise power. For our system, only the true SS bit is of interest; it is only this bit we wish to estimate. In our ordering, the SS bit is the last of the vector of "virtual" CDMA bits. Therefore we focus our efforts on the last row of the matrix in (3),

$$c_m = \frac{1}{\left(1+\sigma^2/w_1\right)\left(1+\sigma^2/w_2\right)-\underline{\rho}^T\underline{\rho}}\left[-\underline{\rho}^T \quad 1+\sigma^2/w_1\right]^T \tag{4}$$

It is this row that we use to generate the MMSE estimate of the SS bit using

$$c_m(t) = \sum_{i=0}^{m} c_{m,i} s_i(t)$$

$$\hat{b}_{SS} = \operatorname{sgn}\left[\int_0^T c_m(t) y(t) dt\right]$$

To find the canonical form [3] of this receiver, we normalize to arrive at a receiver whose inner product with the SS waveform has unit length. The filter then takes the form

$$c_{MMSE}(t) = \frac{1}{1+\sigma^2/w_1-\underline{\rho}^T\underline{\rho}}\cdot\left[\left(1+\sigma^2/w_1\right)s_m(t)-\sum_{i=0}^{m-1}\rho_i s_i(t)\right] \tag{5}$$

In order to determine the probability of error we examine the decision statistic for the SS bit. For convenience we reference all powers to the desired user power, w_2.

Therefore the decision statistic is

$$
\begin{aligned}
D.S. &= \int_0^T c_{MMSE}(t)\frac{y(t)}{\sqrt{w_2}}\,dt \\
&= \frac{1+\sigma'^2/w_1}{1+\frac{\sigma'^2}{w_2}\frac{w_2}{w_1}-\underline{\rho}^T\underline{\rho}}\int_0^T s_m(t)\frac{y(t)}{\sqrt{w_2}}\,dt-\frac{1}{1+\frac{\sigma'^2}{w_2}\frac{w_2}{w_1}-\underline{\rho}^T\underline{\rho}}\sum_{i=0}^{m-1}\rho_i\int_0^T s_i(t)\frac{y(t)}{\sqrt{w_2}}\,dt \\
&= \frac{1+\sigma^2/\alpha^2}{1+\frac{\sigma^2}{\alpha^2}-\underline{\rho}^T\underline{\rho}}\int_0^T s_m(t)\frac{y(t)}{\sqrt{w_2}}\,dt-\frac{1}{1+\frac{\sigma^2}{\alpha^2}-\underline{\rho}^T\underline{\rho}}\sum_{i=0}^{m-1}\rho_i\int_0^T s_i(t)\frac{y(t)}{\sqrt{w_2}}\,dt
\end{aligned}
$$

where α is known as the near far ratio, and σ is the signal-to-Gaussian-noise ratio for the desired user. The received signal, $y(t)$, is the sum of the SS signal and the narrowband user and the additive white Gaussian noise:

$$
\begin{aligned}
\frac{y(t)}{\sqrt{w_2}} &= \frac{1}{\sqrt{w_2}}\left[\sqrt{w_1}\sum_{i=0}^{m-1}b_is_i(t)+\sqrt{w_2}\,b_ms_m(t)+\sigma'n(t)\right] \\
&= \alpha\sum_{i=0}^{m-1}b_is_i(t)+b_ms_m(t)+\sigma n(t)
\end{aligned}
$$

where $n(t)$ is a zero mean, unit variance white Gaussian noise process. The first integral in the decision statistic is given by

$$
\begin{aligned}
&\alpha\sum_{i=0}^{m-1}b_i\int_0^T s_i(t)s_m(t)dt+b_m\int_0^T s_m^2(t)dt+\sigma\int_0^T s_m(t)n(t)dt \\
&= \alpha\sum_{i=0}^{m-1}b_i\rho_i+b_m+\sigma n_m=\alpha\underline{b}^T\underline{\rho}+b_m+\sigma n_m
\end{aligned}
$$

