TOPICS

- Introduction
- Types of slope movements
- **Concepts of Slope Stability Analysis**
- Factor of Safety
- Stability of Infinite Slopes
- Stability of Finite Slopes with Plane Failure Surface
  - Culmann's Method
- Stability of Finite Slopes with Circular Failure Surface
  - Mass Method
  - Method of Slices

Concepts of Slope Stability Analysis

In general we need to check

- ❑ **The stability of a given existed slope**
- ❑ **Determine the inclination angle for a slope that we want to build with a given height**
- ❑ **The height for a slope that we want to build with a given inclination**

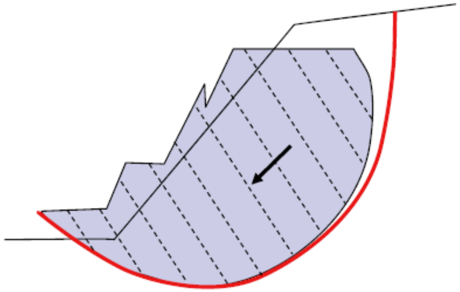
Methodology of Slope Stability Analysis

It is a method to expresses the relationship between resisting forces and driving forces

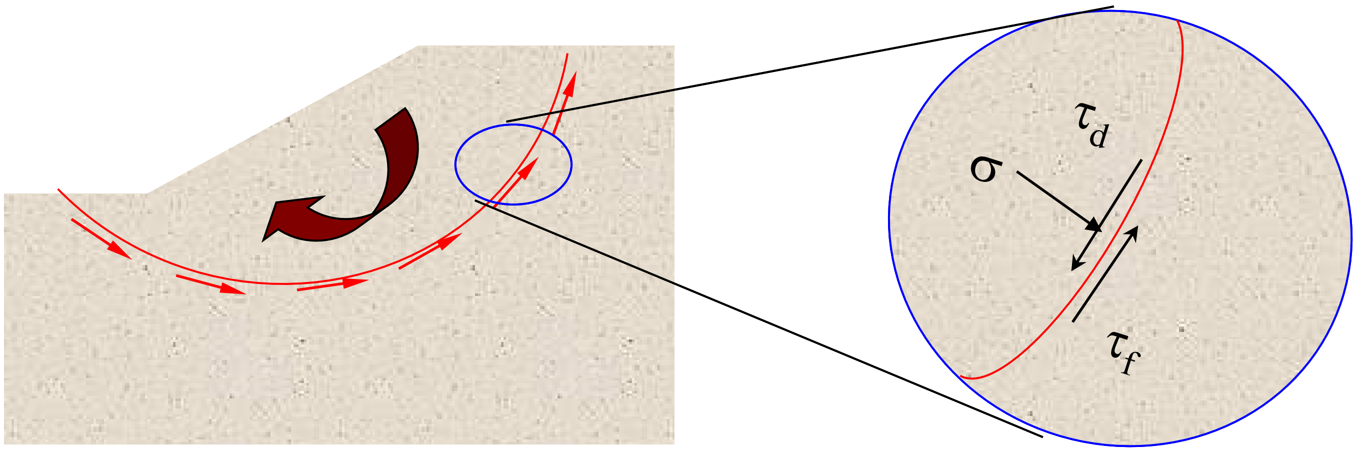
- **Driving forces** – forces which move earth materials downslope. Downslope component of weight of material including vegetation, fill material, or buildings.

- **Resisting forces** – forces which oppose movement. Resisting forces include strength of material

- **Failure** occurs when the driving forces (component of the gravity) overcomes the resistance derived from the shear strength of soil along the potential failure surface.



Methodology of Slope Stability Analysis



The analysis involves determining and comparing the *shear stress developed* along the most likely rupture surface to the *shear strength of soil.*

Slope Stability Analysis Procedure

1. Assume a probable failure surface.
2. Calculate the factor of safety by determining and comparing the shear stress developed along the most likely rupture surface to the shear strength of soil.
3. Repeat steps 1 and 2 to determine the most likely failure surface. The most likely failure surface is the critical surface that has a minimum factor of safety.
4. Based on the minimum F_S, determine whether the slope is safe or not.

Methods of Slope Stability Analysis

- Limit equilibrium method
- Limit analysis method
- Numerical methods

We will consider only the limit equilibrium method, since it is the oldest and the mostly used method in practice.

