A Projected SIC Multi-user Detector for Asynchronous Upstream OCDMA-PON

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***Abstract*—Optical code division multiple access (OCDMA) is a promising multiple access scheme for next generation passive optical networks (NG-PON). It can afford subscribers with a flexible Gb/s bandwidth for the upstream. However, OCDMA suffers from multiple access interference (MAI) that significantly reduces the performance of the system. To alleviate the undesirable impact of the MAI, we propose a projected successive interference cancellation (PSIC) detector that makes use of the non-negativity of the solution to enhance its performance. Using filter factor analysis, we shed light on the PSIC detector's superior performance compared to that of the linear minimum mean square error (LMMSE) detector. Similar to the PPIC detector proposed in [1], the proposed PSIC detector outperforms the conventional correlation receiver, the decorrelator detector, the linear parallel/successive interference cancellation detectors and the LMMSE detector. However, compared to the recently proposed projected PIC (PPIC) detector [1], the proposed PSIC detector is faster, always convergent and doesn't suffer from any oscillatory convergence behavior.**

Keywords-Optical, OCDMA, PIC, SIC, regularization, PON, Multiuser Detection (MUD).

I. INTRODUCTION

The increasing demand for broadband services such as high-definition television (HDTV) and video on demand (VoD) is steadily boosting the rapid growth of optical access systems. OCDMA is a multiple access technique that can provide large bandwidth to all users with asynchronous transmission. OCDMA has the advantage that not only it reduces the packet delay observed in long reach PON (LR-PON) but it allows the deployment of high speed TDM-PON [2].

Despite their numerous advantages, OCDMA systems suffer from MAI which can significantly reduce capacity and degrade system's performance. To mitigate MAI, one has to use advanced interference cancellation (IC) techniques. The optical hard limiter (OHL) is one of the simplest IC detectors and is usually set in front of the conventional correlation receiver (CCR) [3]. Even though it improves the performance of the CCR, the clipping process that it performs makes the assumption of a linear signal model invalid and consequently renders the use of interference cancellation intractable.

Multiuser detection (MUD) is defined as any technique that makes use of spreading codes of users other than the desired user to enhance the quality of the desired user data estimates. The optimal multiuser detector in terms of performance is the maximum likelihood (ML) detector. However, its computational complexity which is exponential in the number of users makes it not amenable to practical implementation [4]. Hence, suboptimal detectors that trade-off between good performance and reasonable complexity such as the parallel interference cancellation (PIC) and serial interference cancellation (SIC) detectors, have been proposed [5, 6].

Recently, a projected PIC (PPIC) detector that exploits the non-negativity of the solution has been proposed [1]. It has been shown that incorporating the non-negativity constraint into the PIC scheme is equivalent to introducing regularization into the solution. Also, it has been shown that the PPIC detector performs better than the CCR, the decorrelator detector and surprisingly the LMMSE detector.

Building upon the results obtained in [1], we propose a new projected successive interference cancellation detector. Similar to the PPIC detector proposed in [1], this scheme also exploits the non-negativity of the solution, however, the proposed scheme outperforms the PPIC scheme on the following aspects: 1) it converges faster than the PPIC detector, 2) it is always convergent and does not need any weighting factor to ensure convergence like the PPIC detector and 3) it converges smoothly and doesn't exhibit any oscillatory behavior. Moreover, using filter factor analysis, we throw light on the superiority of the PSIC detector's performance compared to that of the LMMSE scheme.

The rest of this paper is organized as follows. Section II details the system model for the OCDMA-PON. Section III describes MUD as a constrained optimization problem. Section IV introduces the proposed PSIC detector while section V sheds light on the good performance of the proposed PSIC detector compared to that of the LMMSE detector. Finally, numerical results and discussions are presented in section VI where as conclusions are then given in section VII.

II. SYSTEM MODEL

As shown in Fig.1, an OCDMA system with K users is considered with user 1 arbitrarily chosen as the desired user. Considering Optical Code family as Φ ($N, w, \lambda_a, \lambda_c$), where N is the code length, w is the code weight, λ_a is the maximum out of phase autocorrelation of a code sequence, and λ_c is the maximum cross-correlation of a code sequence with any other code in the set.

