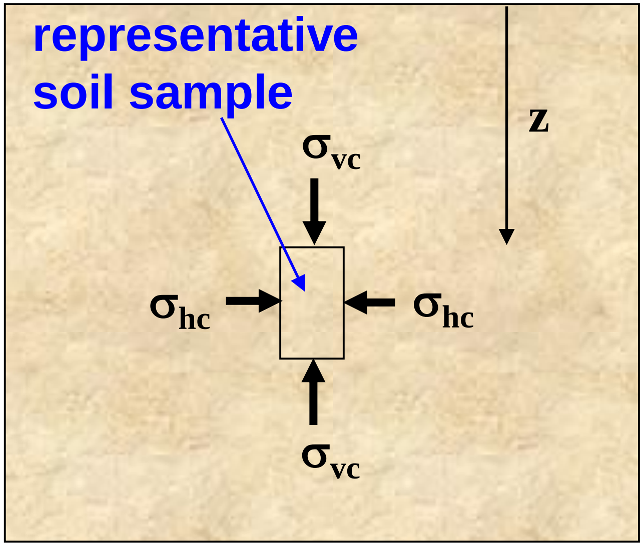
SHEAR STRENGTH OF SOIL

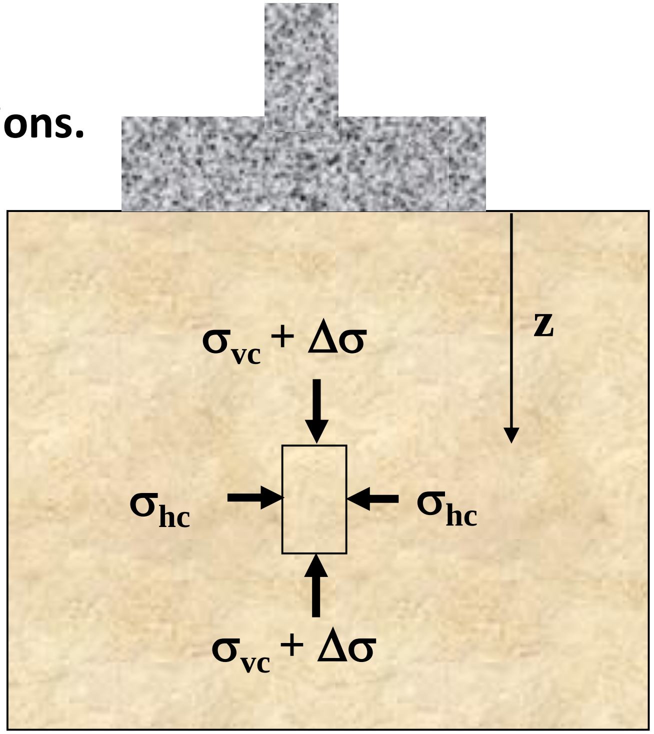
CHAPTER 12

Triaxial Shear Test

- The most reliable method now available for determination of shear strength parameters.
- Entire books have been written on triaxial test .
- The test is used to measure the shear strength of a soil under controlled drainage conditions.
- The test is designed to simulate actual field conditions.



Before construction



After and during construction

Triaxial Shear Test

- The test is called "triaxial" because the three principle stresses are assumed to be known and controlled.
- The triaxial test is much more complicated than the direct shear but also much more versatile.
- The failure plane can occur anywhere and we can control the stress paths to failure reasonably well, which means that complex stress paths in the field can more effectively be modeled in the laboratory with the triaxial test.

Principles of Triaxial Test

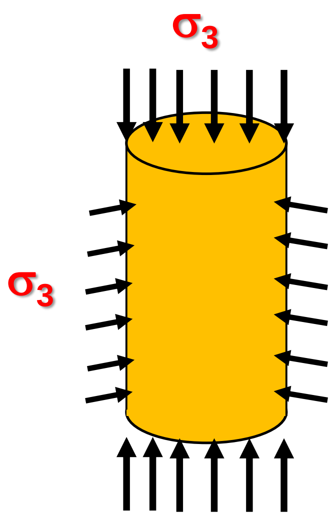
- To simulate field conditions, soil samples is subjected to the following stages:

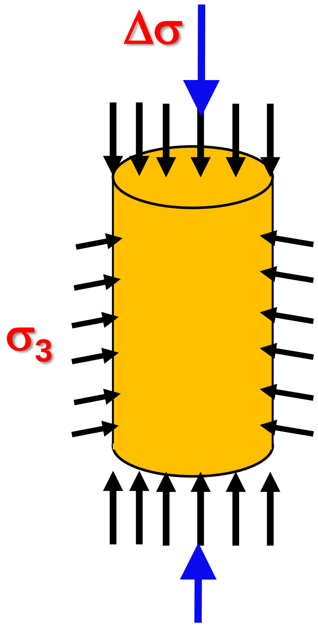
1. Saturation of sample (Check of B value)
2. Applying confining (cell) pressure (σ_3) is applied on the soil sample. The confining pressure is within the range of that subjected in the field.
3. Apply an increasing vertical stress ($\Delta\sigma = \sigma_1 - \sigma_3$) -termed the deviator stress- until failure.
4. The specimen is free to fail on any weak plane or, as sometimes occurs, to simply **BULGE**.

