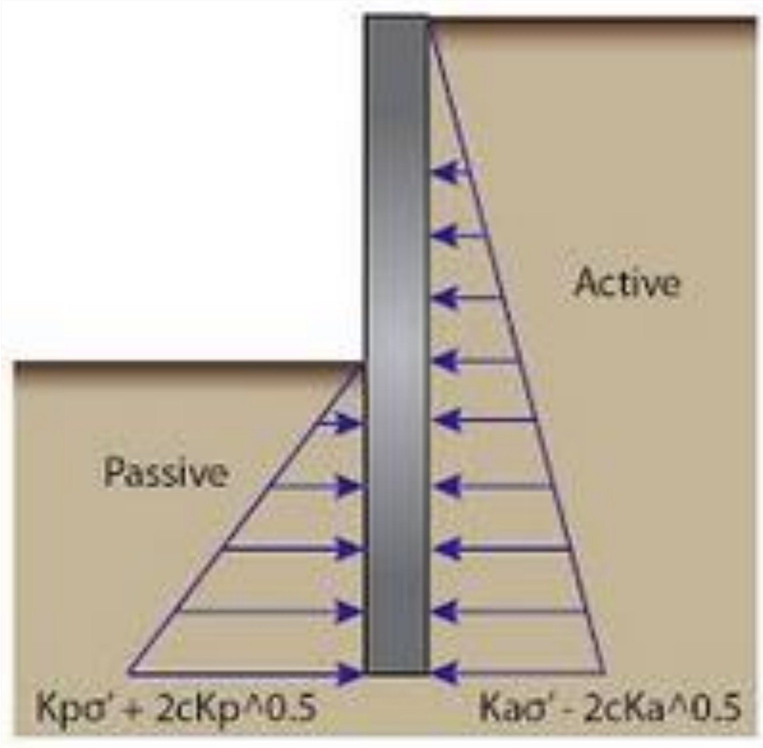
Lateral Earth Pressure

Chapter 13

Omitted sections:
13.5, 13.14, 13.15



TOPICS

- Introduction
- Coefficient of Lateral Earth Pressure
- Types and Conditions of Lateral Earth Pressures
- Lateral Earth pressure Theories
- Rankine's Lateral Earth Pressure Theory
- Lateral Earth Pressure Distribution – Cohesionless Soils
- Lateral Earth Pressure Distribution – C – ϕ Soils
- Coulomb's Lateral Earth Pressure Theory

TOPICS

- ❑ **Introduction**
- ❑ Coefficient of Lateral Earth Pressure
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INTRODUCTION

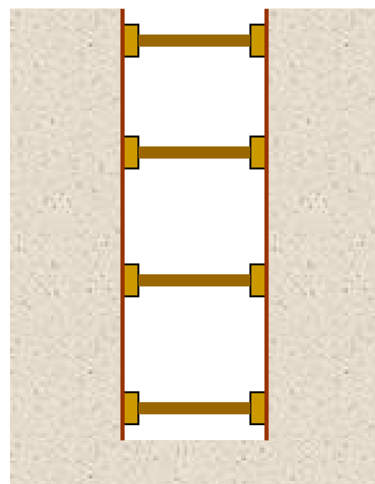
<u>**Proper design and construction of many structures such as:**</u>

- **Retaining walls (basements walls, highways and railroads, platforms, landscaping, and erosion controls)**
- **Braced excavations**
- **Anchored bulkheads**
- **Grain pressure on silo walls and bins**

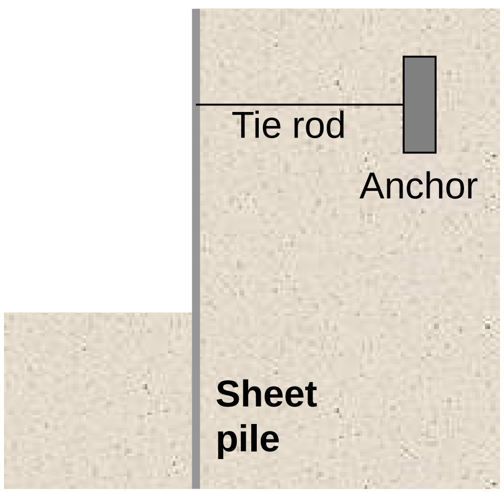
require a thorough knowledge of the lateral forces that act between the retaining structures and the soil masses being retained.

INTRODUCTION

Cantilever
retaining wall



Braced excavation



Anchored sheet pile

INTRODUCTION

- **The lateral forces are caused by lateral earth pressure.**

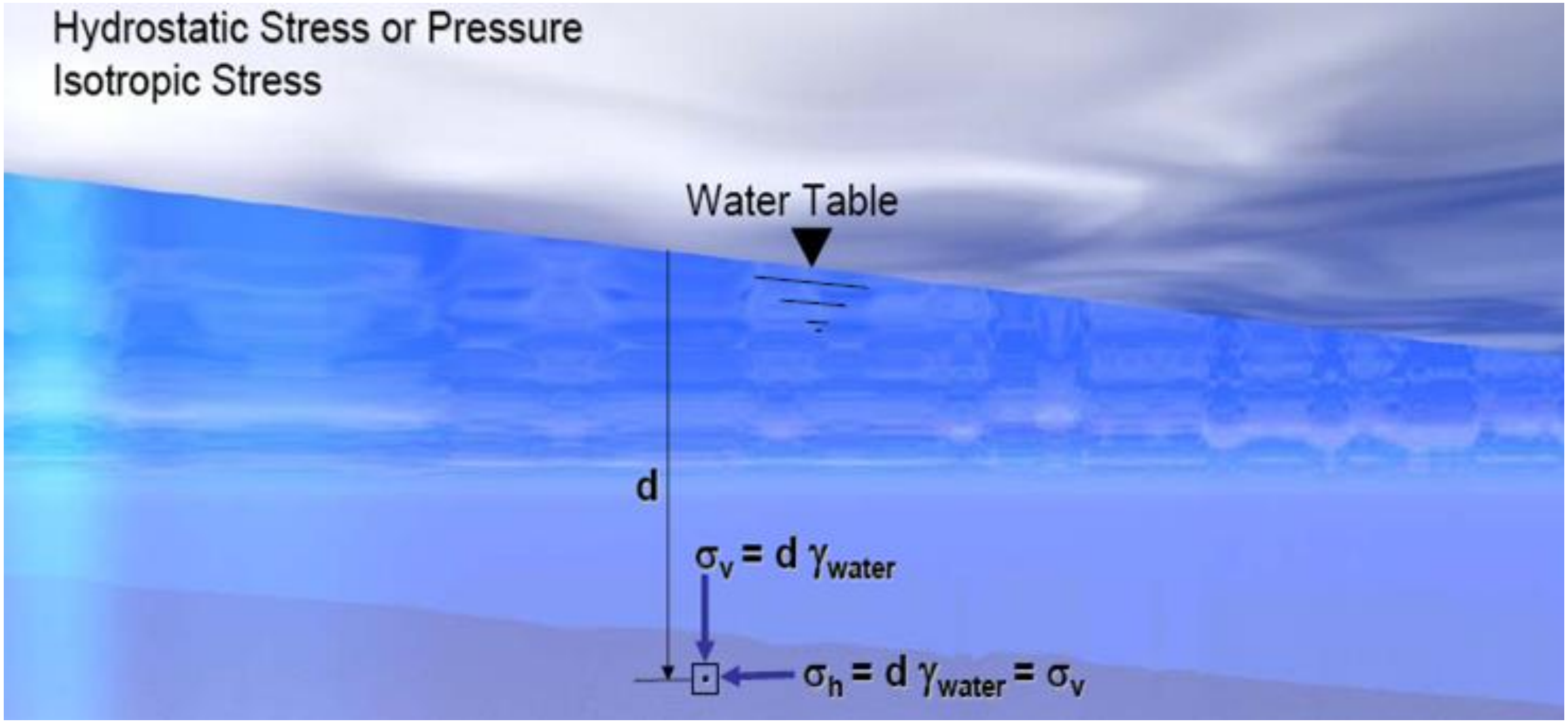
- **We have to estimate the lateral soil pressures acting on these structures, to be able to design them.**

