Inverse QR Decomposition (IQRD) Blind Equalizer for QAM Coherent Optical Systems

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Abstract—**Advanced digital signal processing techniques have a vital role in the development of emerging and next generation ultrahigh speed optical networks. These techniques highly contribute to the improvement of system spectral efficiency. Blind equalizers have recently been used to compensate for linear fiber impairments. In this paper we apply the inverse QR decomposition (IQRD), algorithm for blind equalization in 14Gbaud-16QAM coherent receiver. IQRD is widely used in wireless communications and known for its inherent stability in finite precision environment. We compare its performance with the standard Constant Modulus Algorithm (CMA) and Recursive Least Squares (RLS) algorithm in terms of convergence rate (CR) and bit error rate (BER). Our simulation results show that IQRD algorithm achieves similar CR and BER performance as those of standard RLS algorithm. However, in finite precision environment, which is more appropriate for practical implementation, and results in lower system power consumption (human friendly or Green solution), IQRD completely outperforms the RLS technique. A substantial reduction in BER of two orders of magnitude at 14 bit resolution is achieved for the optical system under consideration.**

Index terms— **Coherent detection, Finite impulse response (FIR) filters, Recursive least squares (RLS), inverse QR decomposition (IQRD).**

I. Introduction

During the last 20 years the world traffic has increased 10,000 times at a rate of 0.2dB per year. The exponential scaling in data rates will cause the world traffic to reach thousands and thousands of Exabyte by the end of 2020. Emerging optical transmission systems are coherent involving multilevel modulation, such as M-ary quadrature amplitude modulation (MQAM), polarization division multiplexing (PDM) and robust digital signal processing (DSP) techniques. This considered as one of the pioneer solutions to increase the spectral efficiency and to achieve the tremendous increase in data rates of the optical communication system. Numerous modulation formats have recently been introduced in the literature in order to enhance the system performance. For example, IEEE P802.3ba standard has recently adopted QPSK with polarization multiplexing (PolMux) for 100 Gbps traffic. Using two polarizations and $M = 4$ symbols (i.e. 2bits/symbol/pol.), only 25GHz bandwidth is needed to carry 100Gbps rate. In next generation systems, per wavelength data rate is expected to achieve 400 Gbps and 1Tbps [1]. Research community is still investigating different possible modulation schemes for these targeted data rates [2]. Unfortunately, the performance of ultrahigh speed systems suffers due to challenges faced at either the optical transmitter and receiver or the fiber channel. In this work, we focus the study on the behavior of the optical coherent receiver under the effect of fiber channel impairments.

There are two categories of linear impairments in the fiber channel. The first is independent of polarization mainly including chromatic dispersion (CD). This causes system Inter-Symbol Interference (ISI) [3]. The second is polarization dependent which originates from optical polarization mode dispersion (PMD) and polarization dependent loss (PDL). This category causes pulse broadening, rotation of the principle state of polarization (PSPs) in the fiber, and polarization dependent optical power fluctuations

Different solutions have been proposed in the literature to combat the linear fiber impairments. Digital finite impulse response (FIR) filters have been used to diminish the effect of CD. 3.7 taps per 1000ps/nm have been used to compensate CD at 10.7Gbaud QPSK transmission [4]. However, the residual effect of CD still remains noticeable. Fractionally spaced equalizer (FSE) [5] with MIMO structure was proposed to not only compensate for the residual CD but also for the PMD. The well-known types of adaptive equalizers used in optical coherent receivers include the constant modulus algorithm (CMA), direct-detection least mean squares (DD-LMS) algorithm [4], and the recursive least squares (RLS) algorithm[6]. These equalizers update their weights blindly, without training sequence, which in turn increases the system spectral efficiency. Fig. 1 shows the block diagram of the compensation system, where the input signal x_{in} is first compensated from the CD effect using FIR filter. Then, the compensated signal x_p is passed through the blind equalizer in order to remove the residual CD distortion and get the estimated signal x_{out}.