- σ_3 = Confining pressure

  = Cell pressure

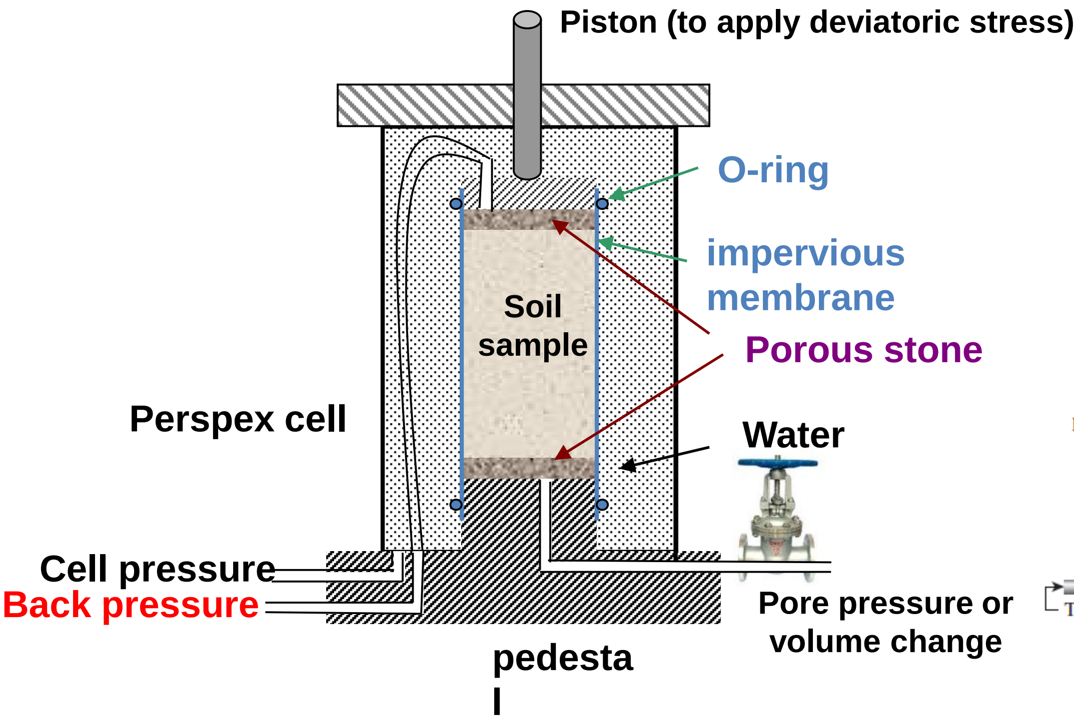
  = All-around pressure

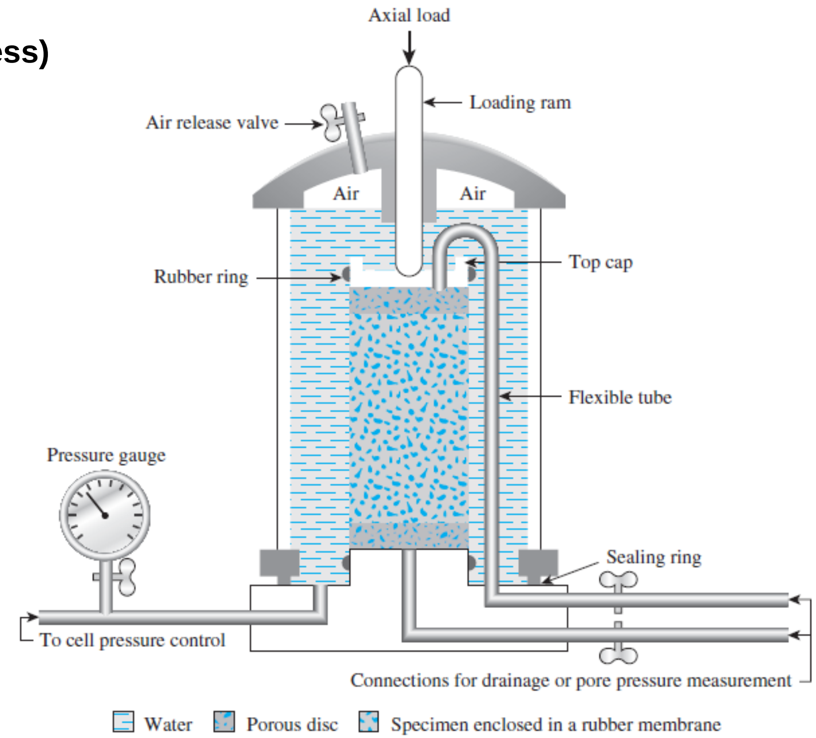




Deviator Principle Stress, $\Delta\sigma$ or $\Delta\sigma_d$ $= \sigma_1 - \sigma_3$

Triaxial Shear Test Device





Two ways for appying axial load

1. Application of dead weights or hydraulic pressure in equal increments until the specimen fails.
2. Application of axial deformation at a constant rate by means of a geared or hydraulic loading press. This is a strain-controlled test.

Specimen Preparation (undisturbed sample)

Edges of the sample are carefully trimmed

Sample is covered with a rubber membrane and sealed

Setting up the sample in the triaxial cell

Cell is completely filled with water

Apparatus Assembly

Proving ring to measure the deviator load

Dial gauge to measure vertical displacement

Types of Triaxial Test

- **Many variations of test procedure are possible with the triaxial apparatus but the three principal types of test are as follows:**

Confining Pressure

- **Consolidated**
- **Consolidated**
- **Unconsolidated**

Shearing

- Drained (CD) Test
- Undrained (CU)Test
- Undrained (UU) Test

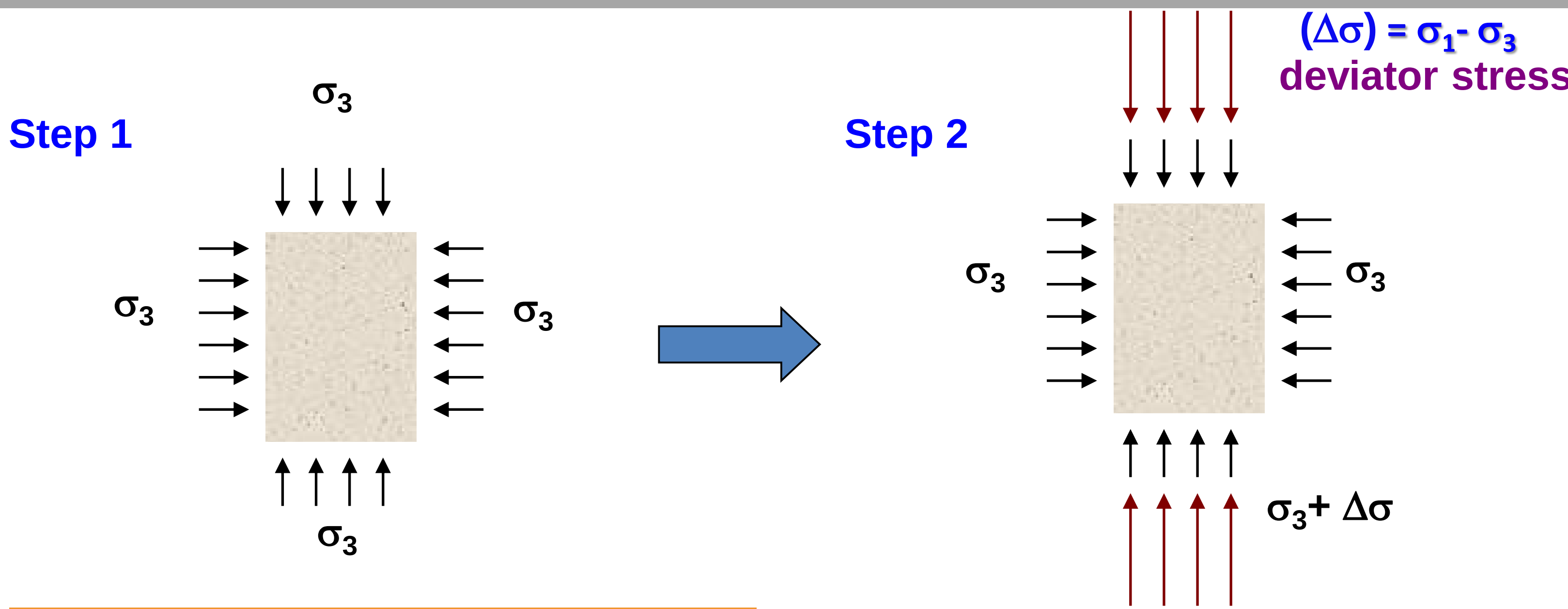
- **Depends on whether drainage is allowed or not during the confining or shearing stage.**

- **The different types of triaxial test are commonly designated by a two-letter symbol. The first letter refers to what happens BEFORE SHEAR that is whether the specimen is consolidated or not. The second letter refers to the drainage conditions during SHEAR, CD, CU, UU.**

Types of Triaxial Test

Test	Drainage during confinement	Drainage during shear	Pore water pressure build up?	Total or Effective	Type of test "duration"
CD	Open	Open	No if the test is slow	Effective	Slow for clay S- test
CU	Open	Closed	Yes	Total Effective if p.w.p is measured	
UU	Closed	Closed	Yes	Total	Fast Q-test

Types of Triaxial Test



Under Confining (cell) pressure σ_3

Shearing (loading)

Is the drainage valve open?

- yes → **Consolidated sample**
- no → **Unconsolidated sample**

Is the drainage valve open?