INTRODUCTION

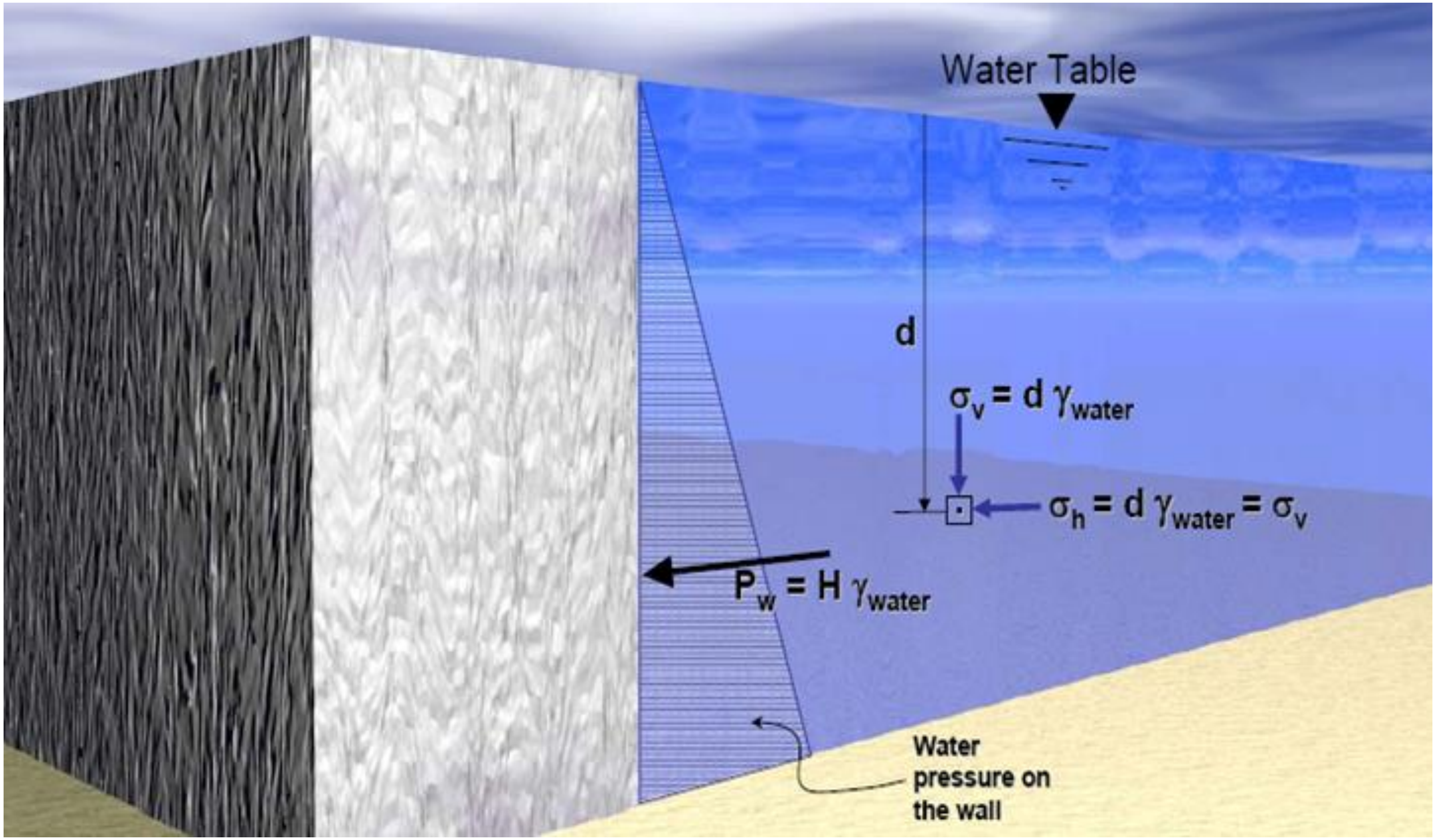
The magnitude and distribution of lateral earth pressure depends on many factors, such as:

- **The shear strength parameters of the soil being retained**
- **The inclination of the surface of the backfill**
- **The height and inclination of the retaining wall at the wall–backfill interface**
- **The nature of wall movement under lateral pressure**
- **The adhesion and friction angle at the wall–backfill interface**