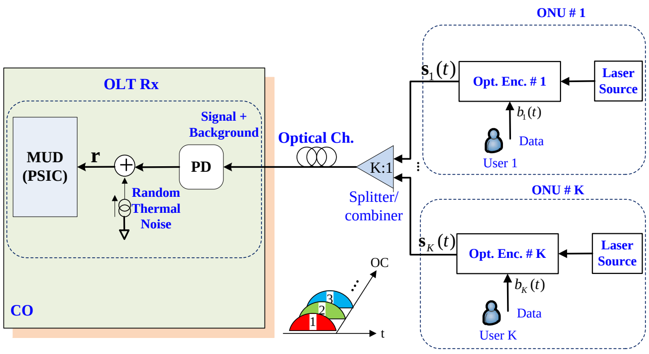

Fig. 1: System model for the asynchronous upstream OCDMA-PON

For the most widely used case of $\lambda_a = \lambda_c = 1$, the number of codes in the set is upper bounded by:

$$\left\lfloor \left\lfloor (N-1)(w-1) \right\rfloor / w \right\rfloor . \tag{1}$$

In this communication, we consider a symbol asynchronous system in which all delays are multiple of the chip period. The discrete time model for the asynchronous OCDMA AWGN channel is given by:

$$\mathbf{r} = \mathbf{SAb} + \mathbf{n} = \bar{\mathbf{S}}\mathbf{b} + \mathbf{n} = \mathbf{r}' + \mathbf{n} \tag{2}$$

where **b** is the vector of the users' data. **S** is the matrix of the spreading codes and each column of **S**: s_k represents the code sequence of the k^{th} user. **A** is the matrix of received amplitudes and **n** is the vector of the independent, identically distributed (i.i.d) AWGN. To get more insight about the structure of these matrices, see [7].

III. MUD AS A CONSTRAINED OPTIMIZATION PROBLEM

In most applications of signal detection and estimation problems, accurate information about statistical properties of the noise is available [8]. Consequently, it suggests looking for statistical approaches to solve these problems. Let us assume that we have a system model described byEq. (2). The problem is to find an estimate $\hat{\mathbf{b}}$ corresponding to users' data $\mathbf{b}$. In case of AWGN channel with zero mean and variance ρ^2 The ML detector selects the data vector $\hat{\mathbf{b}}$ that minimizes the following objective function:

$$\hat{\mathbf{b}}_{ML} = \underset{\mathbf{b}\in\{0,1\}^K}{\arg\min} \frac{1}{2}\left\| \mathbf{r} - \bar{\mathbf{S}}\mathbf{b} \right\|^2 \tag{3}$$

where $\|.\|$ denotes the usual L_2-norm and therefore the ML detection approach coincides with the well-known integer least squares approach.

The optimal solution of (3) achieved by the ML detector is unfortunately not amenable to practical implementation because its complexity is exponential in the number of users. The exponential complexity of the ML detector has inspired a considerable effort in the development of suboptimum receivers with low complexity while still improving the BER as compared to the CCR.

One approach to obtain suboptimal multiuser detectors is to relax the constraints imposed on the set of feasible solutions [9]. For example, by relaxing the constraint on the set of feasible solutions from $\mathbf{b} \in \{0,1\}^K$ to $\mathbf{b} \in R^K$, the decorrelator detector is obtained as the solution to the following optimization problem:

$$\mathbf{y}_{DEC} = \underset{\mathbf{b}\in\mathbb{R}^K}{\arg\min} \frac{1}{2}\left\| \mathbf{r} - \bar{\mathbf{S}}\mathbf{b} \right\|^2 \tag{4}$$

IV. THE PROPOSED PROJECTED SIC DETECTOR

As mentioned in [1], in incoherent Optical CDMA, both the data and the spreading codes are non-negative. The non-negativity of the solution can be exploited to enhance the quality of the solution. This can be done by constraining the set of feasible solutions to span the non-negative orthant $\mathbb{R}_+^K$ instead of $\mathbb{R}^K$ as in the decorrelator detector, that is:

$$\mathbf{b}^* = \underset{\mathbf{b}>0}{\arg\min} \frac{1}{2}\left\| \mathbf{r} - \bar{\mathbf{S}}\mathbf{b} \right\|^2 \tag{5}$$

No closed form solution exists for this optimization problem and therefore iterative methods should be used.