- yes → **Drained loading**
- no → **Undrained loading**

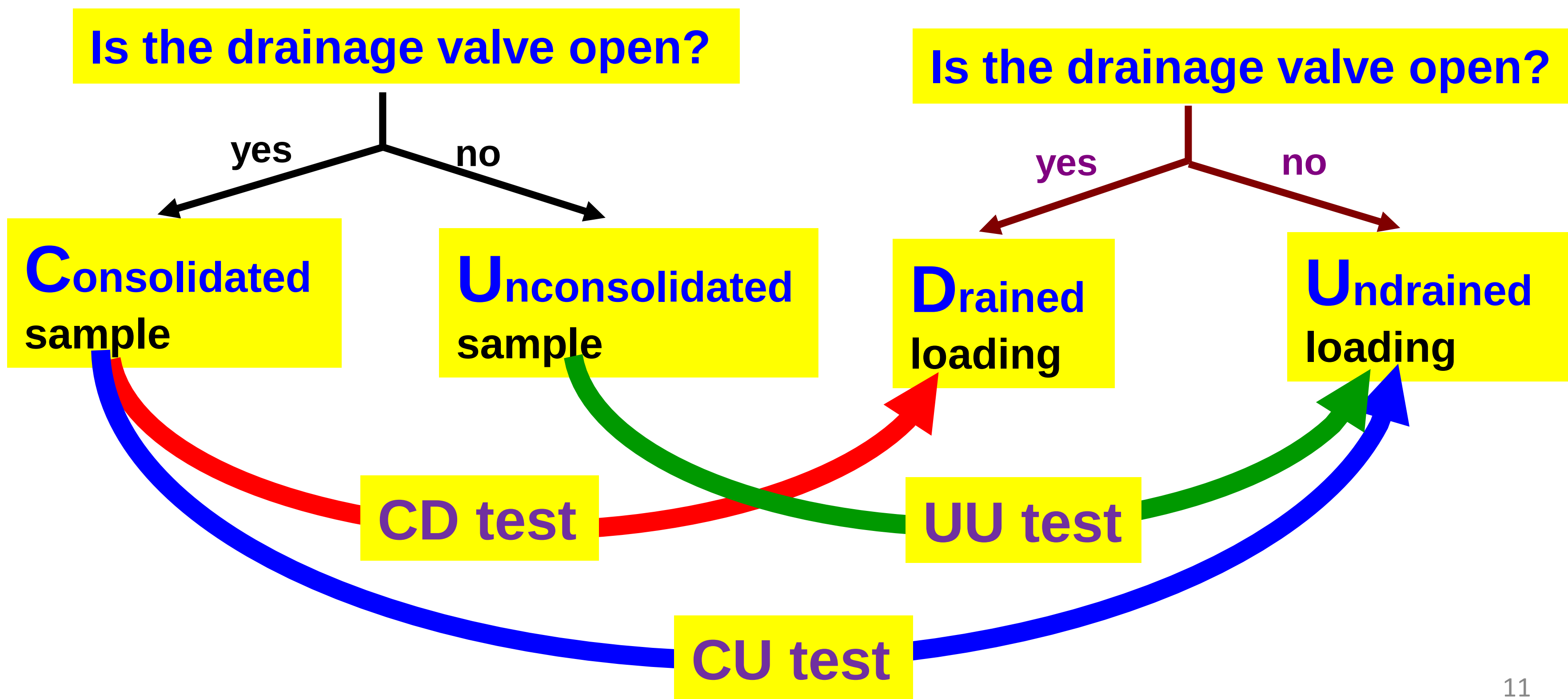
Types of Triaxial Test

Step 1

Under Confining (cell) pressure σ_3

Step 2

Shearing (loading)



I. Consolidated Drained Test (CD Test)

- No excess pore pressure throughout the test

- Very slow shearing to avoid build-up of pore pressure - hence this test is termed the S-test (for "slow" test)

Can be days!

- Gives C′ and ϕ′

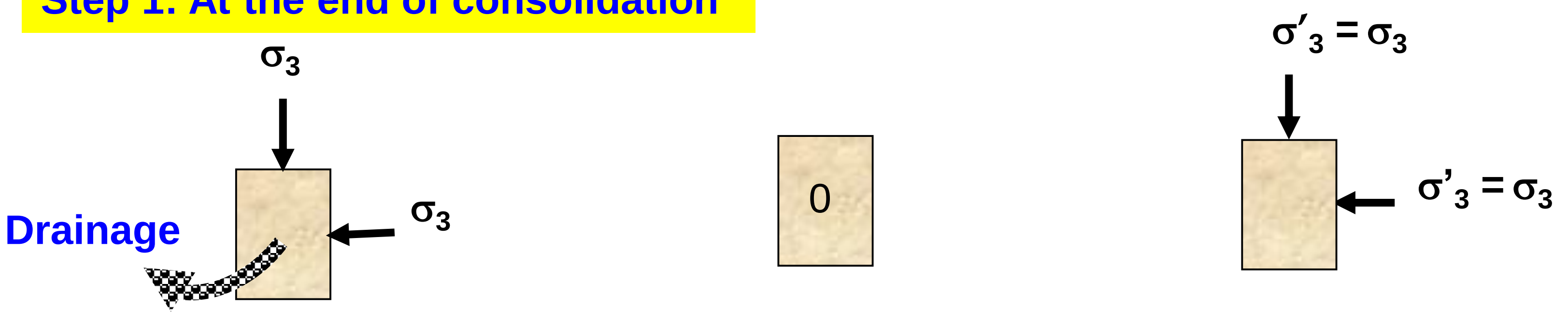
- Note that at all the times during CD test, the pore water pressure is essentially zero.

Use C' and ϕ′ for analysing fully drained Situations (i.e. long term stability, very slow loading)

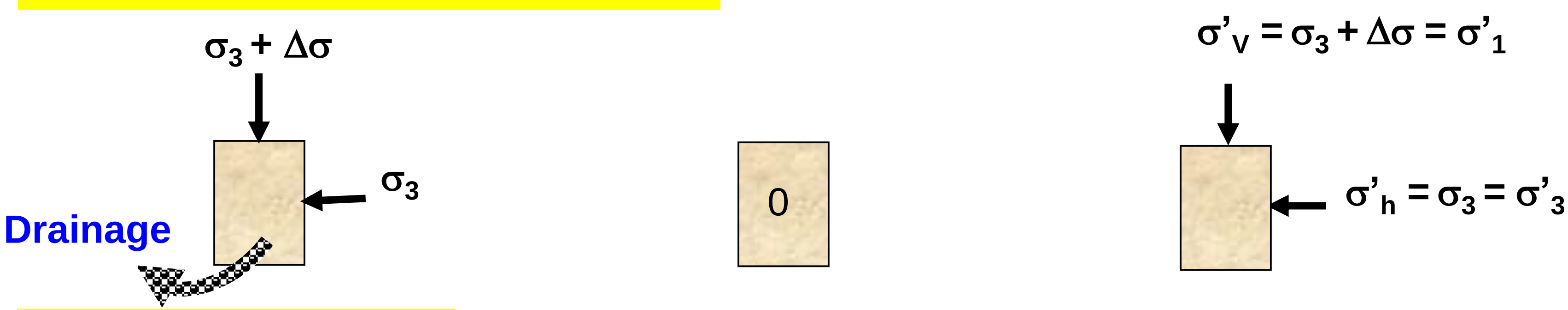
Stress conditions for the consolidated drained test