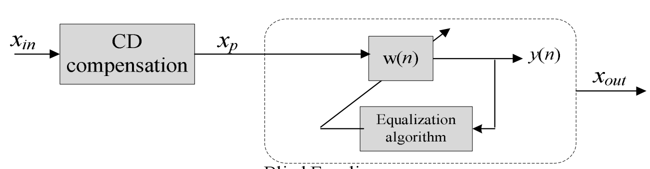

Fig. 1 Impairments compensation diagram

CMA is considered as the simplest algorithm, however it provides low performance for square QAM modulation with M≥8, since the constant modulus property is not satisfied and the error cost function is not minimized [7]. The system performance can be improved by switching the CMA algorithm to the decision directed (DD) mode. In spite of its higher complexity, the RLS shows faster convergence and better BER than the CMA algorithm.

The main contribution of the paper is to apply the inverse QR decomposition (IQRD) in order to blindly adapt the FSE tap weights in 14Gbaud-16QAM coherent receiver and compare its performance with CMA and RLS algorithms. In section IV, we find that the IQRD algorithm can attain the same speed of convergence and BER performance of the standard RLS algorithm; however it completely outperforms the RLS in finite precision environment. The algorithm speed and the computational complexity are totally improved by considering finite precision arithmetic. Moreover, finite precision arithmetic results in a lower system power consumption, which in turn, leads to environmental and human friendly or "Green" solutions.

The organization of the paper is as follows. In Section II, we show the effect of the FIR compensating filter on the CD distortion. In Section III, we introduce the different blind equalizers considered in this work. Simulation results are given in Section IV. Section V concludes the paper.

II. Chromatic Dispersion compensation

Neglecting the effect of fiber nonlinearities and polarization dependent impairments such as PMD and PDL, the effect of CD on the optical signal envelope $E(z,t)$ can be modeled as follows [4],

$$\frac{\partial E(z,t)}{\partial z} = \frac{jD\lambda^2}{4\pi c}\frac{\partial^2 E(z,t)}{\partial t^2} \tag{1}$$

where D is the fiber dispersion parameter, λ is the optical wavelength, c is the speed of light, and z is the fiber length. The solution of the above differential equation can be expressed in the frequency domain as follows.

$$\tilde{E}(z,\omega) = \tilde{E}(0,\omega)e^{-\frac{jD\lambda^2}{4\pi c}\omega^2 z} \tag{2}$$

where $\tilde{E}(z,\omega)$ is the Fourier transform of the optical envelope signal, $E(z,t)$. Consequently, the frequency response of the CD channel is given by

$$H(\omega) = \exp(-\frac{jD\lambda^2}{4\pi c}\omega^2 z) \tag{3}$$

Thus, the impulse response of the CD channel, $h(t)$, can be expressed as

$$h(t) = sqrt\left(\frac{c}{jD\lambda^2 z}\right).\exp\left(\frac{j\pi c}{D\lambda^2 z}t^2\right) \tag{4}$$

and the impulse response of the CD compensating filter can be obtained by inverting the sign of the dispersion parameter D as follows.

$$h_c(t) = sqrt\left(\frac{jc}{D\lambda^2 z}\right).\exp\left(-\frac{j\pi c}{D\lambda^2 z}t^2\right) \tag{5}$$

For practical implementation of the compensating filter, $h_c(t)$ needs to be finite and causal. This can be done by sampling $h_c(t)$ at Nyquist rate (T) and truncating it according to the following criterion [4].

$$-\frac{|D|\lambda^2 z}{2cT} \le t \le \frac{|D|\lambda^2 z}{2cT} \tag{6}$$

Therefore, the FIR filter tap weights are given by

$$w(n) = sqrt\left(\frac{jcT^2}{D\lambda^2 z}\right).\exp\left(-\frac{j\pi cT^2}{D\lambda^2 z}n^2\right) \tag{7}$$

where $-\left\lfloor \frac{N}{2} \right\rfloor \le n \le \left\lfloor \frac{N}{2} \right\rfloor$, and $N = 2\left\lfloor \frac{|D|\lambda^2 z}{2cT^2} \right\rfloor + 1$ is the total number of filter taps.