Assumptions of Stability Analysis

- The problem is considered in two-dimensions
- The failure mass moves as a rigid body
- The shear strength along the failure surface is isotropic
- The factor of safety is defined in terms of the average shear stress and average shear strength along the failure surface

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Factor of Safety

$$\text{Factor of safety} = \frac{\text{Resisting Force}}{\text{Driving Force}}$$

$$= \frac{\text{Shear Strength}}{\text{Shear Stress}}$$

$$F_s = \frac{\tau_f}{\tau_d}$$

τ_f= Avg. Shear strength of soil

τ_d= Avg. Shear stress developed along the failure surface

Factor of Safety

- The most common analytical methods of slope stability use a factor of safety F_S with respect to the limit equilibrium condition,

F_s is the ratio of resisting forces to the driving forces, or

$$F_s = \frac{\tau_f}{\tau_d}$$

Shear strength (resisting movement)
average shear strength of the soil.

$\tau_f = c' + \sigma' \tan \phi'$ (Available)

Shear stress (driving movement)
average shear stress developed along the potential failure surface.

$\tau_d = c'_d + \sigma' \tan \phi'_d$ (developed)

$F_S < 1$ → unstable

$F_S \approx 1$ → marginal

$F_S >> 1$ → stable

Generally, $FS \geq 1.5$ is acceptable for the design of a stable slope

If factor safety F_s equal to or less than 1, the slope is considered in a state of impending failure

Causes of slope failure

1. External causes

These which produce increase of shear stress, like steepening or heightening of a slope, building on the top of the slope

2. Internal causes

These which cause failure without any change in external conditions, like increase in pore water pressure.

Therefore, slopes fail due either to increase in stress or reduction in strength.

Factor of Safety

$$F_s = \frac{c' + \sigma' \tan \phi'}{c'_d + \sigma' \tan \phi'_d}$$

Where:
c' **= cohesion**
ϕ' **= angle of internal friction**
c'_d, ϕ'_d **= cohesion and angle of friction that develop along the potential failure surface**

Other aspects of factor of safety

Factor of safety with respect to cohesion

$$F_{c'} = \frac{c'}{c'_d}$$

Factor of safety with respect to friction

$$F_{\phi'} = \frac{\tan \phi'}{\tan \phi'_d}$$

When the factor of safety with respect to cohesion is equal to the factor of safety with respect to friction, it gives the factor of safety with respect to strength, or

$$\text{When } F_{c'} = F_{\phi'} \quad \text{then} \quad F_s = F_{c'} = F_{\phi'}$$

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Stability of Infinite Slopes

What is an Infinite slope?

- **Slope that extends for a relatively long distance and has consistent subsurface profile can be considered as infinite slope.**
- **Failure plane parallel to slope surface.**
- **Depth of the failure surface is small compared to the height of the slope.**
- **For the analysis, forces acting on a single slice of the sliding mass along the failure surface is considered and end effects is neglected.**

Infinite slope – no seepage

- we will evaluate the factor of safety against a possible slope failure along a plane *AB* located at a depth *H* below the ground surface.
- Let us consider a slope element abcd that has a unit length perpendicular to the plane of the section shown.
- The forces, F, that act on the faces ab and cd are equal and opposite and may be ignored.

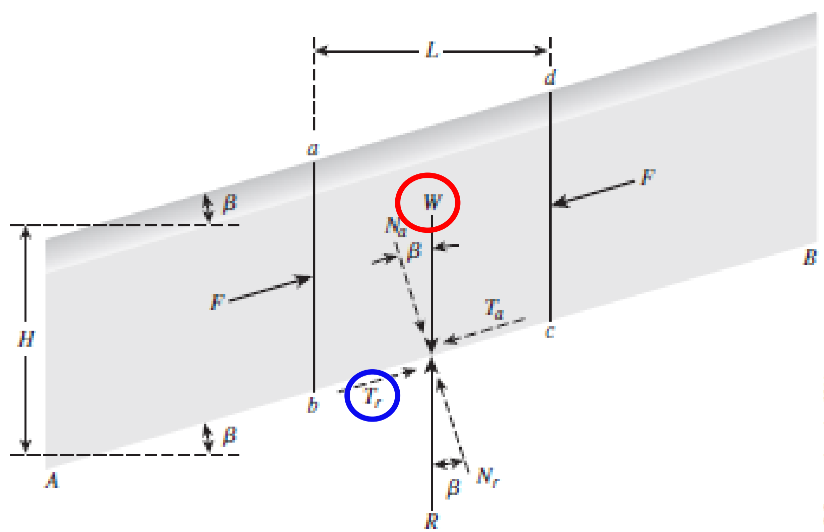
⇨ The shear stress at the base of the slope element can be given by