VERTICAL AND HORIZONTAL STRESS IN WATER



VERTICAL AND HORIZONTAL STRESS IN WATER



STRESS DISTRIBUTION IN SOILS

Geostatic Stresses

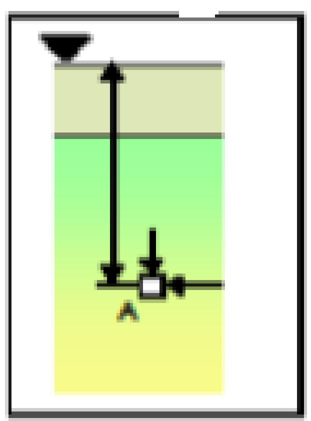
↓

Total Stress
Effective Stress
Pore Water Pressure

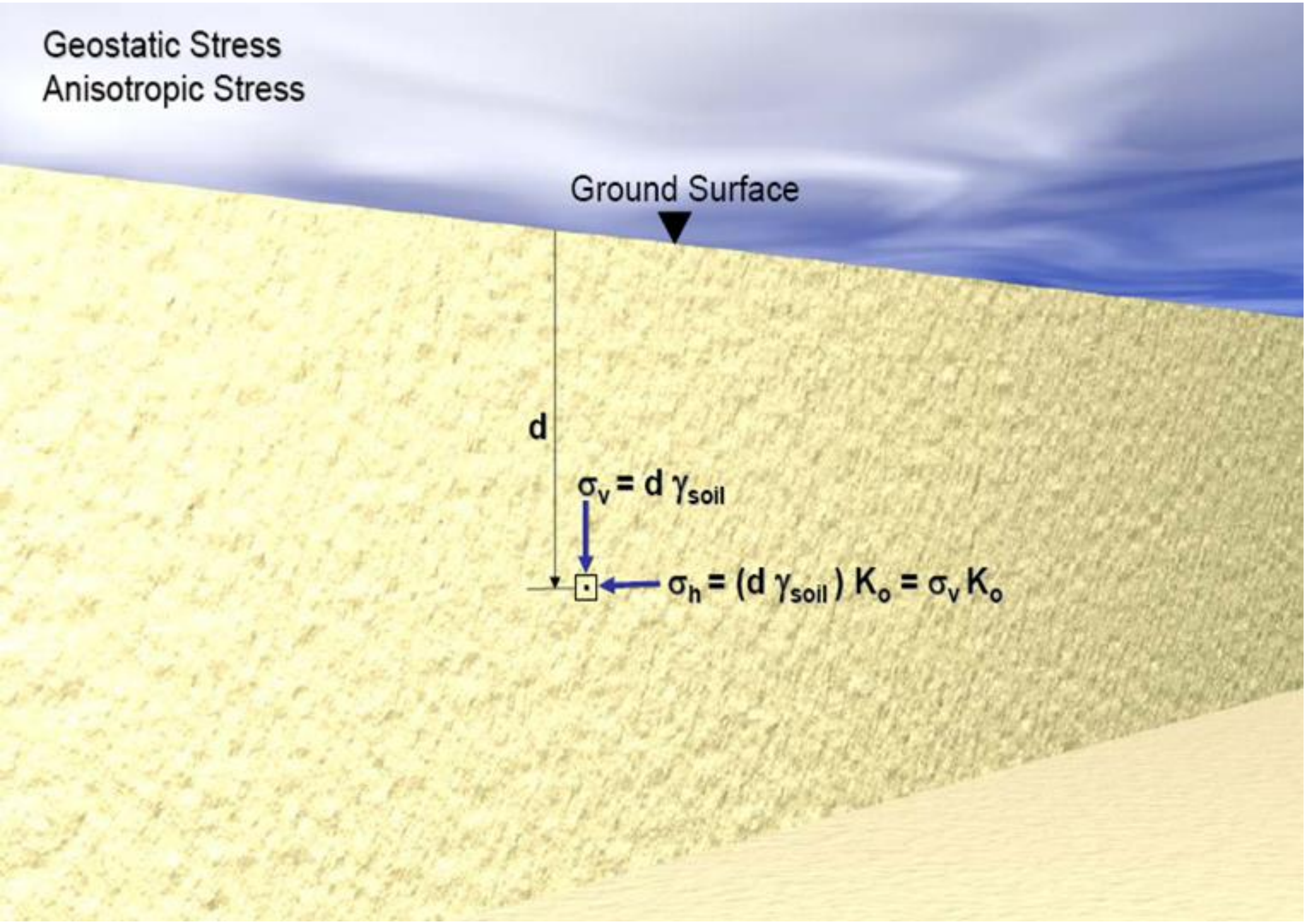
↓

Total Stress= Effective Stress+ Pore Water Pressure

$$\sigma_{total} = \sigma_{eff} + u$$



VERTICAL AND HORIZONTAL STRESS IN SOIL



TOPICS

- Introduction
- **Coefficient of Lateral Earth Pressure**
- Types and Conditions of Lateral Earth Pressures
- Lateral Earth pressure Theories
- Rankine's Lateral Earth Pressure Theory
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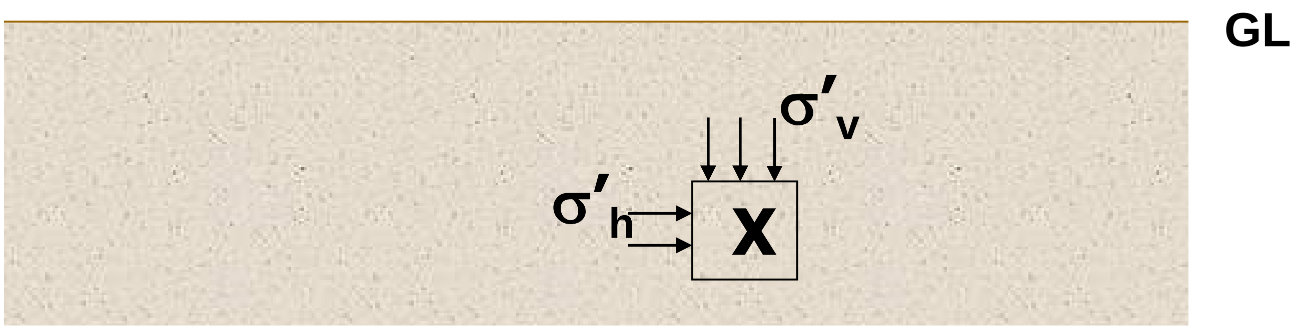
Coefficient of Lateral Earth Pressure

In a homogeneous natural soil deposit,



The ratio σ_h'/σ_v' is a constant known as coefficient of lateral earth pressure.

In other words, it is the ratio of the effective horizontal stress (σ_h') to the effective vertical stress (σ_v'); then

$$K = \frac{\sigma_h'}{\sigma_v'}$$

Or in terms of total stresses

$$K = \frac{\sigma_h}{\sigma_v}$$

- **All in subsequent derivation we use total stress, in the text book the effective stress, treatment is the same.**

TOPICS

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- ❑ Lateral Earth pressure Theories
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Earth Pressure At-Rest

Three possible cases may arise concerning the retaining wall; they are described as follows:

- **Case 1** If the wall AB is static—that is, if it does not move either to the right or to the left of its initial position—the soil mass will be in a state of static equilibrium. In that case, σ_h is referred to as the **at-rest earth pressure**.

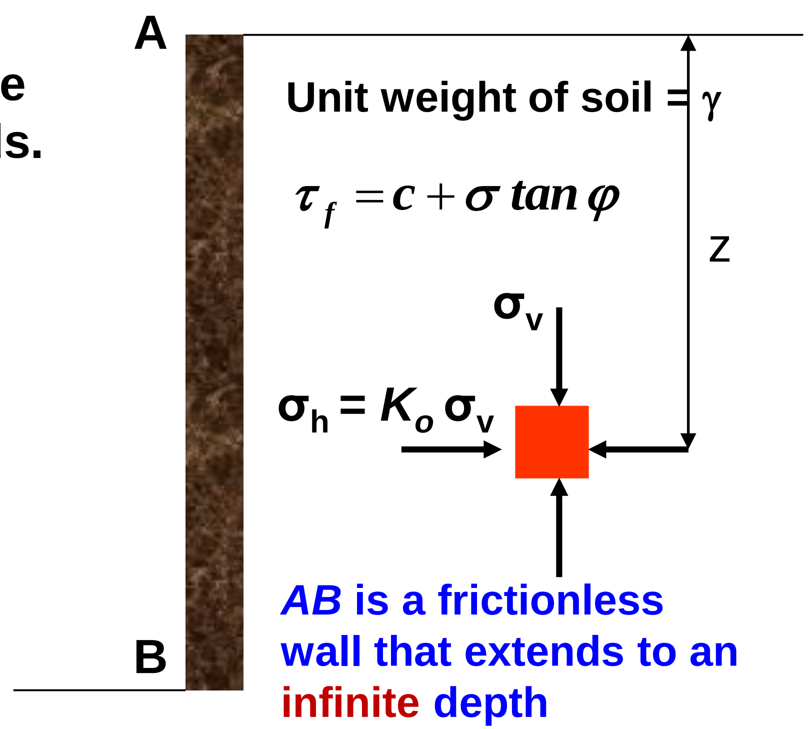
This is also the case **before construction**. The soil in the field by itself with no external loads.

Ratio of *horizontal stress* to *vertical stress* is called **coefficient of earth pressure at-rest, K_o**, or

$$K_o = \frac{\sigma_h}{\sigma_v}$$

$$\sigma_h = K_o\,\sigma_v = K_o \gamma\, z$$

where K_o at-rest earth pressure coefficient.