In the following, we propose a projected SIC detector that approaches the solution of the optimization problem of (5) with increasing number of stages. Compared to the PPIC detector, the PSIC detector exhibits the following desirable features: 1) it is faster because it uses the most recent updates once available, 2) it is stable and 3) its convergence is smooth.

The update equation characterizing the PSIC detector is given by:

$$y_{PSIC}^{p,k} = P_{\Omega}\left(\mathbf{s}_k^T (\mathbf{e}_k^p + \mathbf{I}_k^{p-1}) \right) \tag{6}$$

where $e_1^1 = \mathbf{r}$, $\mathbf{I}_k^p = \mathbf{s}_k P_{\Omega}\left(y_{PSIC}^{p,k} \right)$, $\mathbf{e}_{k+1}^p = \mathbf{e}_k^p + \mathbf{I}_k^{p-1} - \mathbf{I}_k^p$ and P_{Ω} is a projection into $\mathbb{R}_k^+$ defined as: $P_{\Omega}(x) = \max\{0, x\}$.

It has been proven that the SIC detector converges if $\bar{\mathbf{S}}^T\bar{\mathbf{S}}$ is symmetric positive definite regardless the projection operator $P_{\Omega}(\bullet)$ [10]. This condition is satisfied if the spreading codes are independent. This is the case for optical orthogonal codes (OOCs), hence, the PSIC scheme is always convergent.

It is well known that the PIC and SIC detectors exhibit almost the same computational complexity which is in the order of $O(K^2)$ and it is much less than that of the decorrelator detector or the LMMSE detector [7]. The projection operation does not add any extra computational complexity, hence the complexity of the proposed PSIC is of the same order of the conventional SIC and PIC detectors.

V. THE NON-NEGATIVITY CONSTRAINT AS A TRUNCATED SINGULAR VALUE DECOMPOSITION REGULARIZZATION SCHEME

As mentioned in [1], the non-negativity constraint has a regularization effect and stabilizes the solution with respect to noise where it transforms the constrained optimization problem of (5) to the following unconstrained optimization problem [1]:

$$\mathbf{b}^* = \arg\min_{\mathbf{b}\in\mathbb{R}^K} \frac{1}{2}\left\|\mathbf{r} - \bar{\mathbf{S}}\mathbf{D}^*\mathbf{b}\right\|^2 \tag{7}$$

where $\mathbf{D}^* = diag\left(D_1^* \quad D_2^* \quad \cdots \quad D_K^*\right)$ is a K-by-K diagonal matrix of zeros and ones. More details about how to obtain this matrix are provided in [11]. Therefore the solution to this optimization problem is:

$$\mathbf{b}^* = \left(\mathbf{D}^*\bar{\mathbf{S}}^T\bar{\mathbf{S}}\right)^{-1}\bar{\mathbf{S}}^T\mathbf{r} \tag{8}$$

Using the singular value decomposition (SVD), the matrix $\bar{\mathbf{S}}$ can be decomposed as:

$$\bar{\mathbf{S}} = \mathbf{U}\mathbf{\Lambda}\mathbf{V}^T \tag{9}$$

where $\mathbf{U}$ is an N-by-N matrix with columns given by the left singular vectors of $\bar{\mathbf{S}}$; $\mathbf{V}$ is an K-by-K matrix with columns given by the right singular vectors of $\bar{\mathbf{S}}$; and $\mathbf{\Lambda}$ is an N-by-K diagonal matrix with diagonal entries $\sigma_1 \geq \sigma_2 \geq ... \geq \sigma_K \geq 0$ that represent the singular values of $\bar{\mathbf{S}}$.