The second integral is given by

$$
\begin{aligned}
&\alpha\sum_{j=0}^{m-1}b_j\int_0^T s_j(t)s_i(t)dt+b_m\int_0^T s_m(t)s_i(t)dt+\sigma\int_0^T s_i(t)n(t)dt \\
&= \alpha b_i+b_m\rho_i+\sigma n_i
\end{aligned}
$$

as the narrowband bits are orthogonal. Therefore the decision statistic is given by

$$
\begin{aligned}
D.S. &= \frac{\left(1+\sigma^2/\alpha^2\right)\cdot\left[\alpha\underline{b}^T\underline{\rho}+b_m+\sigma n_m\right]-\sum_{i=0}^{m-1}\rho_i\left[\alpha b_i+b_m\rho_i+\sigma n_i\right]}{1+\frac{\sigma^2}{\alpha^2}-\underline{\rho}^T\underline{\rho}} \\
&= b_m+\frac{\frac{\sigma^2}{\alpha}\underline{b}^T\underline{\rho}++\sigma\left(1+\frac{\sigma^2}{\alpha^2}\right)n_m-\sigma\underline{n}^T\underline{\rho}}{1+\frac{\sigma^2}{\alpha^2}-\underline{\rho}^T\underline{\rho}}
\end{aligned}
$$

We can write the decision statistic equivalently as

$$D.S. = b_m + \frac{\frac{\sigma^2}{\alpha}\underline{b}^T\underline{\rho} + \tilde{n}}{1 + \frac{\sigma^2}{\alpha^2} - \underline{\rho}^T\underline{\rho}}$$

where the effective additive white Gaussian noise, $\tilde{n}$, has zero mean and variance

$$\tilde{\sigma}^2 = \sigma^2\left[\left(1 + \frac{\sigma^2}{\alpha^2}\right)^2 - \left(1 + 2\frac{\sigma^2}{\alpha^2}\right)\underline{\rho}^T\underline{\rho}\right]$$

The probability of error using this decision statistic against a zero threshold is

$$P_e = \frac{1}{2}\Pr\left(\tilde{n} > 1 + \frac{\sigma^2}{\alpha^2} - \underline{\rho}^T\underline{\rho} - \frac{\sigma^2}{\alpha}\underline{\rho}^T\underline{b}\right) + \frac{1}{2}\Pr\left(\tilde{n} < -1 - \frac{\sigma^2}{\alpha^2} + \underline{\rho}^T\underline{\rho} - \frac{\sigma^2}{\alpha}\underline{\rho}^T\underline{b}\right)$$

$$= \frac{1}{2^{m+1}}\sum_{i=0}^{2^m-1} Q\left(\frac{1 + \frac{\sigma^2}{\alpha^2} - \underline{\rho}^T\underline{\rho} - \frac{\sigma^2}{\alpha}\underline{\rho}^T\underline{b}^i}{\tilde{\sigma}}\right) + \frac{1}{2^{m+1}}\sum_{i=0}^{2^m-1} Q\left(\frac{1 + \frac{\sigma^2}{\alpha^2} - \underline{\rho}^T\underline{\rho} + \frac{\sigma^2}{\alpha}\underline{\rho}^T\underline{b}^i}{\tilde{\sigma}}\right)$$

where $\{b^i\}$ is an ordering of the m narrowband bits and the function Q is defined by

$$Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-v^2/2}dv$$

From these equations we also arrive at some asymptotic results. As the near far ratio goes to zero, the probability of error for the MMSE detector approaches that of the single user case, $Q(1/\sigma)$. As the near far ratio goes to infinity, the probability of error is equal to that of the decorrelating detector,

$$(P_e)_{dec} = Q\left(\frac{\sqrt{1 - \underline{\rho}^T\underline{\rho}}}{\sigma}\right)$$

For weak interferers the MMSE receiver has an advantage over the decorrelating detector, but this is not a scenario of interest. For very small cross correlations, the performance of both the MMSE and the decorrelating detector approaches that of the single-user system.

Simulations

Figure 1 shows the probability of error versus the near far ratio for four different receivers: the conventional (matched filter) detector, the decorrelating detector, and the MMSE detector. We use a noise power that is 6 dB above the spread signal, as suggested by field tests [4] of overlay spread spectrum systems. An *m*-sequence of

length 63 was used as the spreading code. Square waves at baseband were used for bit and chip waveforms.