Earth pressure at-rest

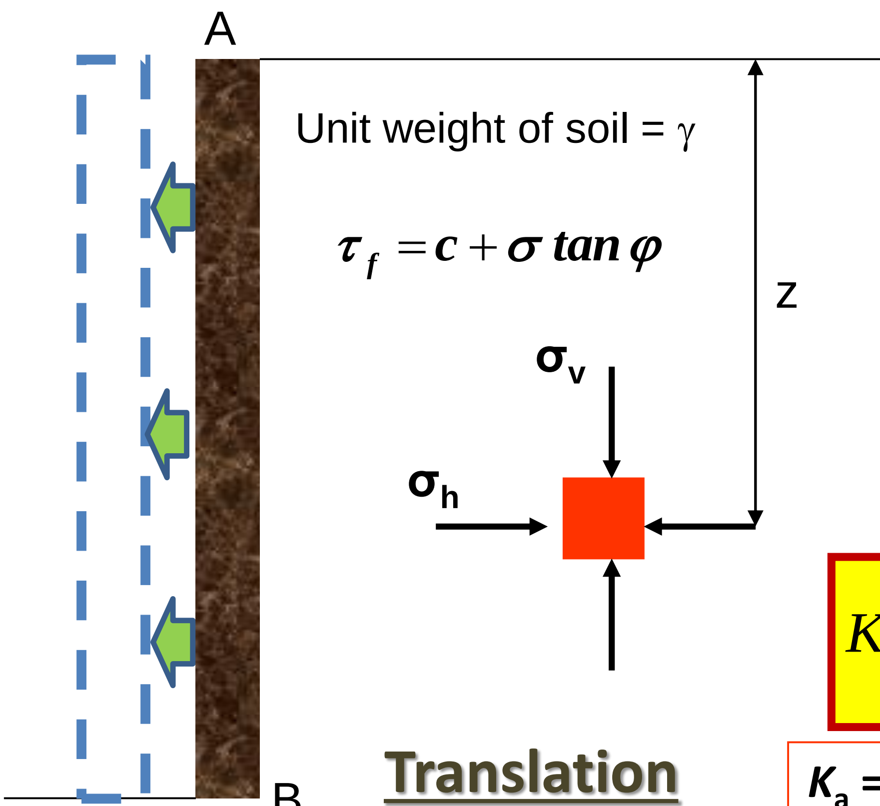
Active Earth Pressure

Case 2: If the frictionless wall rotates sufficiently about its bottom to a position of A′B, then a triangular soil mass ABC′ adjacent to the wall will reach a state of *plastic equilibrium* and will fail sliding down the plane **BC′**.

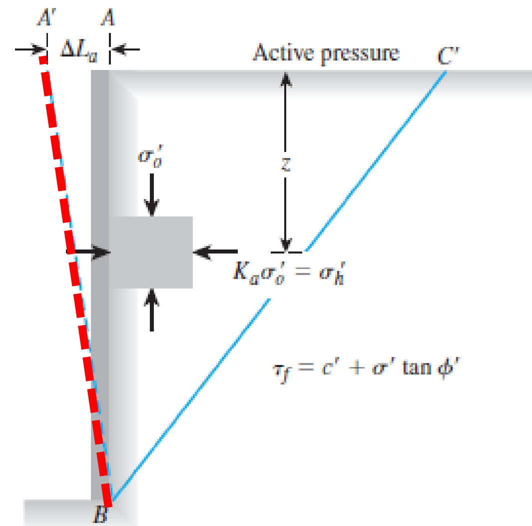
Equally if wall AB is allowed to move away from the soil mass gradually, *horizontal stress* will decrease, and the shearing resistance of the soil is mobilized.

In this case the soil is the ACTUATING ELEMENT

Plastic equilibrium in soil refers to the condition where every point in a soil mass is on the verge of failure.



Translation



Rotation

$$K_a = \frac{\sigma_h}{\sigma_v}$$

K_a = Coefficient of active earth pressure

Passive Earth Pressure

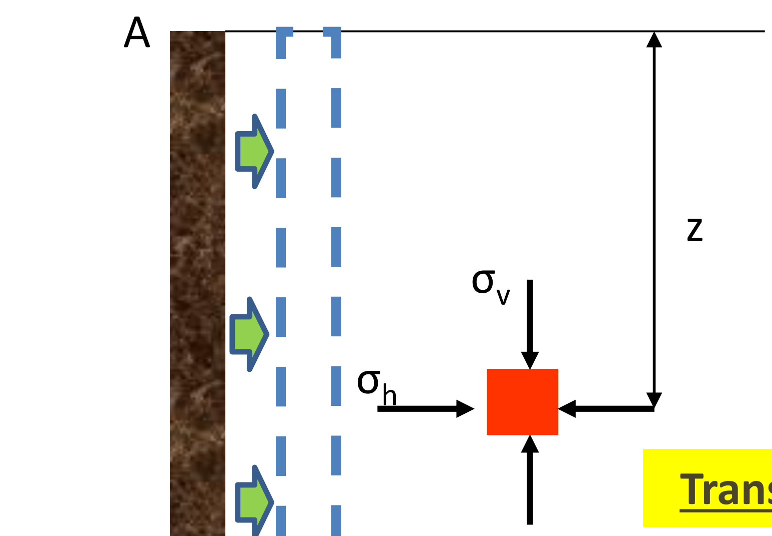
Case 3 :If the frictionless wall rotates sufficiently about its bottom to a position of A′B′′ then a triangular soil mass ABC′′ adjacent to the wall will reach a state of *plastic equilibrium* and will fail sliding upward the plane **BC′′.**

- **If the wall is pushed into the soil mass, σ_h will increase and the shearing resistance of the soil is mobilized.**

In this case the retaining wall is the ACTUATING ELEMENT and the soil provided the resistance for maintaining stability