Total, σ = **Neutral, u** + **Effective, σ’**

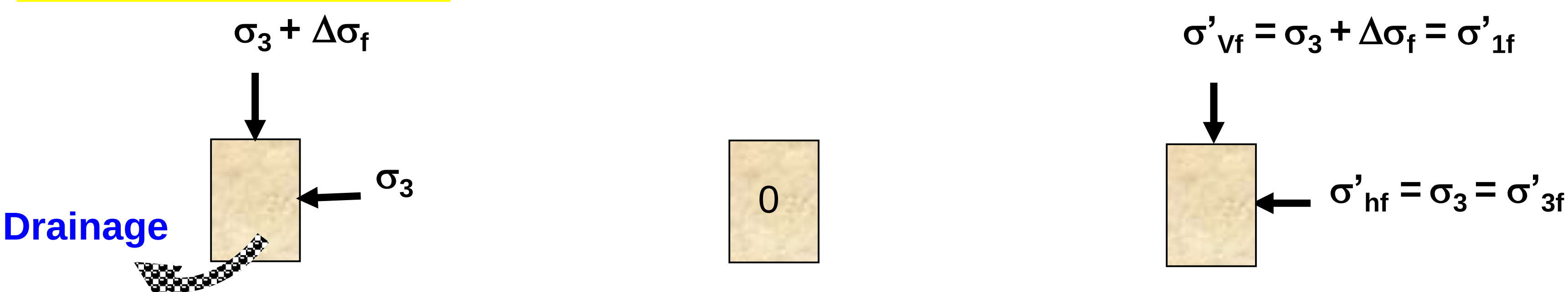
Step 1: At the end of consolidation



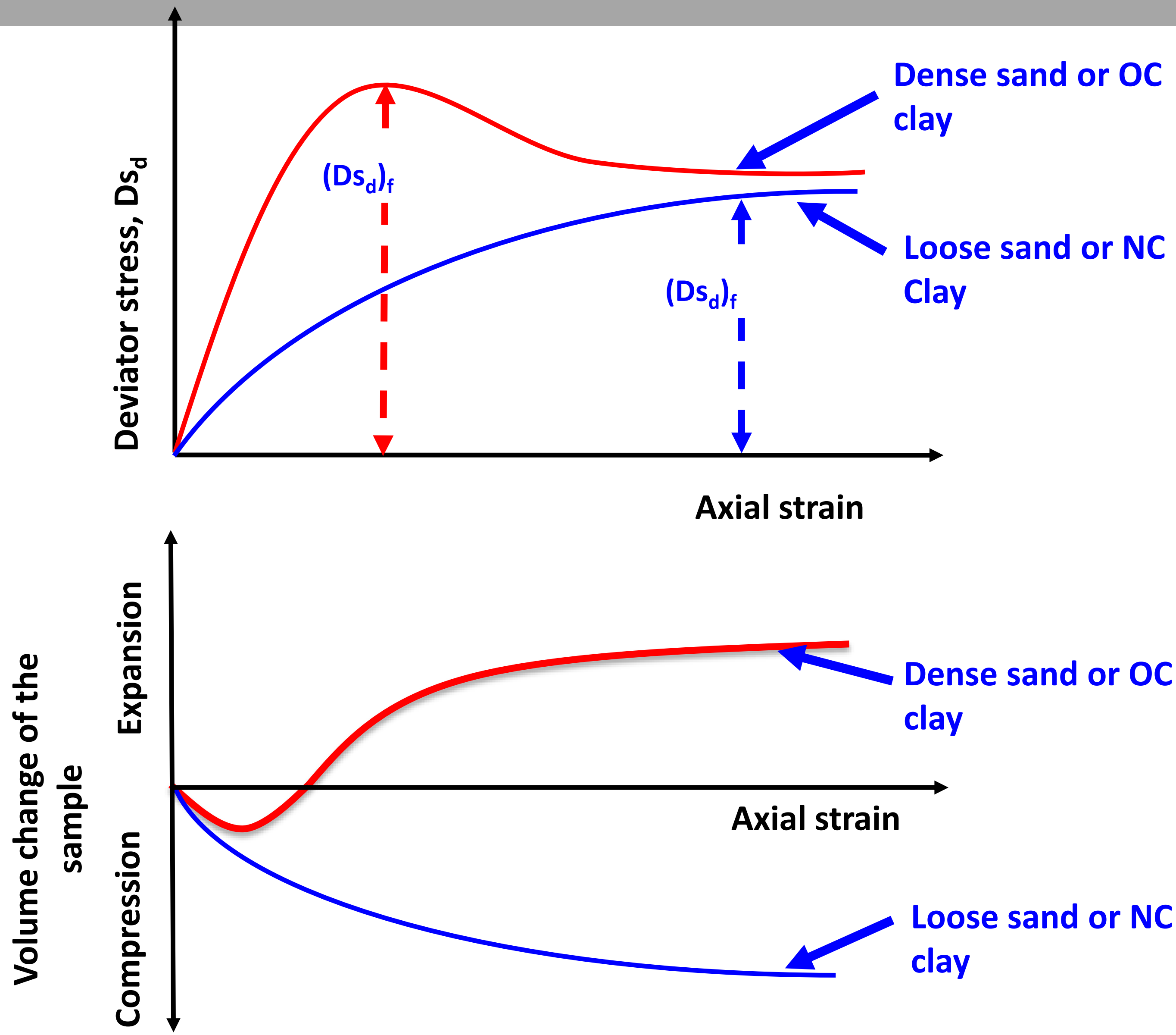
Step 2: During axial stress increase



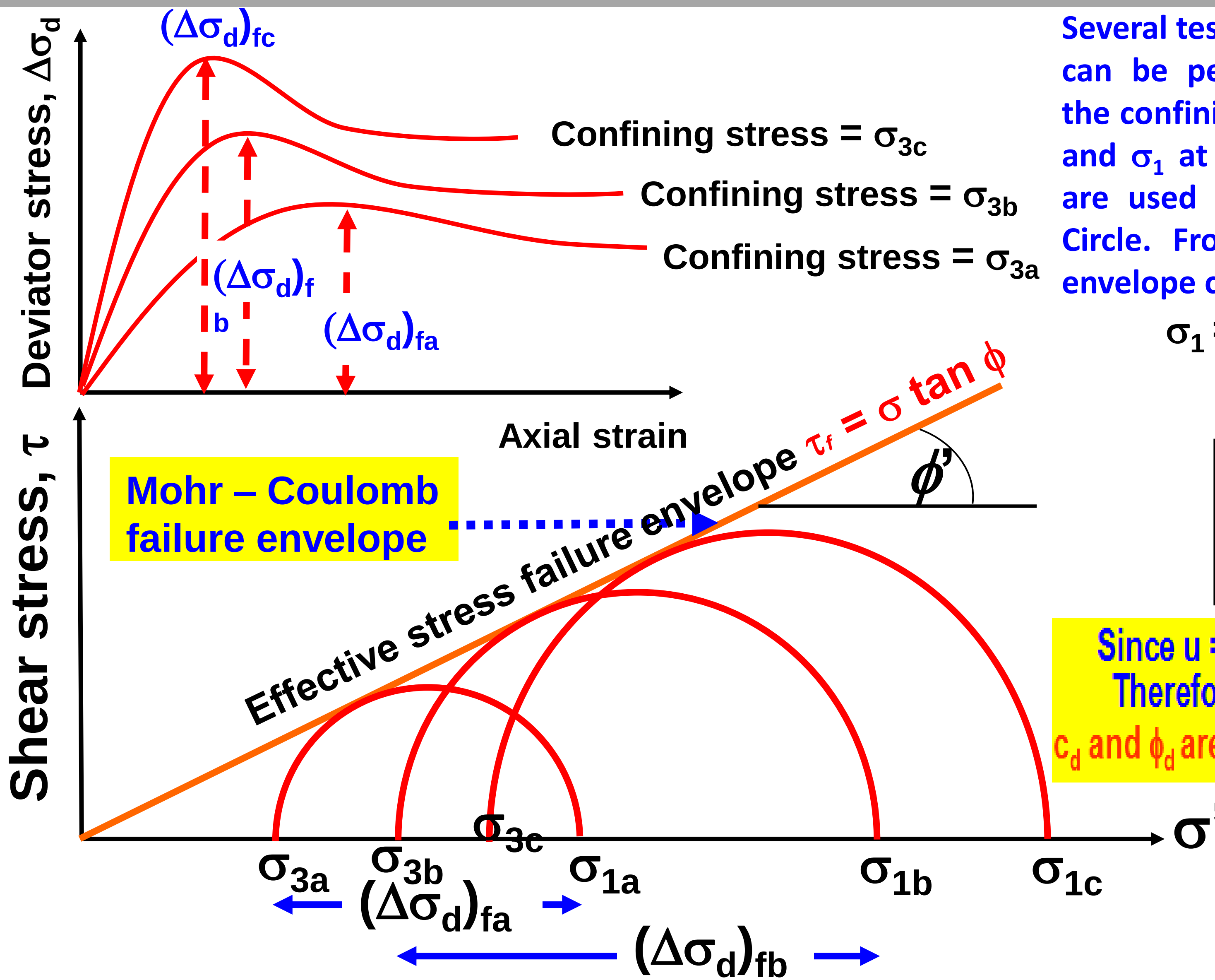
Step 3: At failure



Stress-strain relationship during shearing (CD Test)

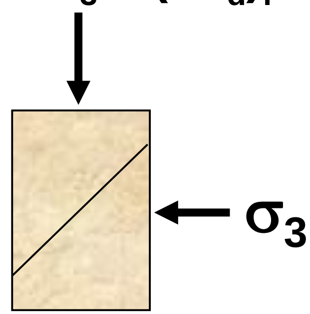


Consolidated Drained Test (CD Test)



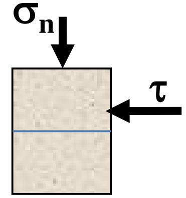
Several tests on similar samples can be performed by varying the confining pressure. Then σ_3 and σ_1 at failure for each test are used to construct Mohr's Circle. From that M-C failure envelope can be obtained.

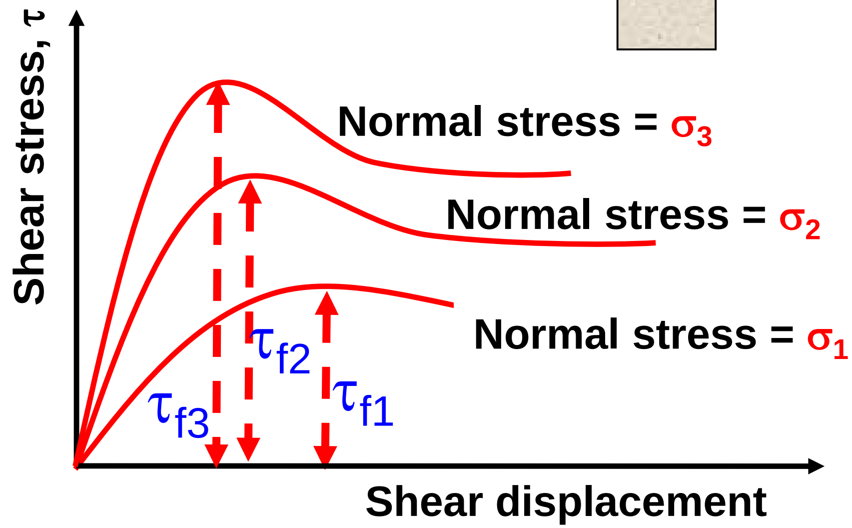
$$\sigma_1 = \sigma_3 + (\Delta\sigma_d)_f$$