In [4], the compensation of CD for 10Gbaud-QPSK optical system operating at 42.8Gbps has been considered. In this section, we extend the work done in [4] to study the effectiveness of the FIR equalizer in compensating the CD for transmission over 1000km fiber with dispersion parameter of 17ps/nm.km. The modulation format selected is 16QAM-PolMux with 14Gbaud symbol rate that allows for 112Gbps transmission. Fig. 2 shows the back to back performance of the simulated 16QAM as well as its performance over 1000km. A 107-tap FIR filter is used to compensate for the CD effect, with 6.2 taps per 1000ps/nm. Fig. 2 shows that the system penalty is 2 dB at BER 10^{-3} for 14Gbaud and 1000 km transmission.

In practice, it is desirable to reduce the number of filter taps. Fig. 3 shows the system quality versus the normalized number of filter taps. It is clear that for 60% reduction in the total number of filter taps (i.e. 3.7 taps per 1000ps/nm), the system penalty is no more than 0.45dBQ, where Q represents the system performance ($Q = \sqrt{2}erfcinv(2BER)$)

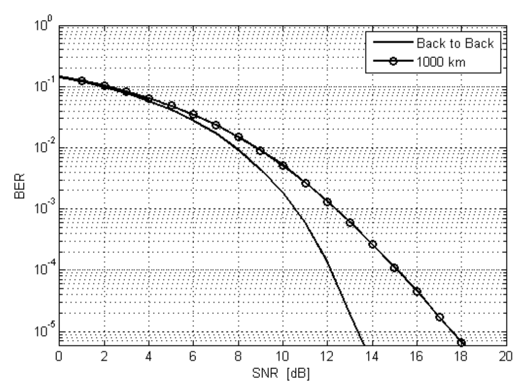

Fig. 2 SNR per bit versus BER for back to back and 1000km system

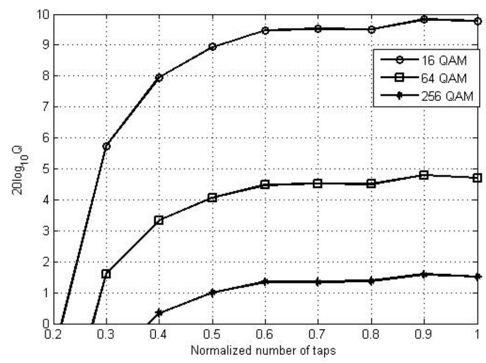

Fig. 3 System performance versus normalized number of taps

III. BLIND EQUALIZATION

Equalization in single carrier optical communications could be based on a training sequence sent before the actual data transmission. However, this reduces the spectral efficiency of the optical system. Blind equalization however doesn't require any training sequence. This reduces the system overhead and makes it spectrally more efficient. In the following, we introduce some of the most used blind equalization techniques in the literature to compensate for the residual CD as well as the proposed IQRD algorithm.

A. Constant modulus algorithm (CMA)

The CMA proposed by Godard in 1980's [8] is one of the most famous blind equalization techniques. The adaptation criterion of the CMA is based on minimizing the cost function given by

$$J(n) = E\left[\left(\left|y(n)\right|^2 - R_2\right)^2\right] \tag{8}$$

where $y(n)$ is the equalizer response, E is the statistical expectation operator, and R_2 is a positive real constant that depends on the transmitted symbols and is given by

$$R_2 = \frac{E\left[\left|s(n)\right|^4\right]}{E\left[\left|s(n)\right|^2\right]} \tag{9}$$

where s(n) is the transmitted symbols. Let **w** denotes the equalizer tap weights (or equivalently impulse response), the equalizer output can be written as

$$y(n) = \mathbf{w}^H \mathbf{X} \tag{10}$$

where $\mathbf{w}=[w_0(n).w_1(n),...,w_{L-1}(n)]^T$ is the equalizer tap weight vector and $\mathbf{X}=[x(n),x(n-1),...,x(n-L+1)]^T$ is the equalizer input data vector. T and H denote the transpose and complex transpose, respectively. L is the length of the filter. The equalizer coefficients are adapted according to the stochastic gradient algorithm (SGA) as follows