The lateral earth pressure, σ_h, is called passive earth pressure

K_p = coefficient of passive earth pressure

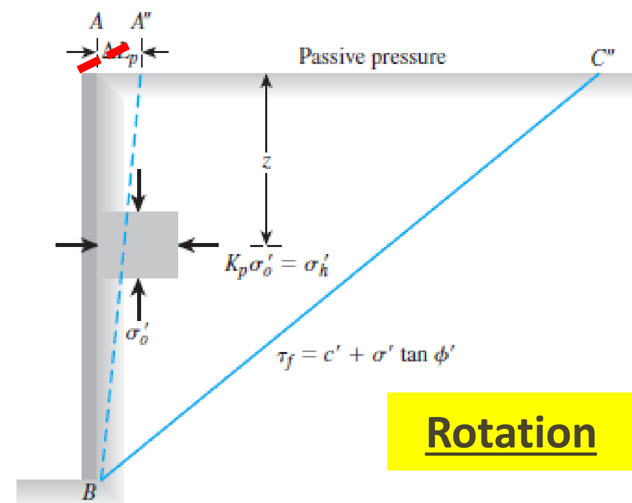


$$K_p = \frac{\sigma_h}{\sigma_v}$$

Translation

Unit weight of soil = γ

$$\tau_f = c + \sigma \tan \varphi$$



Rotation

CASES

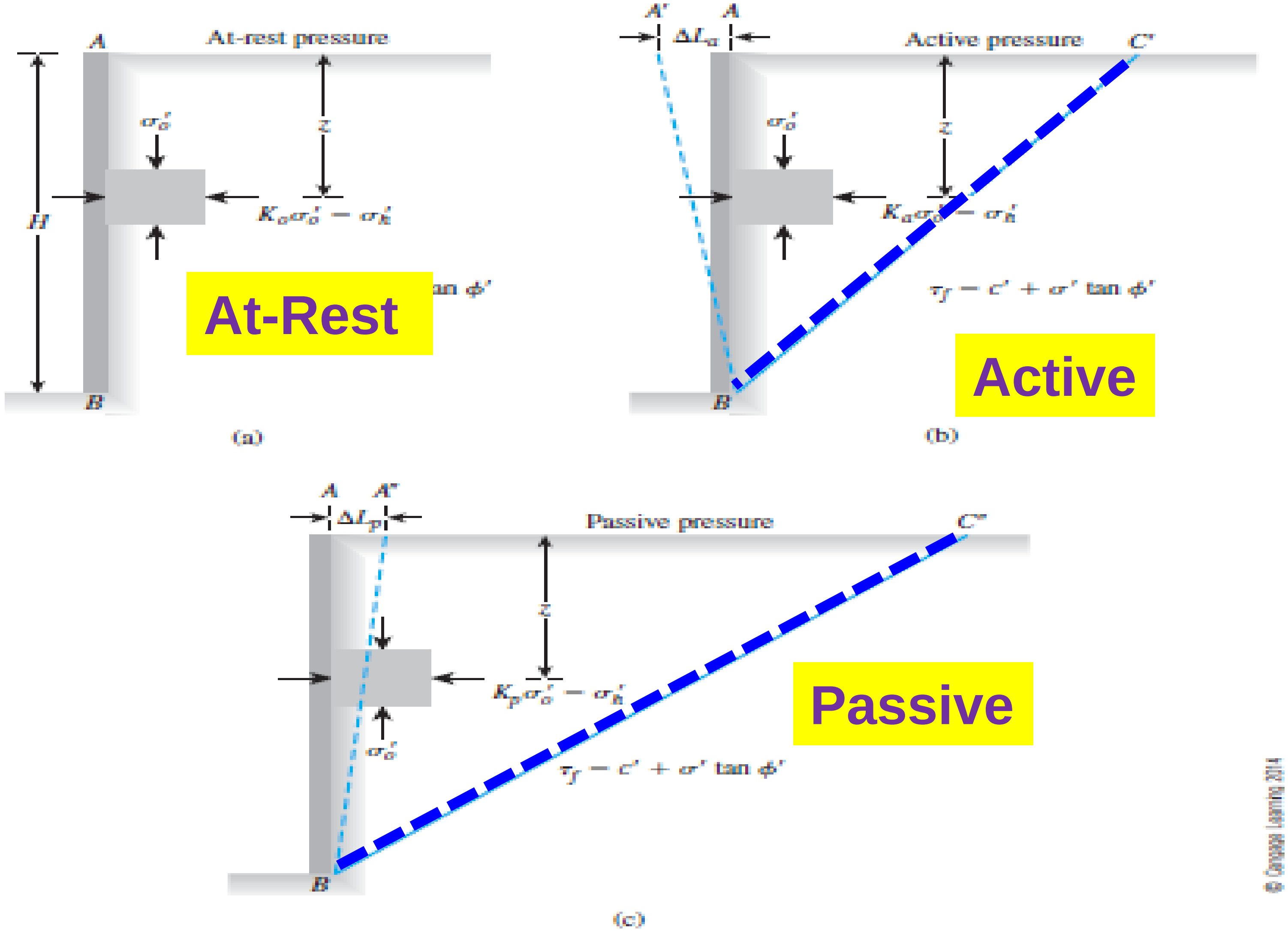
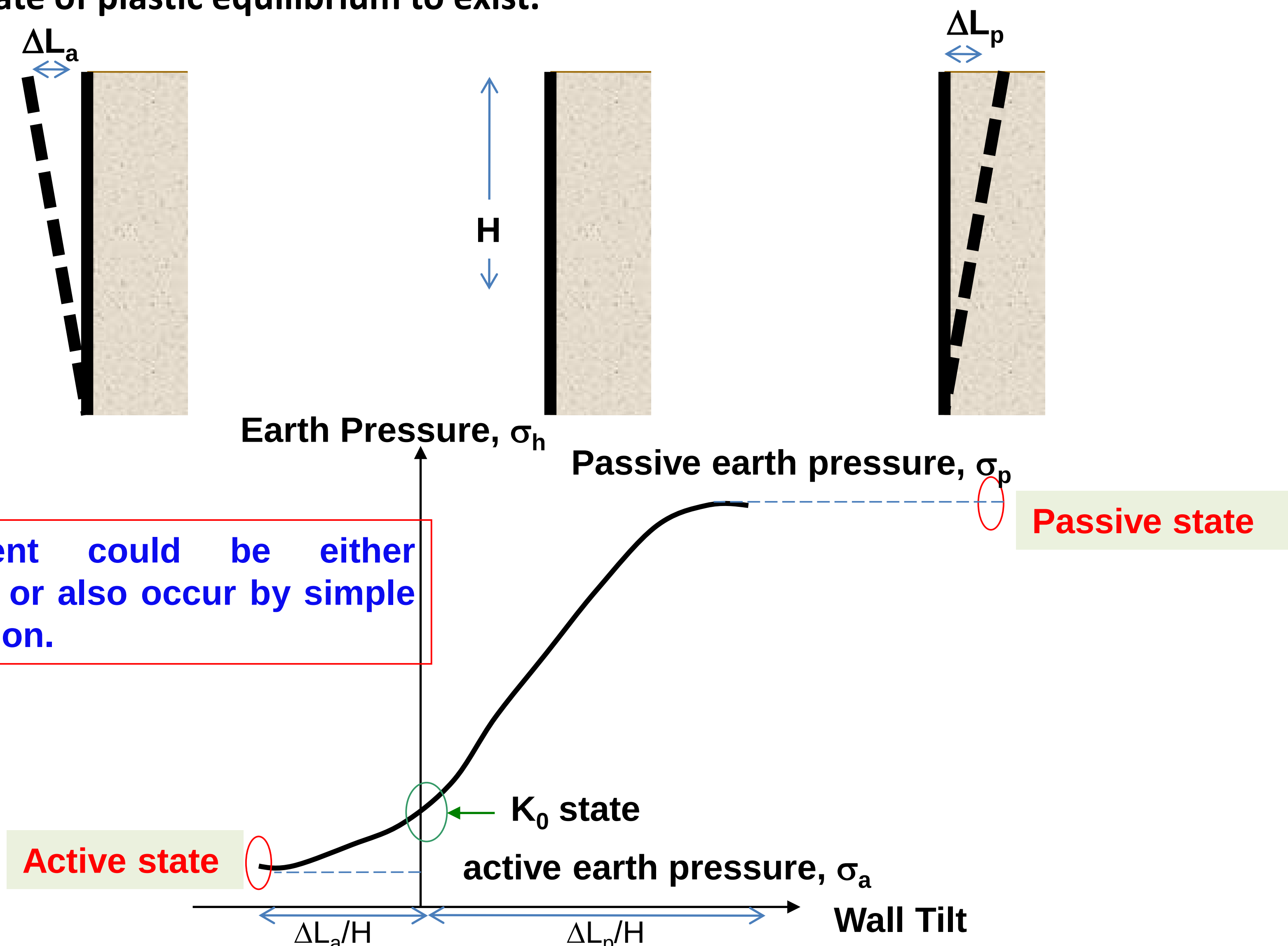


Figure 13.1 Definition of at-rest, active, and passive pressures (*Note:* Wall *AB* is frictionless)