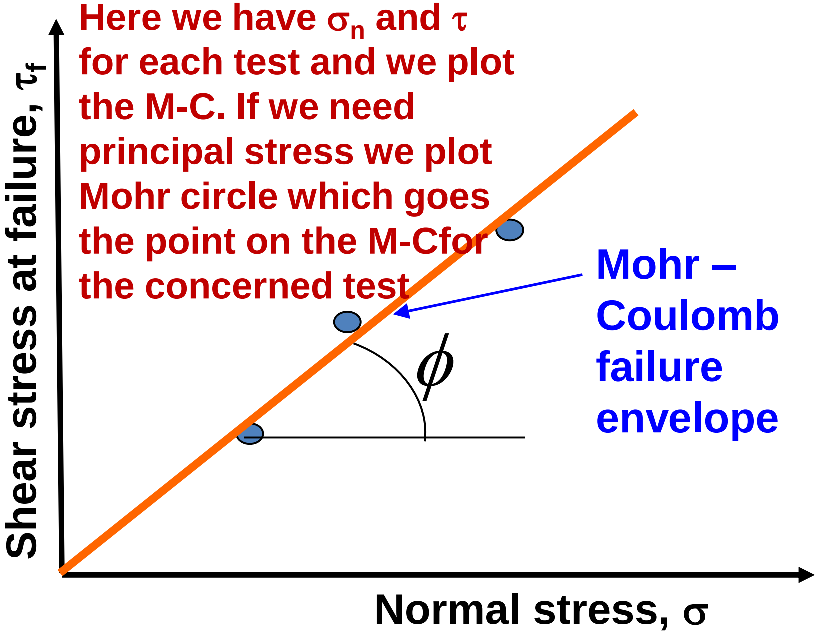
Since u = 0 in CD tests, σ = σ'
Therefore, c = c' and ϕ = ϕ'
c_d and ϕ_d are used to denote them

Can you think about Direct shear and Triaxial Test w.r.t. analysis of results



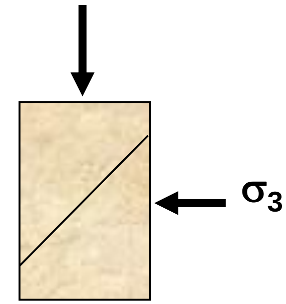


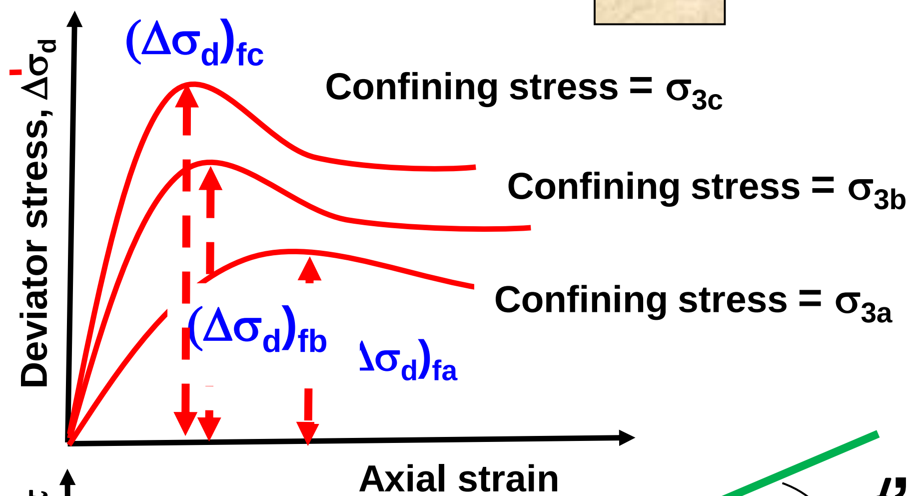
Here we have σ_n and τ for each test and we plot the M-C. If we need principal stress we plot Mohr circle which goes the point on the M-Cfor the concerned test

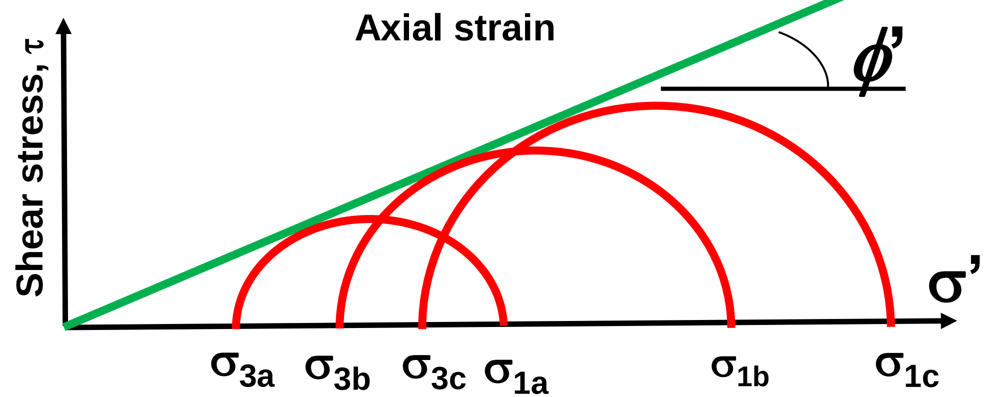


σ_1 and σ_3 for each test from that we plot Mohr circle and then we plot tangent to the circles (if is assumed = 0) only we need one circle)

$\sigma_1 = \sigma_3 + (\Delta\sigma_d)_f$







Example 12.3

Example 12.3

A consolidated-drained triaxial test was conducted on a normally consolidated clay. The results are

$\sigma_3 = 140\ \text{kN/m}^2$
$(\Delta\sigma_d)_f = 104\ \text{kN/m}^2$

Determine:

a. Angle of friction, ϕ'
b. Angle θ that the failure plane makes with the major principal plane

Solution

For normally consolidated soil, the failure envelope equation is

$$\tau_f = \sigma' \tan\phi' \quad (\text{because } c' = 0)$$

For the triaxial test, the effective major and minor principal stresses at failure are

$$\sigma_1' = \sigma_1 = \sigma_3 + (\Delta\sigma_d)_f = 140 + 104 = 244\ \text{kN/m}^2$$

and

$$\sigma_3' = \sigma_3 = 140\ \text{kN/m}^2$$

Part a

The Mohr's circle and the failure envelope are shown in Figure 12.26. From Eq. (12.21),

$$\sin\phi' = \frac{\sigma_1' - \sigma_3'}{\sigma_1' + \sigma_3'} = \frac{244 - 140}{244 + 104} = 0.333$$

or

$$\phi' = \mathbf{17.46°}$$

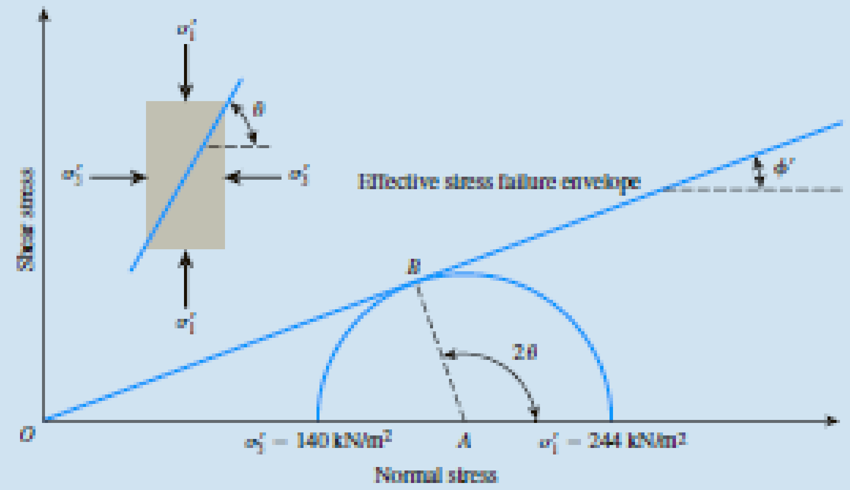


Figure 12.26

Part b

From Eq. (12.4),

$$\theta = 45 + \frac{\phi'}{2} = 45° + \frac{17.46°}{2} = \mathbf{53.73°}$$

Example 12.4

Example 12.4

Refer to Example 12.3.

a. Find the normal stress σ' and the shear stress τ_f on the failure plane.
b. Determine the effective normal stress on the plane of maximum shear stress.