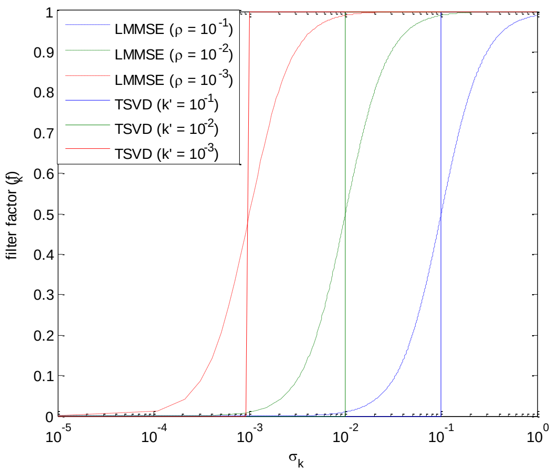

Fig. 2: Filter factors of the LMMSE and TSVD regularization methods

As detailed in [12], there exists a permutation matrix $\mathbf{Q}$ such that the first k' diagonal entries of the diagonal matrix $\mathbf{D}^*$ = $\mathbf{Q}^T\mathbf{D}^*\mathbf{Q}$ are one, with the remaining diagonal entries zero. Therefore without loss of generality, we may write the matrix $\mathbf{D}^*$ as:

$$\mathbf{D}^* = \begin{bmatrix} \mathbf{I}_{k',k'} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \tag{10}$$

where $\mathbf{I}_{k',k'}$ is a k'-by-k' diagonal matrix of ones. Thus, the solution in (8) can be written in terms of the SVD of $\bar{\mathbf{S}}$ as:

$$\begin{aligned} \mathbf{b}^* &= \left(\mathbf{D}^*\left(\mathbf{U}\mathbf{\Lambda}\mathbf{V}^T\right)^T\left(\mathbf{U}\mathbf{\Lambda}\mathbf{V}^T\right)\right)^{-1}\left(\mathbf{U}\mathbf{\Lambda}\mathbf{V}^T\right)^T\mathbf{r} \\ &= \mathbf{V}\left(\mathbf{D}^*\mathbf{\Lambda}\right)^{-1}\mathbf{U}^T\mathbf{r} = \sum_{k=1}^{k'}\frac{\mathbf{u}_k^T\mathbf{r}}{\sigma_k}\mathbf{v}_k \end{aligned} \tag{11}$$

Hence, the filter factors of this scheme are given by:

$$f_k = \begin{cases} 1 & \text{if} \quad k \leq k' \\ 0 & \text{if} \quad k > k' \end{cases}, 1 \leq k \leq K \tag{12}$$

which are in fact the filter factors of the truncated singular values decomposition regularization (TSVD) scheme.

On the other hand, the filter factors of the LMMSE regularization scheme are given by [13]:

$$f_k = \frac{\sigma_n^2}{\sigma_n^2 + \rho^2} \approx \begin{cases} 1 & \text{if} \quad \sigma_k \gg \rho \\ 0 & \text{if} \quad \sigma_k \ll \rho \end{cases}; \ 1 \leq k \leq K \tag{13}$$

The filter factors of the TSVD and the LMMSE regularization schemes are depicted in Fig. (2) for different values of the noise level ρ.

The filter factors of the LMMSE regularization scheme behaves like a smooth low-pass filter of the solution terms damping solution components pertaining to singular values less than the noise level ρ whereas those of the TSVD regularization scheme behave like a sharp low-pass filter and truncate all singular values below the cut-off threshold set by the truncation parameter k'.

Even though, no direct relation exist between the truncation parameter k' and the non-negativity constraint, however, if we set the truncation parameter k' equal to the noise level as in the LMMSE detector (see Fig. 2), the filter factors of the TSVD seem to better filter-out small singular values that are polluted with noise and retain large singular values that are essential in reconstructing good solutions compared to the filter factors of the LMMSE detector.

VI. Numerical Resutls and Discussions

In our numerical simulation, OOCs of the set Φ (511, 3, 1, 1) are used. There are 85 distinct codes available in this set. We set the SNR to 14dB and K to 85 users and investigate the convergence behavior of the PPIC, the PSIC, the CCR, the decorrelator and the LMMSE detectors by plotting the average BER versus the number of stages and depict the results in Fig. 3.