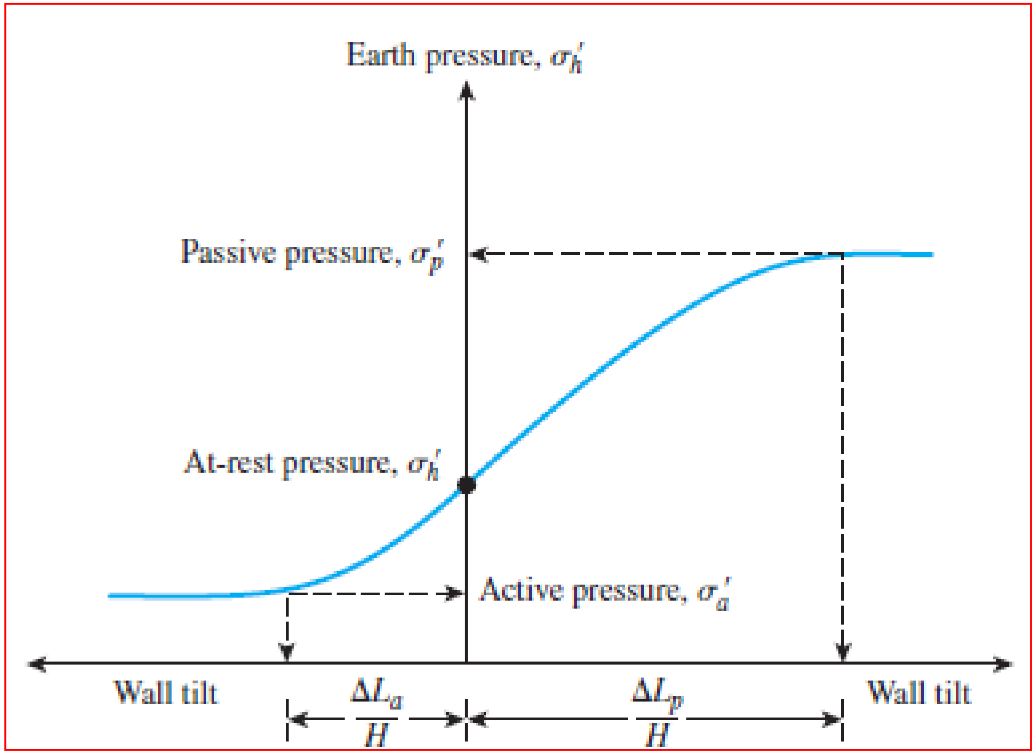
Variation of the Magnitude of Lateral Earth Pressure with Wall Tilt

For the active and passive (Rankine cases), a sufficient yielding of the wall is necessary for a state of plastic equilibrium to exist.

Movement could be either rotation or also occur by simple translation.



REMARKS



Active or passive condition will only be reached if the wall is allowed to yield sufficiently. The amount of wall necessary depends on:-

- **Soil type (sand vs. clay)**
- **Soil density (Loose vs. dense)**
- **Pressure (Active vs. passive)**

Table 13.1 Typical Values of $\Delta L_a/H$ and $\Delta L_p/H$

Soil type	$\Delta L_a/H$	$\Delta L_p/H$
Loose sand	0.001–0.002	0.01
Dense sand	0.0005–0.001	0.005
Soft clay	0.02	0.04
Stiff clay	0.01	0.02

REMARKS

- **If the lateral strain in the soil is ZERO the corresponding lateral pressure is called the earth pressure at-rest. This is the case before construction.**

- **In the case of active case the soil is the actuating element and in the case of passive the wall is the actuating element.**

- **For either the active or passive states to develop, the wall must MOVE. If the wall does not move, an intermediate stress state exists called earth pressure at rest. (i.e. zero lateral strain).**

- **For greatest economy, retaining structures are designed only sufficiently strong to resist ACTIVE PRESSURE. They therefore must be allowed to move.**

- **It may at first seem unlikely that a wall ever would be built to PUSH into the soil and mobilize passive earth pressure.**

Coefficient of Lateral Earth Pressure

K_0

Coefficient of Lateral Earth Pressure K_0

$K_0 = 1 - \sin\phi'$ (Jaky formula)

- **Gives good results when the backfill is loose sand.**
- **For a dense, compacted sand backfill, may grossly underestimate the lateral earth pressure at rest.**

$$K_o = (1 - \sin\phi) + \left[\frac{\gamma_d}{\gamma_{d(\min)}} - 1\right]5.5$$

where γ_d = actual compacted dry unit weight of the sand behind the wall
$\gamma_{d(\min)}$ = dry unit weight of the sand in the loosest state (Chapter 3)

For normally consolidated clays, $K_0 = 0.95 - \sin\phi'$

$$K_o = 0.44 + 0.42\left[\frac{PI\,(\%)}{100}\right]$$

Fine-grained soils

From elastic analysis,

$$K_0 = \frac{\upsilon}{1-\upsilon}$$

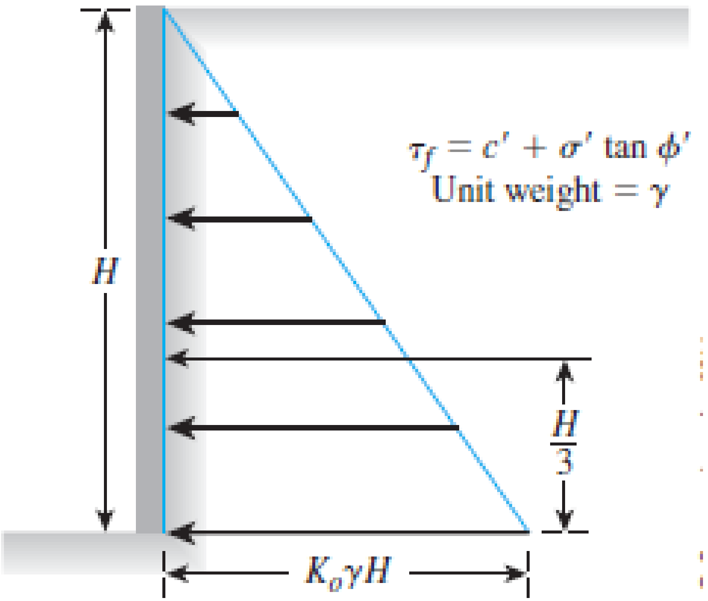
$$K_{o(\text{overconsolidated})} = K_{o(\text{normally consolidated})}\sqrt{OCR}$$

$$K_o = (1 - \sin\phi')(OCR)^{\sin\phi'}$$

Distribution of Lateral Earth Pressure at Rest on a Wall

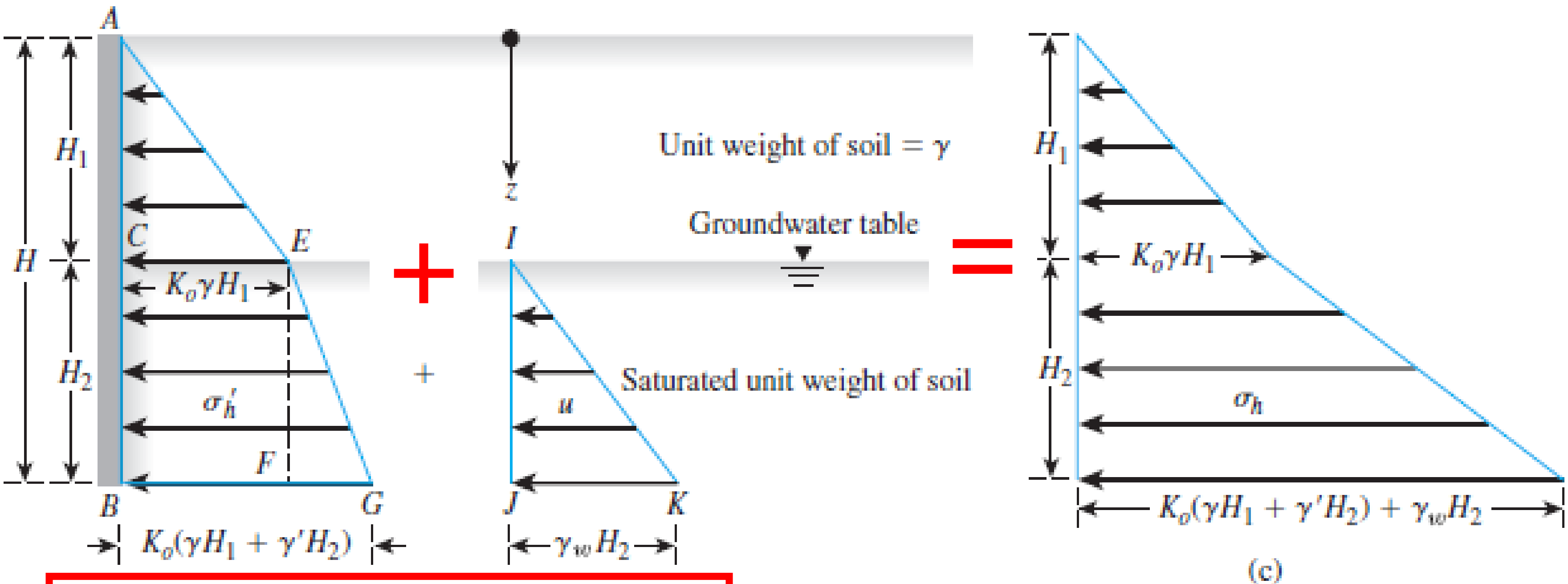
The total force per unit length of the wall, P_o