$$\tau = \frac{T_a}{\text{Area of base}}$$

Force parallel to the plane AB

$T_a = W \sin\beta = \gamma L H \sin\beta$

$$\tau = \frac{T_a}{\text{Area of base}} = \frac{\gamma L H \sin\beta}{\left(\frac{L}{\cos\beta}\right)} = \gamma H \cos\beta \sin\beta \quad (*)$$



⇨ The resistive shear stress is given by

$$\tau_d = c'_d + \sigma' \tan\phi'_d$$

Infinite slope – no seepage

The effective normal stress at the base of the slope element is given by

$$\sigma' = \frac{N_a}{\text{Area of base}} = \frac{\gamma LH \cos\beta}{\left(\dfrac{L}{\cos\beta}\right)} = \underline{\gamma H \cos^2\beta}$$

$$\tau_d = c'_d + \gamma H \cos^2\beta \tan\phi'_d \qquad (^{**})$$

Equating R.H.S. of Eqs. (*) and (**) gives

$$\gamma H \sin\beta \cos\beta = c'_d + \gamma H \cos^2\beta \tan\phi'_d$$

$$F_s = F_{c'} = F_{\phi'} \Rightarrow \tan\phi'_d = \frac{\tan\phi'}{F_s} \quad \text{and} \quad c'_d = \frac{c'}{F_s}$$

$$F_s = \frac{c'}{\gamma H \cos^2\beta \tan\beta} + \frac{\tan\phi'}{\tan\beta} \qquad (^{***})$$

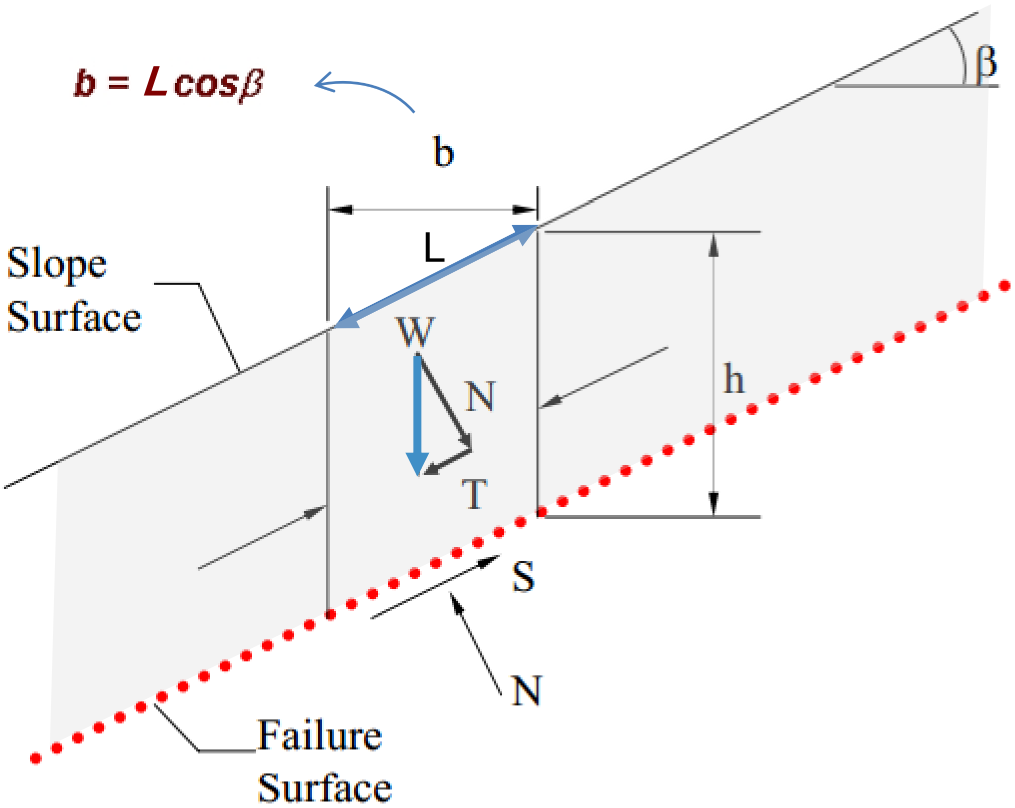
For Granular Soil (i.e., c = 0)