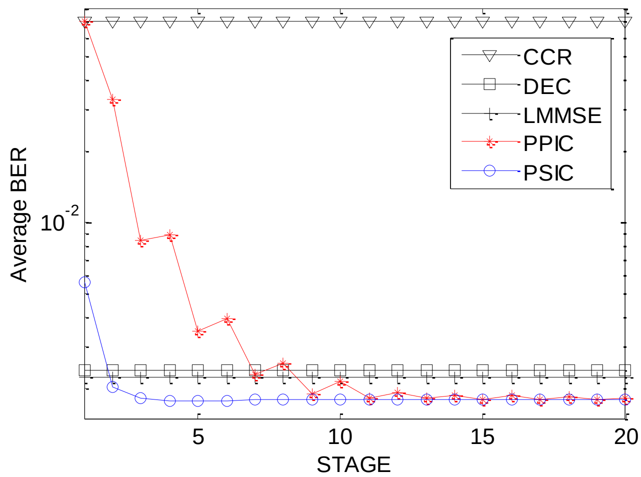

Fig. 3: Average BER vs. number of stages of different detectors.

It is clear that the PSIC and PPIC detectors converge to the same average BER performance, however, the PSIC detector converges faster and it needs roughly 4 stages for convergence compared to 16 stages for the PPIC detector. Additionally, the PSIC does not suffer from the oscillatory behavior observed in the PPIC detector. It is also important to mention that the PSIC detector achieves a better average BER compared to the LMMSE detector which indicates that exploiting the non-negativity information (as in PSIC) is better than exploiting the noise variance information (as in LMMSE).

Next, we fix the number of stages to 4 and investigate the effect of increasing SNR on the average BER performance and depict the results in Fig. 4. It can be easily seen, that the PSIC detector again performs better than the other detectors especially at high SNR which is due to the non-negativity projection

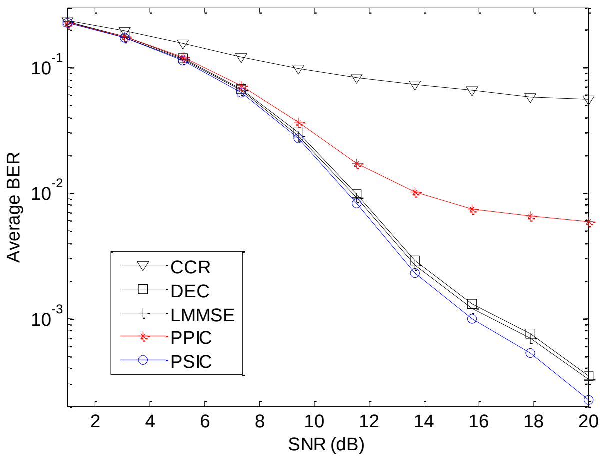

Fig. 4: Average BER vs. SNR of different detectors.

Finally, we fix the SNR to 14dB and the number of stages to 4 and we investigate the average BER performance versus the number of active users within the system. Simulation results for the PSIC detector are plotted in Fig. 5

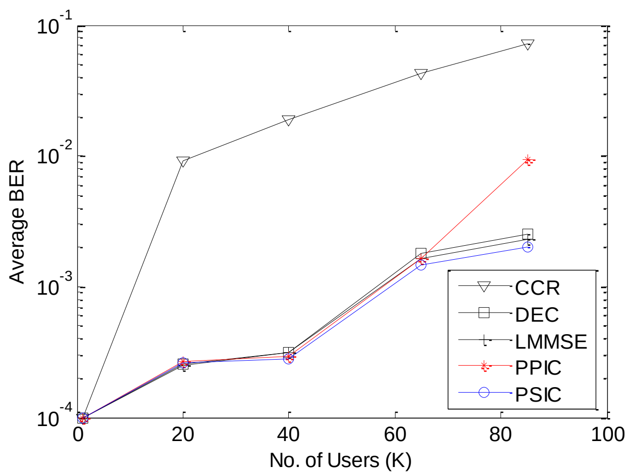

Fig. 5: Average BER vs. number of users (K) of different detectors.

Again the PSIC detector outperforms the other detectors especially at medium and highly loaded systems.

VII. Conclusion

In this work, a new interference cancellation detector that exploits the non-negativity of the solution is proposed to mitigate the MAI. This detector is always convergent and exhibits a fast and smooth convergence behavior. Using filter factor analysis, we put insight on the superior performance of the PSIC compared to that of the LMMSE detector. Numerical results supported well the theory and illustrated the important performance gain one can get by incorporating additional information about the solution such the non-negativity constraint. As a matter of fact, the PSIC detector achieved more than 1 dB enhancement in SNR at 10^{-3} average BER for 85 active users compared to the LMMSE detector.

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