$$P_o = \frac{1}{2} K_o \gamma H^2$$



Distribution of Lateral Earth Pressure at Rest on a Wall

P_0 for Partially Submerged Soil

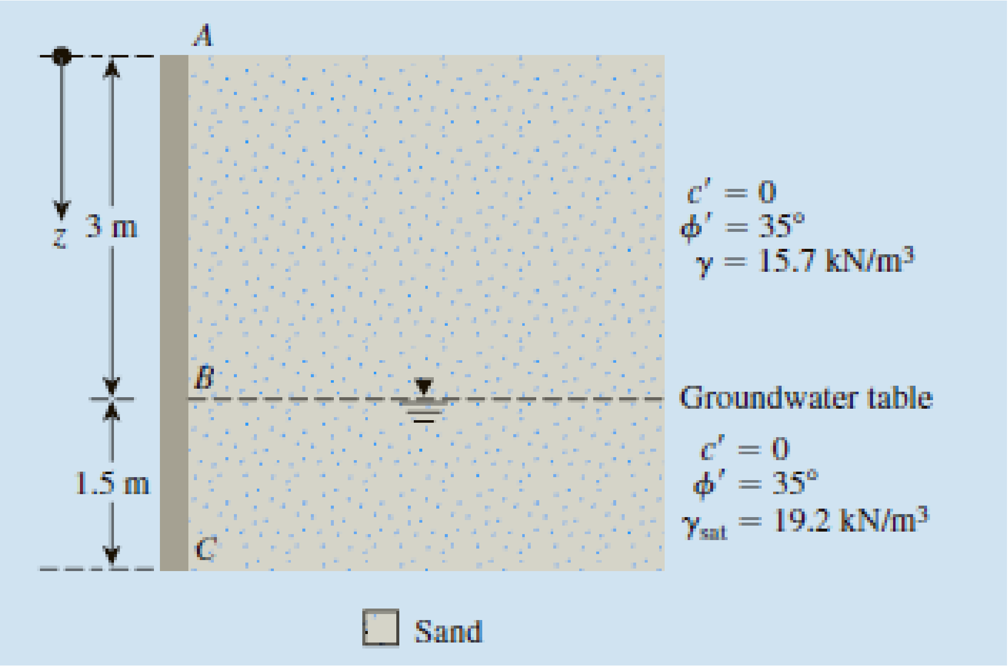


$$P_o = \underbrace{\tfrac{1}{2}K_o\gamma H_1^2}_{\text{Area } ACE} + \underbrace{K_o\gamma H_1 H_2}_{\text{Area } CEFB} + \underbrace{\tfrac{1}{2}(K_o\gamma' + \gamma_w)H_2^2}_{\text{Areas } EFG \text{ and } IJK}$$

EXAMPLE 13.1

Example 13.1

Figure 13.6a shows a 4.5-m-high retaining wall. The wall is restrained from yielding. Calculate the lateral force P_o per unit length of the wall. Also, determine the location of the resultant force. Assume that for sand $OCR - 1.5$.



EXAMPLE 13.1

Solution

$$K_o = (1 - \sin\phi')(OCR)^{\sin\phi'}$$
$$= (1 - \sin 35)(1.5)^{\sin 35} = 0.538$$

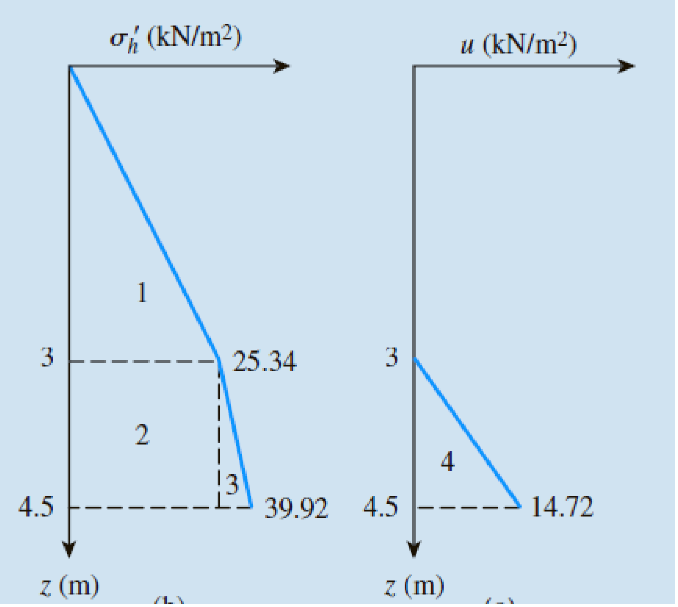
At $z = 0$: $\sigma'_o = 0$; $\sigma'_h = 0$; $u = 0$

At $z = 3$ m: $\sigma'_o = (3)(15.7) = 47.1\ \text{kN/m}^2$

$$\sigma'_h = K_o\sigma'_o = (0.538)(47.1) = 25.34\ \text{kN/m}^2$$
$$u = 0$$

At $z = 4.5$: $\sigma'_o = (3)(15.7) + (1.5)(19.2 - 9.81) = 61.19\ \text{kN/m}^2$

$$\sigma'_h = K_o\sigma'_o = (0.538)(61.19) = 39.92\ \text{kN/m}^2$$
$$u = (1.5)(\gamma_w) = (1.5)(9.81) = 14.72\ \text{kN/m}^2$$



Lateral force P_o = Area 1 + Area 2 + Area 3 + Area 4

or

$$P_o = \left(\frac{1}{2}\right)(3)(25.34) + (1.5)(25.34) + \left(\frac{1}{2}\right)(1.5)(14.58) + \left(\frac{1}{2}\right)(1.5)(14.72)$$
$$= 38.01 + 38.01 + 10.94 + 11.04 = 98\ \text{kN/m}$$