Solution

Part a

From Eqs. (10.8) and (10.9),

$$\sigma'(\text{on the failure plane}) = \frac{\sigma_1' + \sigma_3'}{2} + \frac{\sigma_1' - \sigma_3'}{2} \cos 2\theta$$

and

$$\tau_f = \frac{\sigma_1' - \sigma_3'}{2} \sin 2\theta$$

Substituting the values of $\sigma_1' = 244$ kN/m², $\sigma_3' = 140$ kN/m², and $\theta = 53.36°$ into the preceding equations, we get

$$\sigma' = \frac{244 + 140}{2} + \frac{244 - 140}{2} \cos(2 \times 53.73) = \mathbf{176.36\ kN/m^2}$$

and

$$\tau_f = \frac{244 - 140}{2} \sin(2 \times 53.73) = \mathbf{49.59\ kN/m^2}$$

Part b

From Eq. (10.9), it can be seen that the maximum shear stress will occur on the plane with $\theta = 45°$. From Eq. (10.8),

$$\sigma' = \frac{\sigma_1' + \sigma_3'}{2} + \frac{\sigma_1' - \sigma_3'}{2} \cos 2\theta$$

Substituting $\theta = 45°$ into the preceding equation gives

$$\sigma' = \frac{244 + 140}{2} + \frac{244 - 140}{2} \cos 90 = \mathbf{192\ kN/m^2}$$

Example 12.5

Example 12.5

The equation of the effective stress failure envelope for normally consolidated clayey soil is $\tau_f = \sigma' \tan 28°$. A drained triaxial test was conducted with the same soil at a chamber-confining pressure of 100 kN/m². Calculate the deviator stress at failure.

Solution

For normally consolidated clay, $c' = 0$. Thus, from Eq. (12.8),

$$\sigma_1' = \sigma_3' \tan^2\left(45 + \frac{\phi'}{2}\right)$$

$$\phi' = 28°$$

$$\sigma_1' = 100 \tan^2\left(45 + \frac{28}{2}\right) = 277 \text{ kN/m}^2$$

So,

$$(\Delta\sigma_d)_f = \sigma_1' - \sigma_3' = 277 - 100 = \mathbf{177\ kN/m^2}$$

Example 12.6

Example 12.6

The results of two drained triaxial tests on a saturated clay follow:

Specimen I:

$$\sigma_3 = 70 \text{ kN/m}^2$$

$$(\Delta\sigma_d)_f = 130 \text{ kN/m}^2$$

Specimen II:

$$\sigma_3 = 160 \text{ kN/m}^2$$

$$(\Delta\sigma_d)_f = 223.5 \text{ kN/m}^2$$

Determine the shear strength parameters.

Solution

Refer to Figure 12.27. For Specimen I, the principal stresses at failure are

$$\sigma_3' = \sigma_3 = 70 \text{ kN/m}^2$$

and

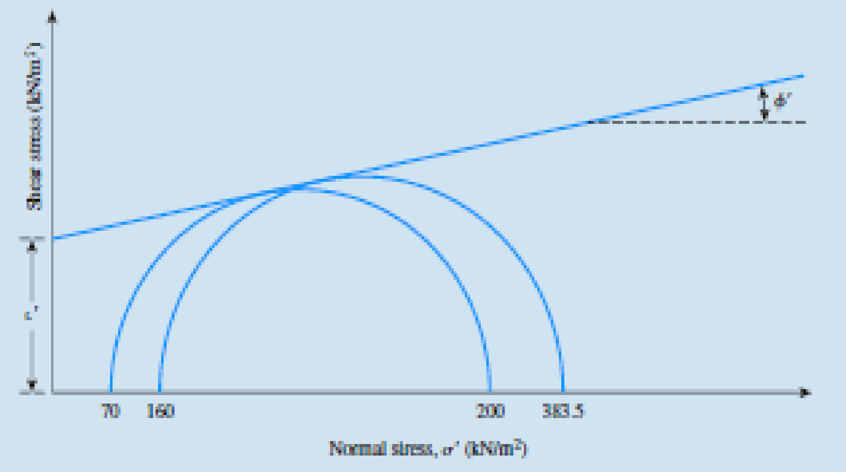
$$\sigma_1' = \sigma_1 = \sigma_3 + (\Delta\sigma_d)_f = 70 + 130 = 200 \text{ kN/m}^2$$

Similarly, the principal stresses at failure for Specimen II are

$$\sigma_3' = \sigma_3 = 160 \text{ kN/m}^2$$

and

$$\sigma_1' = \sigma_1 = \sigma_3 + (\Delta\sigma_d)_f = 160 + 223.5 = 383.5 \text{ kN/m}^2$$



Now, from Eq. (12.25),

$$\phi_1' = 2\left\{\tan^{-1}\left[\frac{\sigma_{1(I)}' - \sigma_{1(II)}'}{\sigma_{3(I)}' - \sigma_{3(II)}'}\right]^{0.5} - 45°\right\}$$

$$= 2\left\{\tan^{-1}\left[\frac{200 - 383.5}{70 - 160}\right]^{0.5} - 45°\right\} = 20°$$

Again, from Eq. (12.26),

$$c' = \frac{\sigma_{1(I)}' - \sigma_{3(I)}' \tan^2\left(45 + \frac{\phi_1'}{2}\right)}{2 \tan\left(45 + \frac{\phi_1'}{2}\right)} = \frac{200 - 70 \tan^2\left(45 + \frac{20}{2}\right)}{2 \tan\left(45 + \frac{20}{2}\right)} = 20 \text{ kN/m}^2$$

Example 1

For a normally consolidated clay specimen, the results of a drained triaxial test are as follows:

- Chamber-confining pressure = 125 kN/m^2
- Deviator stress at failure = 175 kN/m^2

Determine the soil friction angle, ϕ'

Solution

$\sigma_1 = 175+125 = 300$ kN/m^2

$\sin \phi' = (\sigma_1 - \sigma_3)/(\sigma_1 + \sigma_3)$

$\phi' = 24.3^o$

Graphical Solution → Check it?

Example 2

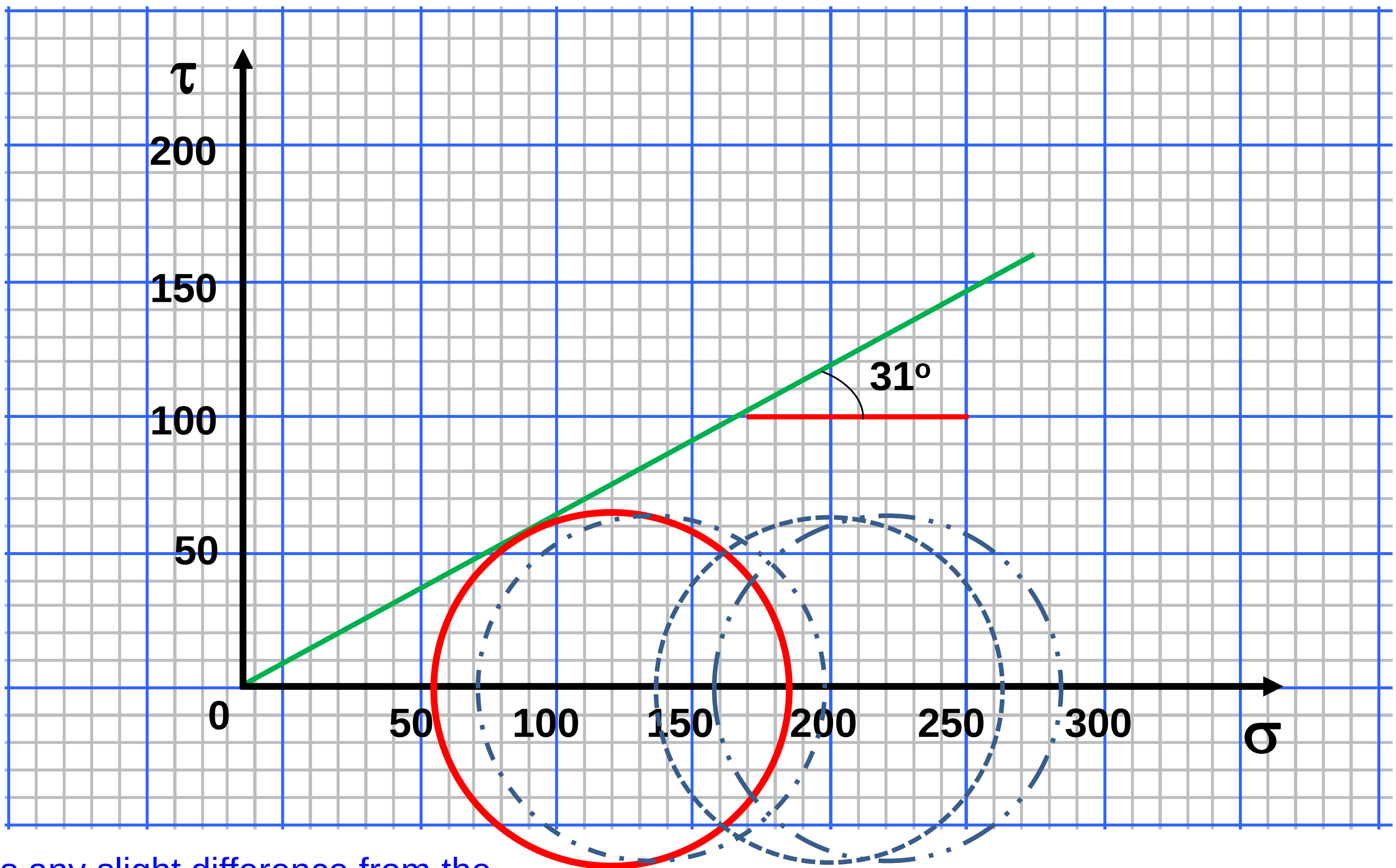
In a consolidated-drained triaxial test on a clay, the specimen failed at a deviator stress of 124 kN/m^2. If the effective stress friction angle is known to be 31°, what was the effective confining pressure at failure?