For the case *m=1*, one narrowband bit per spread spectrum bit, the cross correlation is equal to the auto-correlation of the *m*-sequence. This value is very low, and we see that the MMSE and decorrelating detector each have performance equal to that of the single-user case. For larger values of *m* the cross correlations become larger and performance falls off slightly. For all the simulated systems, however, the MMSE and the decorrelating detectors show the same performance and their curves overlap.

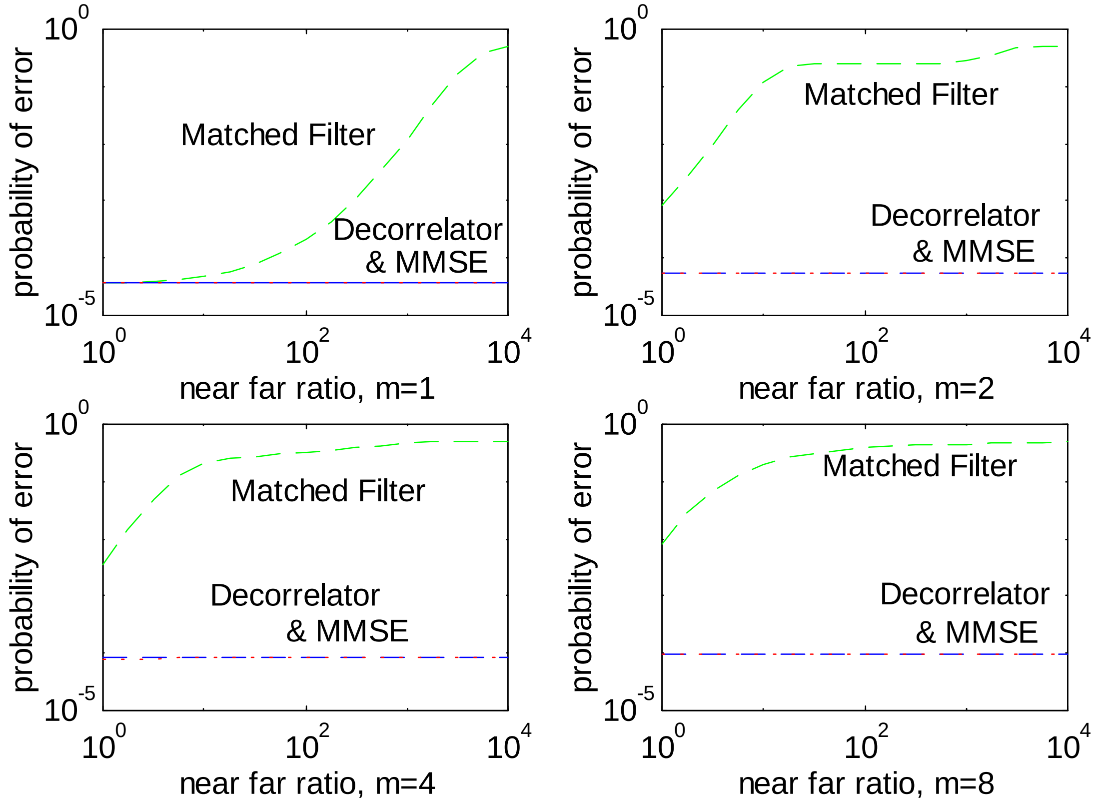

Figure 1 Results for Matched Filter, Decorrelating and MMSE detector

Conclusions

We have demonstrated that the MMSE detector can be very effective in removing the a binary narrowband interference signal from a direct sequence spread spectrum signal overlaid on the narrowband system. We presented the MMSE receiver and its probability of error in closed-form, as a function of

1. the bit waveform of narrowband signal,
2. the cross correlation with the spread spectrum signal, and
3. the relative powers of the signals and noise.

The adaptive version of this filter enjoys similar performance, while no longer requiring this information to form the receiver. Study of the dynamics of the adaptive algorithm (appropriate step sizes, convergence rates, etc.) is the subject of ongoing research.

References

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