EXAMPLE 13.1

The location of the resultant, measured from the bottom of the wall, is

$$\bar{z} = \frac{\Sigma \text{ moment of pressure diagram about } C}{P_o}$$

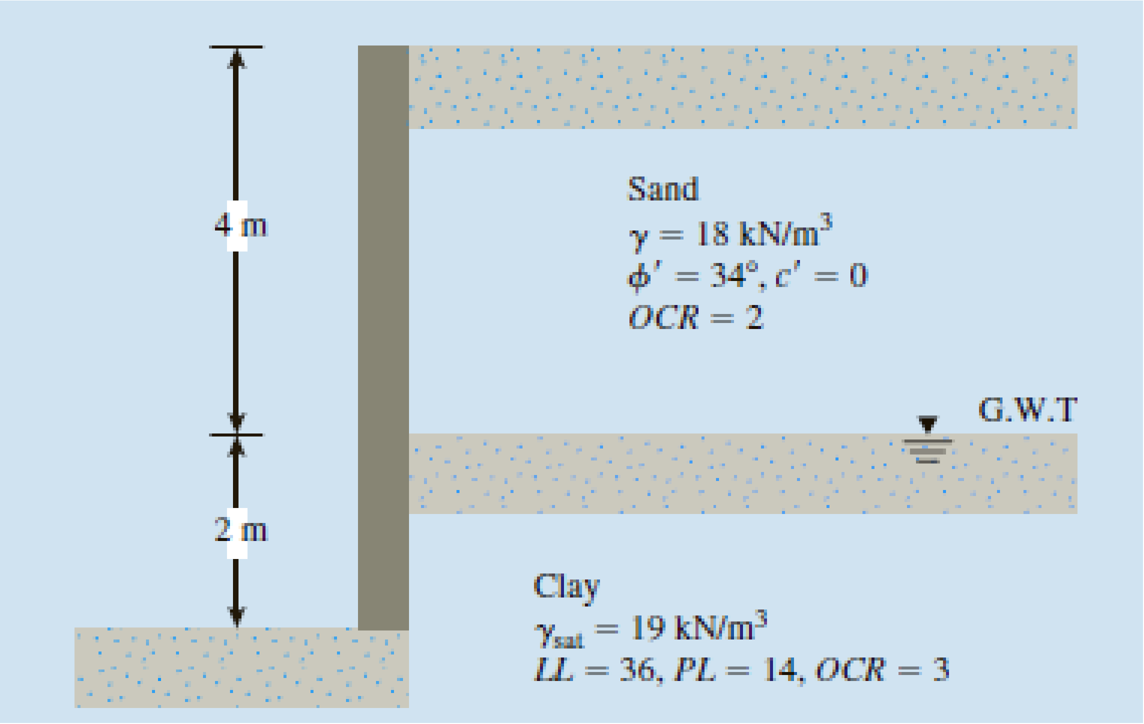
or

$$\bar{z} = \frac{(38.01)\left(1.5 + \frac{3}{3}\right) + (38.01)\left(\frac{1.5}{2}\right) + (10.94)\left(\frac{1.5}{3}\right) + (11.04)\left(\frac{1.5}{3}\right)}{98} = \mathbf{1.76\ m}$$

EXAMPLE 13.2

Example 13.2

Figure 13.7a shows a non-yielding vertical wall retaining a sandy backfill underlain by clay. Determine the magnitude of the resultant at-rest force per unit length on the wall, P_o.



EXAMPLE 13.2

Solution

For sand, $\phi' = 34°$ and OCR = 2. From Eq. (13.7),

$$K_{o(\text{sand})} = (1 - \sin\phi')(OCR)^{\sin\phi'} = (1 - \sin 34)(2)^{\sin 34} \approx 0.65$$

For clay, $LL = 36$ and $PL = 14$. So, $PI = 36 - 14 = 22$. From Eqs. (13.8) and (13.9),

$$K_{o(\text{clay})} = \left\{0.44 + 0.42\left[\frac{PI(\%)}{100}\right]\right\}(OCR)^{0.5} = \left[0.44 + (0.42)\left(\frac{22}{100}\right)\right](3)^{0.5} = 0.922$$

At $z = 0$: $\sigma'_o = 0$

$u = 0$

At $z = 4$ m(−): $\sigma'_o = 4 \times 18 = 72\ \text{kN/m}^2$

$\sigma'_h = K_{o(\text{sand})}\sigma'_o = (0.65)(72) = 46.8\ \text{kN/m}^2$

$u = 0$

At $z = 4$ m(+): $\sigma'_h = K_{o(\text{clay})}\sigma'_o = (0.922)(72) = 66.38\ \text{kN/m}^2$

$u = 0$

At $z = 6$ m: $\sigma'_o = (18 \times 4) + (19 - 9.81)(2) = 72 + 18.38 = 90.38\ \text{kN/m}^2$

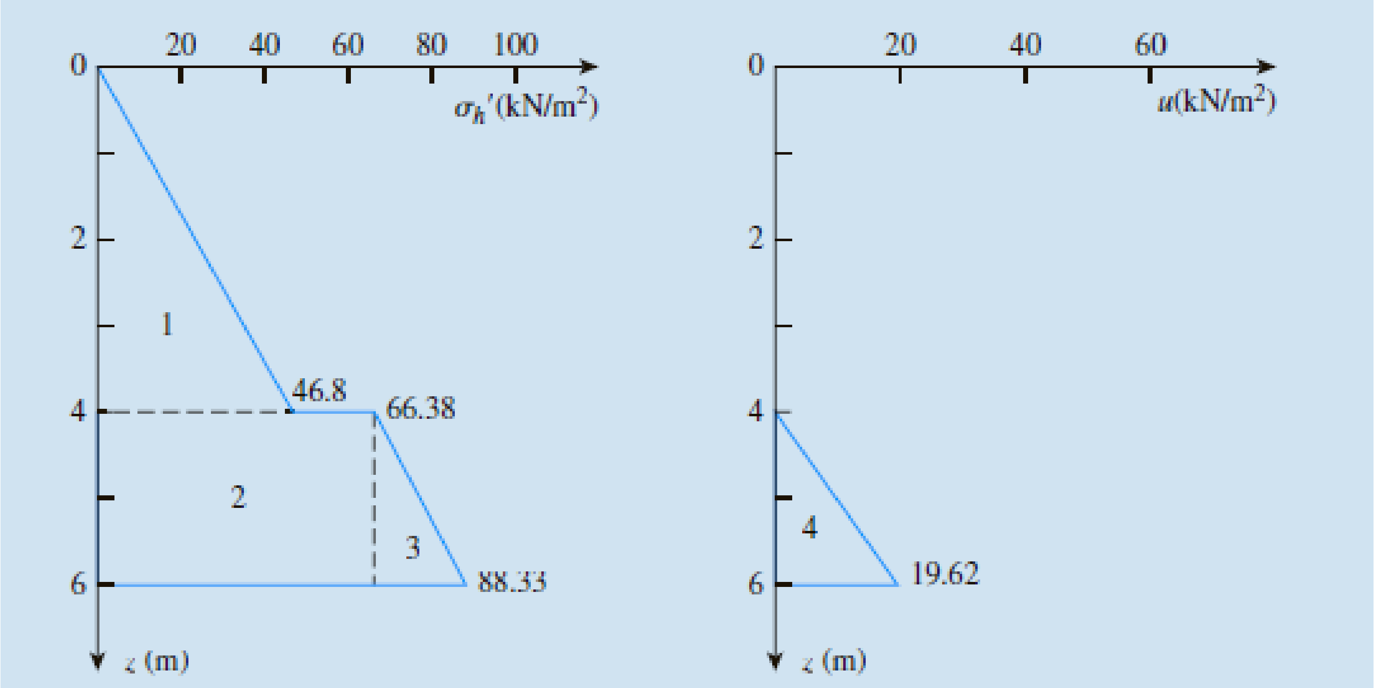
$\sigma'_h = K_{o(\text{clay})}\sigma'_o = (0.922)(90.38) = 83.33\ \text{kN/m}^2$

$u = 2\gamma_w = (2)(9.81) = 19.62\ \text{kN/m}^2$

EXAMPLE 13.2

The variations of σ'_h and u with z are shown in Figures 13.7b and 13.7c, respectively. So,

$$P_o = \text{Area 1} + \text{Area 2} + \text{Area 3} + \text{Area 4}$$
$$= \left(\tfrac{1}{2}\right)(4)(46.8) + (2)(66.38) + \left(\tfrac{1}{2}\right)(88.33 - 66.38)(2) + \left(\tfrac{1}{2}\right)(2)(19.62)$$
$$= 93.6 + 132.76 + 21.95 + 19.62 = \mathbf{267.93\ kN/m}$$



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