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu.\mathbf{x}(n).e^*(n) \tag{11}$$

where μ is the equalizer step size, e(n) is the error signal given by

$$e(n) = y(n)\left[R_2 - \left|y(n)\right|^2\right] \tag{12}$$

Table I
Complete RLS-CMA Algorithm

Initialization	$\mathbf{P}(0) = \delta^{-1}\mathbf{I}$, δ is small positive constant, $0 \le \lambda \le 1$
RLS-CMA Update	$\mathbf{z}(n) = \mathbf{x}(n)\mathbf{x}^H(n)\mathbf{w}(n-1)\,\lvert\mathbf{x}^H(n)\mathbf{w}(n-1)\rvert^{q-2}$
	$\mathbf{h}(n) = \mathbf{z}^H(n)\mathbf{P}(n-1)$
	$\mathbf{g}(n) = \mathbf{P}(n-1)\mathbf{z}(n) / (\lambda+\mathbf{h}(n)\mathbf{z}(n))$
	$\mathbf{P}(n) = (\mathbf{P}(n-1)-\mathbf{g}(n)\mathbf{h}(n)) / \lambda$
	$e(n) = 1 - \mathbf{w}^H(n-1)\mathbf{z}(n)$
	$\mathbf{w(n) = w}(n-1) + \mathbf{g(n)}\, e^*(n)$

Table II
Complete IQRD-CMA Algorithm

Initialization	$\mathbf{U}^{-1}(0) = \delta^{-1}\mathbf{I}$, δ is small positive constant, $0 \le \lambda \le 1$
IQRD-CMA Update	$\mathbf{z}(n) = (\mathbf{w}^H(n)\mathbf{x}(n))^* \mathbf{x}(n)$
	$\mathbf{a}(n) = \boldsymbol{\lambda}^{-1/2}\,\mathbf{U}^{-H}(n-1)\,\mathbf{z}(n)$
	Obtaining $\mathbf{Q}_\theta(n)$ and γ(n)
	$\begin{bmatrix} 1/\gamma(n) \\ \mathbf{0} \end{bmatrix} = \mathbf{Q}_\theta(n)\begin{bmatrix} 1 \\ -\mathbf{a}(n) \end{bmatrix}$
	Obtaining **u**(n) and updating $\mathbf{U}^{-H}(n)$
	$\begin{bmatrix} \mathbf{u}^H(n) \\ \mathbf{U}^{-H}(n) \end{bmatrix} = \mathbf{Q}_\theta(n)\begin{bmatrix} \mathbf{0}^T \\ \lambda^{-1/2}\mathbf{U}^{-H}(n-1) \end{bmatrix}$
	Obtaining e(n)
	$e(n) = 1 - \mathbf{w}^H(n-1)\mathbf{z}(n)$
	Updating the filter coefficients
	$\mathbf{w}(n) = \mathbf{w}(n-1) - \gamma(n)e^*(n)\mathbf{u}(n)$

It is worthy to note that, although CMA is simple and robust algorithm, it is not efficient for QAM modulation with M ≥ 8, since the constant modulus property is not satisfied [3, 9].

B. Recursive least squares-CMA (RLS-CMA)

RLS-CMA algorithm is another type of the blind equalization techniques. It is characterized by the faster convergence property than the SGA-based blind algorithms, since it utilizes information contained in the input data extending back to the instant of time when the algorithm is initiated [10]. The complete updating algorithm of the RLS-CMA is summarized in Table I. The parameter λ is called the forgetting factor ($0 \le \lambda \le 1$). According to [11], the parameter $q = 2$ achieves the best convergence property. The cost function of the RLS-CMA for $q = 2$ can be written as [6]

$$J(n) = \sum_{m=1}^{n} \lambda^{n-m} \left|\mathbf{w}(n)\mathbf{z}(m) - 1\right|^2 \tag{13}$$

and the output of the equalizer is given by

$$y(n) = \mathbf{w}^H \mathbf{X} \tag{14}$$

C. Inverse QR decomposition (IQRD)-CMA

The IQRD algorithm attains the same convergence speed as that of the RLS; however it provides better performance than the RLS algorithm, specifically in finite precision environment. The IQRD algorithm computes the inverse QR decomposition of the input data matrix using *Givens* rotation matrices and solves the least square (LS) problem without back substitution [12]. The cost function for obtaining the optimal weight vector is given by

$$J(n) = \sum_{m=1}^{n} \lambda^{n-m} \left|1 - \mathbf{w}^H(n)\mathbf{z}(m)\right|^2 \tag{15}$$