$$F_s = \frac{\tan\phi'}{\tan\beta}$$

This means that in case of infinite slope in sand, the value of F_s is independent of the height H and the slope is stable as long as $\beta < \phi'$

Case of Granular soil – Derivation From Simple Statics

$b = L\cos\beta$



Equilibrium of forces on a slice:

$$W = \gamma\, b\, h$$

$$N = W \cos\beta$$

$$T = W \sin\beta$$

$$S = N \tan\phi$$

$$FS = \frac{\text{Resisting Forces}}{\text{Driving Forces}}$$

$$FS = \frac{S}{T} = \frac{N \tan\phi}{W \sin\beta} = \frac{\tan\phi}{\tan\beta}$$

Infinite slope – no seepage

Critical Depth, H_{cr}

The depth of plane along which critical equilibrium occurs is obtained by substituting $F_s = 1$ and $H = H_{cr}$ into Eq. (***)

$$F_s = \frac{c'}{\gamma H \cos^2\beta \tan\beta} + \frac{\tan\phi'}{\tan\beta}$$

$$H_{cr} = \frac{c'}{\gamma} \frac{1}{\cos^2\beta(\tan\beta - \tan\phi')}$$

EXAMPLE 15.1

Example 15.1

For the infinite slope shown in Figure 15.8 (consider that there is no seepage through the soil), determine:

a. The factor of safety against sliding along the soil–rock interface
b. The height, H, that will give a factor of safety (F_s) of 2 against sliding along the soil–rock interface

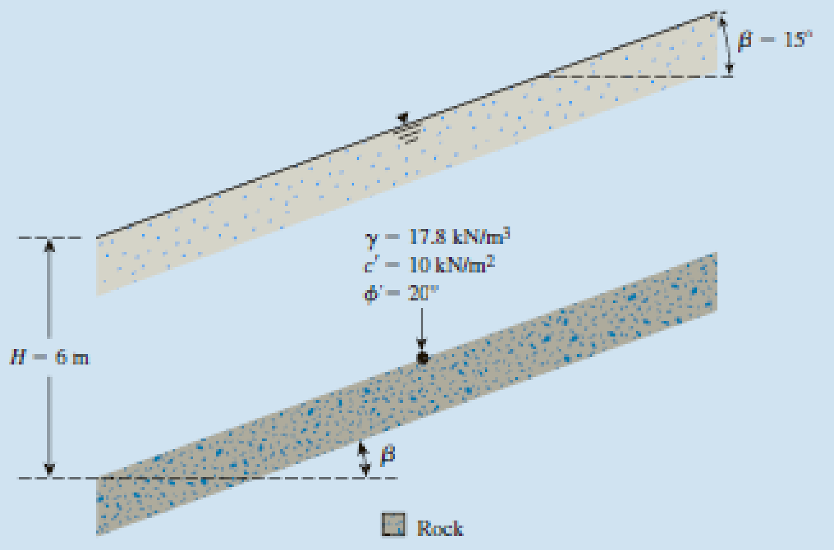


Figure 15.8

EXAMPLE 15.1

Solution

Part a

From Eq. (15.15),

$$F_s = \frac{c'}{\gamma H \cos^2\beta \tan\beta} + \frac{\tan\phi'}{\tan\beta}$$

Given: $c' = 10\ \text{kN/m}^2$, $\gamma = 17.8\ \text{kN/m}^3$, $\phi' = 20°$, $\beta = 15°$, and $H = 6$ m, we have

$$F_s = \frac{10}{(17.8)(6)(\cos^2 15)(\tan 15)} + \frac{\tan 20}{\tan 15} = \mathbf{1.73}$$