Solution

Sin 31 = $(\sigma_1 - \sigma_3)/(\sigma_1 + \sigma_3)$ = 124/Sin 31
$(\sigma_1 + \sigma_3)$=240 kN/m^2
$(\sigma_1 - \sigma_3)$ =124 kN/m^2
Two eqs. Two unknowns → σ_1=182 kN/m^2, σ_3= 58 kN/m^2

We can solve it graphically. We know ϕ so we plot M-C envelope.
$(\sigma_1 - \sigma_3)/2$ equal the radius of Mohr circle
Through trial we plot the circle that touch the M-C.
From that we know σ_1 and σ_3

Graphical Solution



If there is any slight difference from the analytical solution it is because the grid is not perfect square

Draw a circle with a radius of 62 and move it left until toughing the M-C envelope

Example 3

Samples of dry sand are to be tested in a direct shear and a triaxial test. In the triaxial test the sample fails when the major and minor principal stresses are 450 kN/m² and 150 kN/m², respectively. What shear strength be expected in the direct shear test when the normal loading is equal to a stress of 80 kN/m².

Solution

$\sin \phi' = (\sigma_1 - \sigma_3)/(\sigma_1 + \sigma_3) = (450-150)/(450+150) = 0.5$

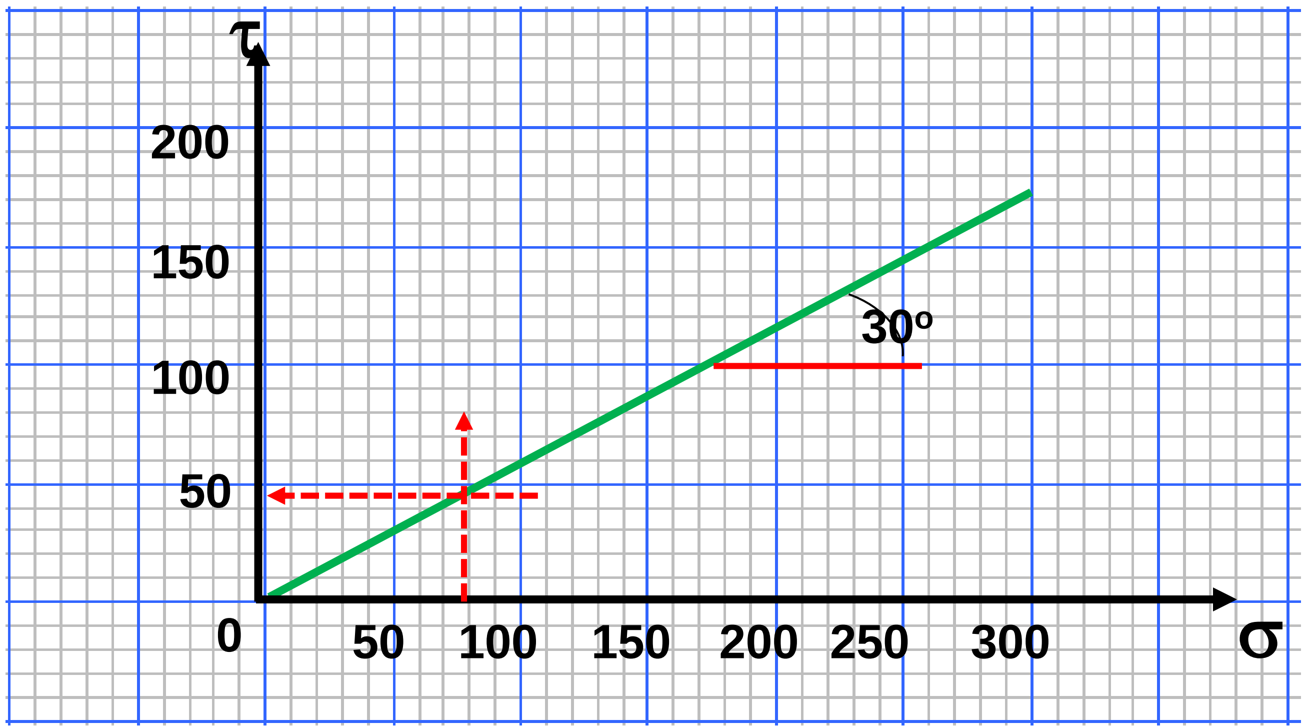
Hence $\phi' = 30^o$

$\tau = \sigma_n \tan \phi'$

$\tau = 80 \tan 30$

$\tau = 46.2\ kN/m^2$

Graphical Solution



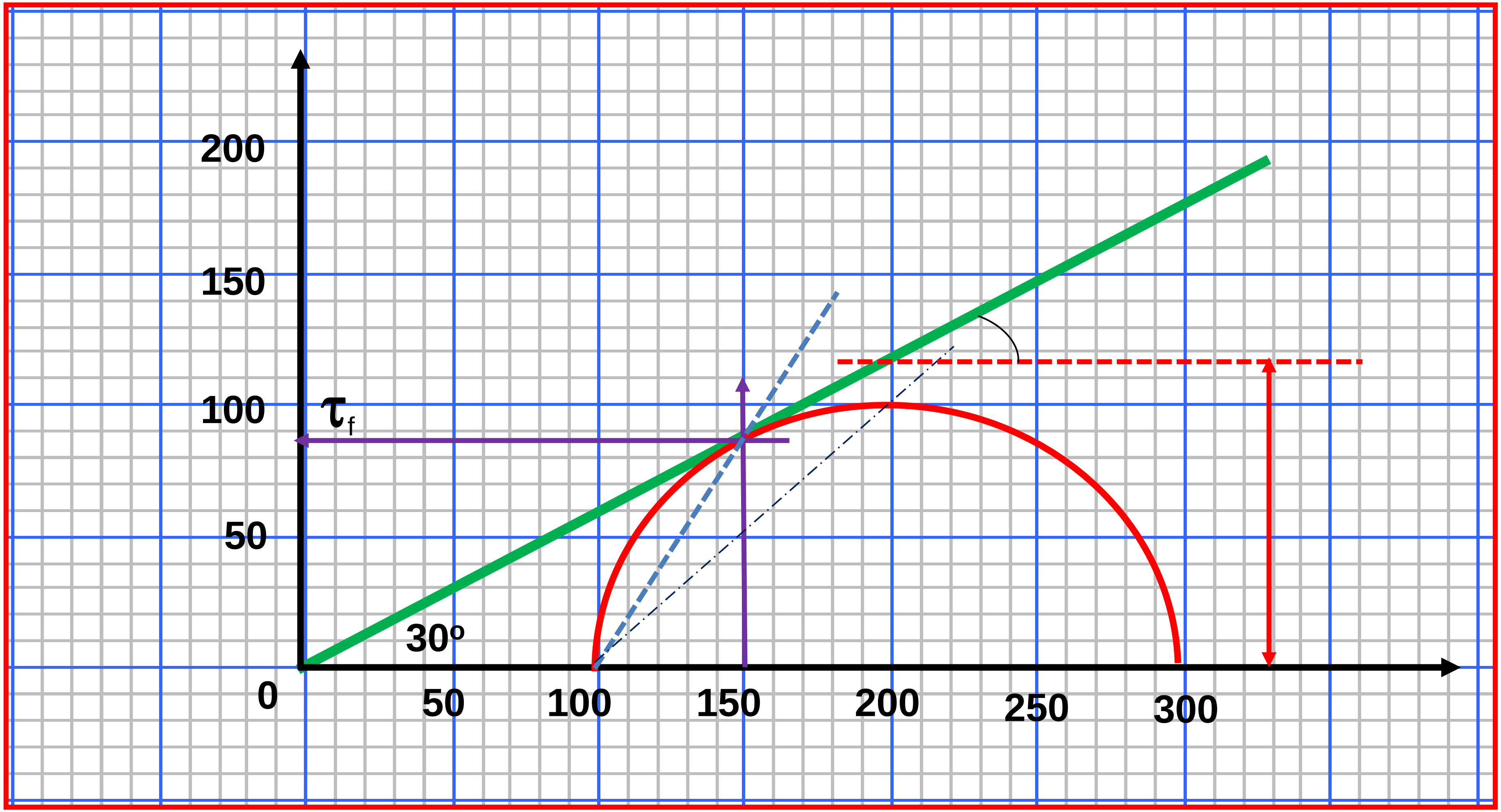
Example 4

A conventional consolidated-drained (CD) triaxial test is conducted on a sand. The cell pressure is 100 kN/m^2, and the applied axial stress at failure is 200 kN/m^2.