where

$$z(n) = \left(\mathbf{w}^H(n)\mathbf{x}(n)\right)^* \mathbf{x}(n) \quad (16)$$

represents an intermediate data vector, and λ is the forgetting factor that controls the speed of convergence. The triangular factorization of the weighted intermediated data vector is performed using the orthogonal rotation matrix $\mathbf{Q}$(n) as follows

$$\mathbf{Q}(n)Z(n) = \begin{bmatrix} 0 \\ \mathbf{U}(n) \end{bmatrix} \quad (17)$$

where $\mathbf{U}$(n) is a triangular matrix. The IQRD starts by obtaining the *Givens* rotation matrices $\mathbf{Q}_\theta$(n) defined as

$$\mathbf{Q}_\theta(n) = \begin{bmatrix} \gamma(n) & \mathbf{g}^T(n) \\ \mathbf{f}(n) & \mathbf{E}(n) \end{bmatrix} \quad (18)$$

where γ(n) is a scalar, and **g**, **f**, and **E** are matrices that depend on the type of triangularization (upper or lower). Then, the IQRD updates the matrix $\mathbf{U}$(n) as follows.

$$\begin{bmatrix} \mathbf{u}^H(n) \\ \mathbf{U}^{-H}(n) \end{bmatrix} = \mathbf{Q}_\theta(n) \begin{bmatrix} \mathbf{0}^T \\ \lambda^{-1/2}\mathbf{U}^{-H}(n-1) \end{bmatrix} \quad (19)$$

The optimal weight taps vector is given by

$$\mathbf{w}(n) = \mathbf{w}(n-1) - \gamma(n)e^*(n)\mathbf{u}(n) \quad (20)$$

where the vector $\mathbf{u}$(n) is defined as

$$\mathbf{u}(n) = -\lambda^{-1/2}\gamma(n)\mathbf{U}^{-1}(n-1)\mathbf{a}(n) \quad (21)$$

The complete IQRD-RLS algorithm is summarized in Table II.

IV. SIMULATION RESULTS

The blind equalizers, discussed in the previous section, are used to compensate for the residual CD in 14Gbaud 16QAM-PolMux coherent optical system operating at 112Gbps. The data stream is generated using rectangular pulses that are filtered using square root raised cosine filters with roll-off factor α=1, at both the transmitter and the receiver. The step size parameter of the CMA equalizer is chosen as $\mu = 10^{-4}$. The forgetting factor of both the RLS and IQRD equalizers is chosen as λ=0.999. The total number of the equalizers taps is fixed at 7. The three equalizers taps are initiated with zeros except the first tap set to one. That is, $\mathbf{w} = [1,0,0,0,0,0,0]^T$.

Fig. 4 shows the convergence speed of the different equalizers at SNR = 14dB and CD of 500 ps/nm. It is clear that both the RLS and IQRD equalizers have faster convergence speed and lower BER than the CMA equalizer. However, they have computational complexity in the order of $O(N^2)$ compared to $O(N)$ for the CMA equalizer. Fig. 5 shows the performance of the RLS and IQRD algorithms with respect to the resolution of analog to digital converter (ADC) at the receiver. The IQRD algorithm achieves the targeted BER with lower number of bits resolution than the RLS. In Fig. 6 we show the BER versus the SNR with infinite precision operation for both RLS and IQRD equalizers. Both techniques achieve the same BER performance. However, in Fig. 7 we show the performance of both RLS and IQED in case of finite precision operation for different number of bits resolution. The RLS algorithm suffers from rounding error propagation in data and coefficients that causes the system to lose its stability and hence to diverge. However, the IQRD algorithm maintains the system stability at relatively low numbers of bits resolution. At 14 bit resolution the IQRD outperforms the RLS by 2 orders of BER magnitudes. Fig. 8 shows the received constellation diagrams for both RLS and IQRD at 14 bits resolution and SNR=14 dB .