Part b

From Eq. (15.15),

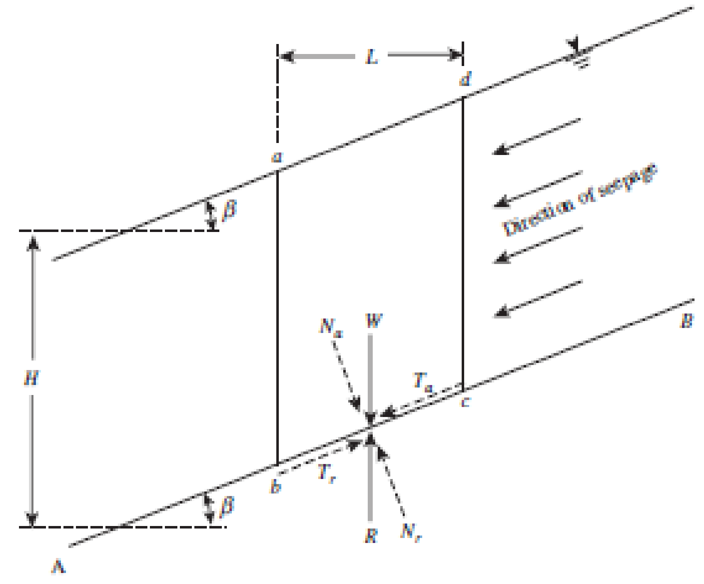
$$F_s = \frac{c'}{\gamma H \cos^2\beta \tan\beta} + \frac{\tan\phi'}{\tan\beta}$$

$$2 = \frac{10}{(17.8)(H)(\cos^2 15)(\tan 15)} + \frac{\tan 20}{\tan 15}$$

$$H = \mathbf{3.5\ m}$$

Infinite slope – with steady state seepage

Seepage is assumed to be parallel to the slope and that the ground water level coincides with the ground surface.



(a)

- The shear stress at the base of the slope element can be given

$$\tau = \frac{T_r}{\left(\frac{L}{\cos\beta}\right)} = \gamma_{sat} H \cos\beta \sin\beta \quad (*)$$

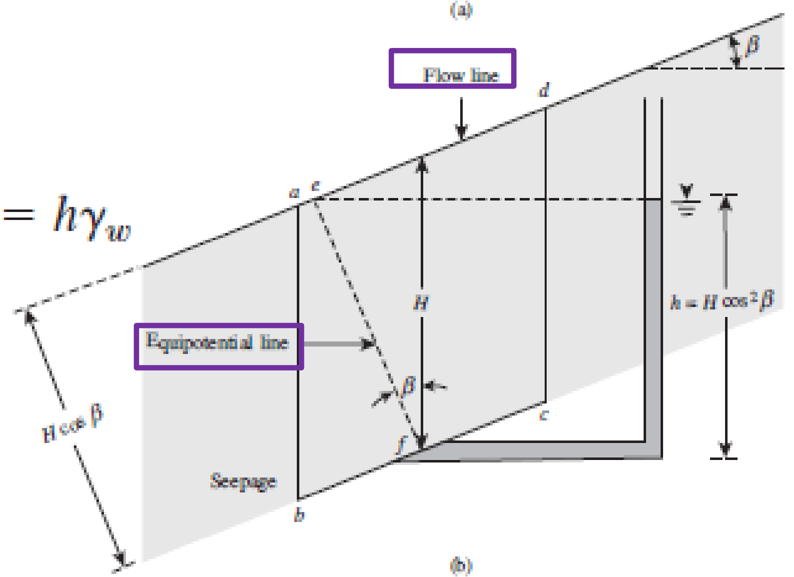
- The resistive shear stress developed at the base of the element is given by

$$\tau_d = c'_d + \sigma' \tan\phi_d = c'_d + (\sigma - u)\tan\phi'_d$$

u = (Height of water in piezometer placed at f)(γ_w) = $h\gamma_w$

$$u = \gamma_w H \cos^2\beta$$

$$\tau_d = c'_d + (\gamma_{sat} H \cos^2\beta - \gamma_w H \cos^2\beta)\tan\phi'_d$$
$$= c'_d + \gamma' H \cos^2\beta \tan\phi'_d \quad (**)$$



(b)