Required:

a. **Plot the Mohr circle for both the initial and failure stress conditions.**
b. **The friction angle (Assume c = 0)**
c. **The shear stress on the failure plane at failure.**
d. **The theoretical angle of the failure plane in the specimen.**
e. **The orientation of the plane of the maximum obliquity**
f. **The maximum shear stress at failure and the angle of the plane on which it acts.**
g. **The available shear strength on the plane of maximum shear and the factor of safety on this plane.**

Example 4



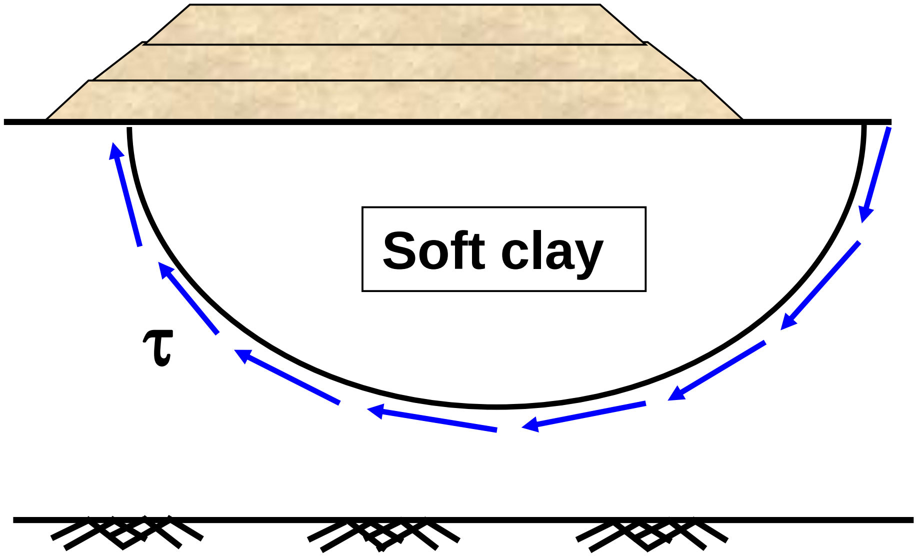
Get the required from the graph yourself

Some practical applications of CD analysis for clays

- The limiting drainage conditions modeled in the triaxial test refer to real filed situations.
- CD conditions are the most critical for the long-term steady seepage case for embankment dams and the long-term stability of excavations or slopes in both soft and stiff clays.

EXAMPLES OF CD ANALYSIS

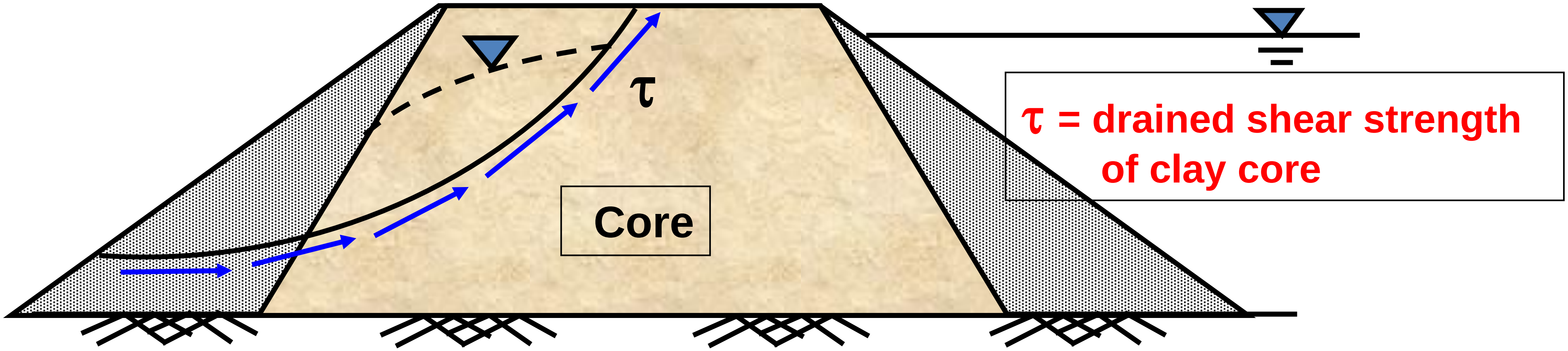
1. Embankment constructed very slowly, in layers, over a soft clay deposit



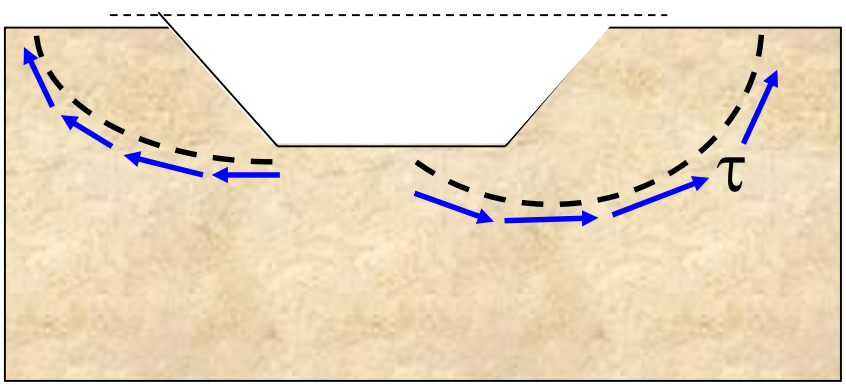
τ = in situ drained shear strength

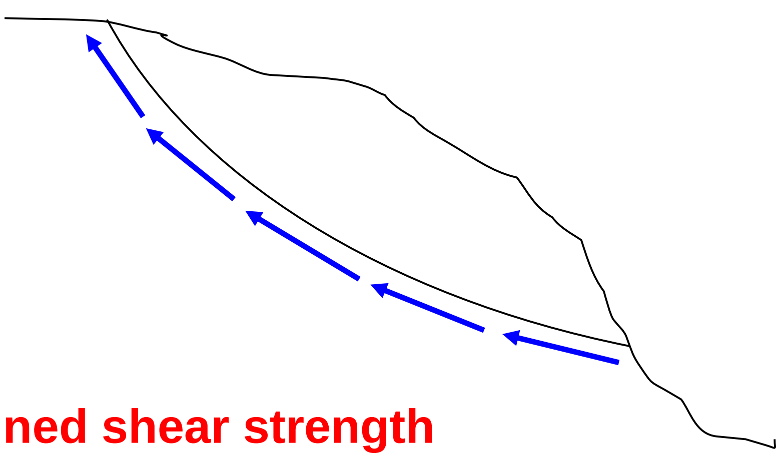
Some practical applications of CD analysis for clays

2. Earth dam with steady state seepage



3. Excavation or natural slope in clay





τ = In situ drained shear strength

Note on CD test

- CD test simulates the long term condition in the field. Thus, C_d and ϕ_d should be used to evaluate the long term behavior of soils.
- CD test is called s-test because the stress difference is applied very slowly to ensure that no p.w.p. develops during the test.
- Time to failure ranges from a day to several weeks for fine-grained soils.
- Such a long time leads to practical problems in the laboratory such as leakage of valves, seals, and the membrane that surrounds the sample.
- Therefore CD triaxial test is uncommon and not a popular in most soil laboratories.