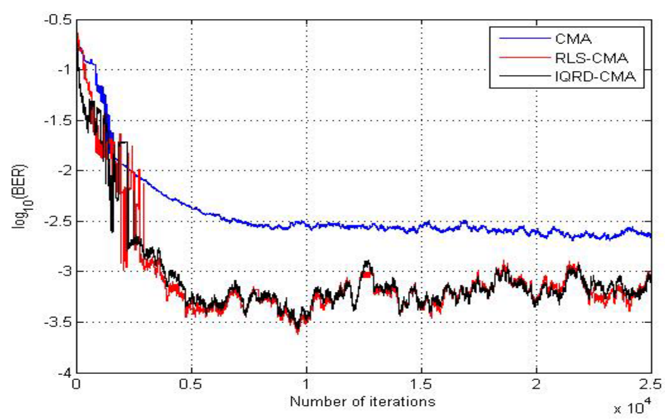

Fig. 4 Convergence time versus BER of different equalizers for SNR=14dB. CMA in blue, RLS in black and IQRD in red

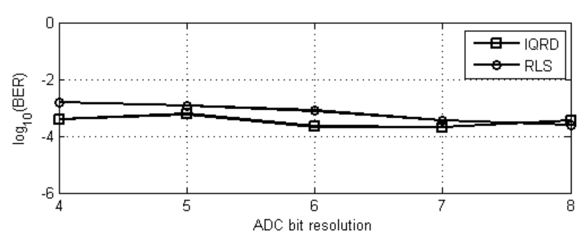

Fig. 5 Performance of RLS and IQRD with ADC

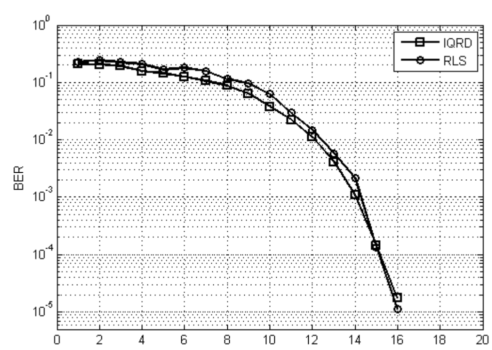

Fig. 6 BER vs SNR of the RLS and IQRD with infinite precision

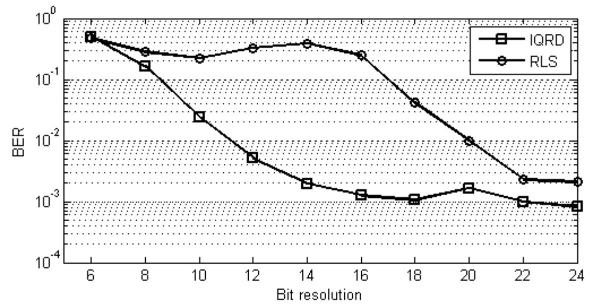

Fig. 7 BER vs. number of bit resolution of fixed point precision for SNR = 14dB

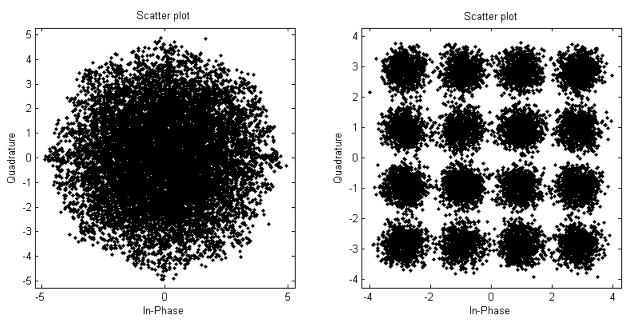

Fig. 8 Received constellation diagrams for both RLS and IQRD at 14 bits resolution and SNR = 14dB

V. CONCLUSION

Blind equalization and CD compensation have been considered in 14Gbaud 16QAM coherent optical system. 6.2 FIR filter taps has been shown to compensate 1000ps/nm of CD with 2dB power penalty. CMA, RLS, and IQRD algorithms have been investigated to compensate up to 500 ps/nm of residual CD. The IQRD algorithm has shown similar performance as that of RLS in terms of convergence speed and BER. The RLS algorithm suffers from numerical instability in finite precision arithmetic, however the IQRD algorithm can maintain the system stability at relatively low numbers of bits resolution; an enhancement of two order of BER magnitude is achieved by IQRD over RLS at 14 bits resolution. This results in a lower system power consumption, which in turn, leads to environmental and human friendly or “Green” solutions.

VI. ACKNOWLEDGMENT

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