Infinite slope – with steady state seepage

Equating the right-hand sides of Eq. (*) and Eq. (**) yields

$$\gamma_{sat} H \cos\beta \sin\beta = c'_d + \gamma' H \cos^2\beta \tan\phi'_d \quad (***)$$

Recall

$$\tan\phi'_d = \frac{\tan\phi'}{F_s} \quad \text{and} \quad c'_d = \frac{c'}{F_s} \quad (****)$$

Substituting Eq. (****) Into Eq. (***) and solving for F_s gives

$$F_s = \frac{c'}{\gamma_{sat} H \cos^2\beta \tan\beta} + \frac{\gamma' \tan\phi'}{\gamma_{sat} \tan\beta}$$

$$F_s = \frac{c'}{\gamma H \cos^2\beta \tan\beta} + \frac{\tan\phi'}{\tan\beta}$$

No seepage

Stability of Infinite Slopes

Cohesive Soils

With seepage

$$\tan\phi_d' = \frac{\tan\phi'}{F_s} \qquad c_d' = \frac{c'}{F_s}$$

$$F_s = \frac{c'}{\gamma_{sat} H \cos^2\beta \tan\beta} + \frac{\gamma'}{\gamma_{sat}} \frac{\tan\phi'}{\tan\beta}$$

$$H_{cr} = \frac{c'}{\cos^2\beta(\gamma_{sat}\tan\beta - \gamma'\tan\phi')}$$

No seepage

$$\tan\phi_d' = \frac{\tan\phi'}{F_s} \qquad c_d' = \frac{c'}{F_s}$$

$$F_s = \frac{c'}{\gamma H \cos^2\beta \tan\beta} + \frac{\tan\phi'}{\tan\beta}$$

$$H_{cr} = \frac{c'}{\gamma} \frac{1}{\cos^2\beta(\tan\beta - \tan\phi_d')}$$

Stability of Infinite Slopes

Granular Soils

With seepage

$$c' = 0.0$$

$$F_S = \frac{c'}{\gamma_{sat} H \cos^2 \beta \tan \beta} + \frac{\gamma'}{\gamma_{sat}} \frac{\tan \phi'}{\tan \beta}$$

$$F_S = \frac{\gamma'}{\gamma_{sat}} \frac{\tan \phi'}{\tan \beta}$$

Independant of H

No seepage

$$c' = 0.0$$

$$F_S = \frac{c'}{\gamma H \cos^2 \beta \tan \beta} + \frac{\tan \phi'}{\tan \beta}$$

$$F_S = \frac{\tan \phi'}{\tan \beta}$$

Slope is stable as long as $\beta < \phi$

EXAMPLE 15.2

Example 15.2

A cut is to be made in a soil having $\gamma = 16.5\ \mathrm{kN/m^3}$, $c' = 28.75\ \mathrm{kN/m^2}$, and $\phi' = 15°$. The side of the cut slope will make an angle of 45° with the horizontal. What should be the depth of the cut slope that will have a factor of safety (F_s) of 3?

EXAMPLE 15.2

Solution

Given: $\phi' = 15°$; $c' = 28.75$ kN/m². If $F_s = 3$, then $F_{c'}$ and $F_{\phi'}$ should both be equal to 3.

$$F_{c'} = \frac{c'}{c'_d}$$

or

$$c'_d = \frac{c'}{F_{c'}} = \frac{c'}{F_s} = \frac{28.75}{3} = 9.58 \text{ kN/m}^2$$

Similarly,

$$F_{\phi'} = \frac{\tan \phi'}{\tan \phi'_d}$$

$$\tan \phi'_d = \frac{\tan \phi'}{F_{\phi'}} = \frac{\tan \phi'}{F_s} = \frac{\tan 15}{3}$$

or

$$\phi'_d = \tan^{-1}\left[\frac{\tan 15}{3}\right] = 5.1°$$

Substituting the preceding values of c'_d and ϕ'_d in Eq. (15.40),

$$H = \frac{4c'_d}{\gamma}\left[\frac{\sin \beta \cos \phi'_d}{1 - \cos(\beta - \phi'_d)}\right]$$

$$= \frac{4 \times 9.58}{16.5}\left[\frac{\sin 45 \cos 5.1}{1 - \cos(45 - 5.1)}\right]$$

$$= \mathbf{7.03\ m}$$

The end