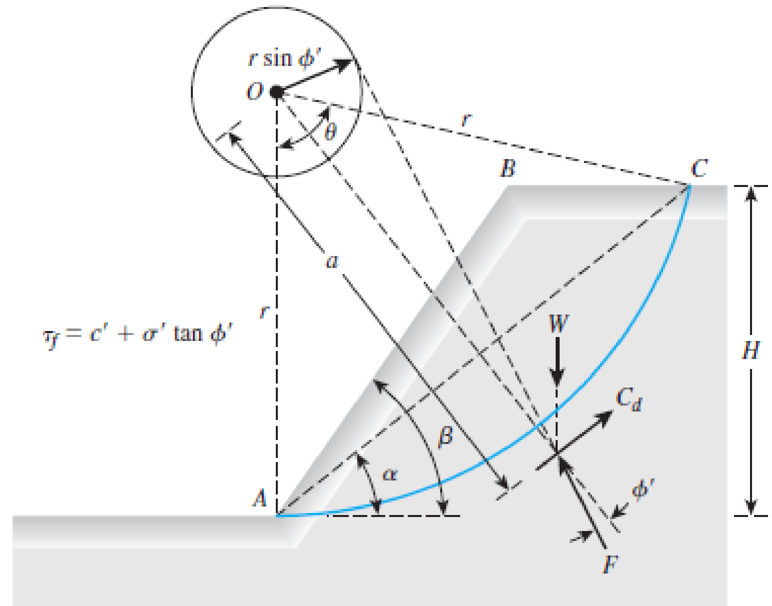
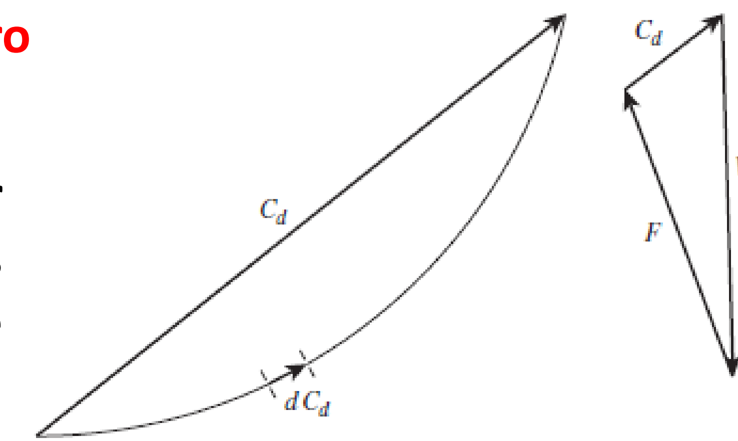
Slopes in Homogeneous C' – ϕ' Soils

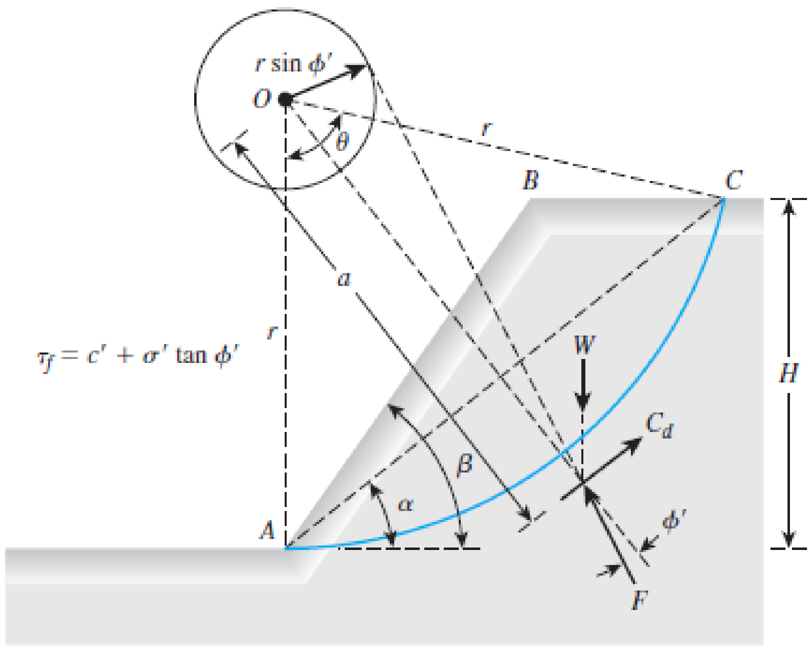
- Here the situation is more complicated than for purely cohesive soils.
- The Friction Circle method (or the ϕ-Circle Method) is very useful for homogenous slopes. The method is generally used when both cohesive and frictional components are to be used.
- $\widehat{AC}$ is a trial circular arc that passes through the toe of the slope, and O is the center of the circle.



- The pore water pressure is assumed to be zero
- F—the resultant of the normal and frictional forces along the surface of sliding. For equilibrium, the line of action of F will pass through the point of intersection of the line of action of W and C_d.

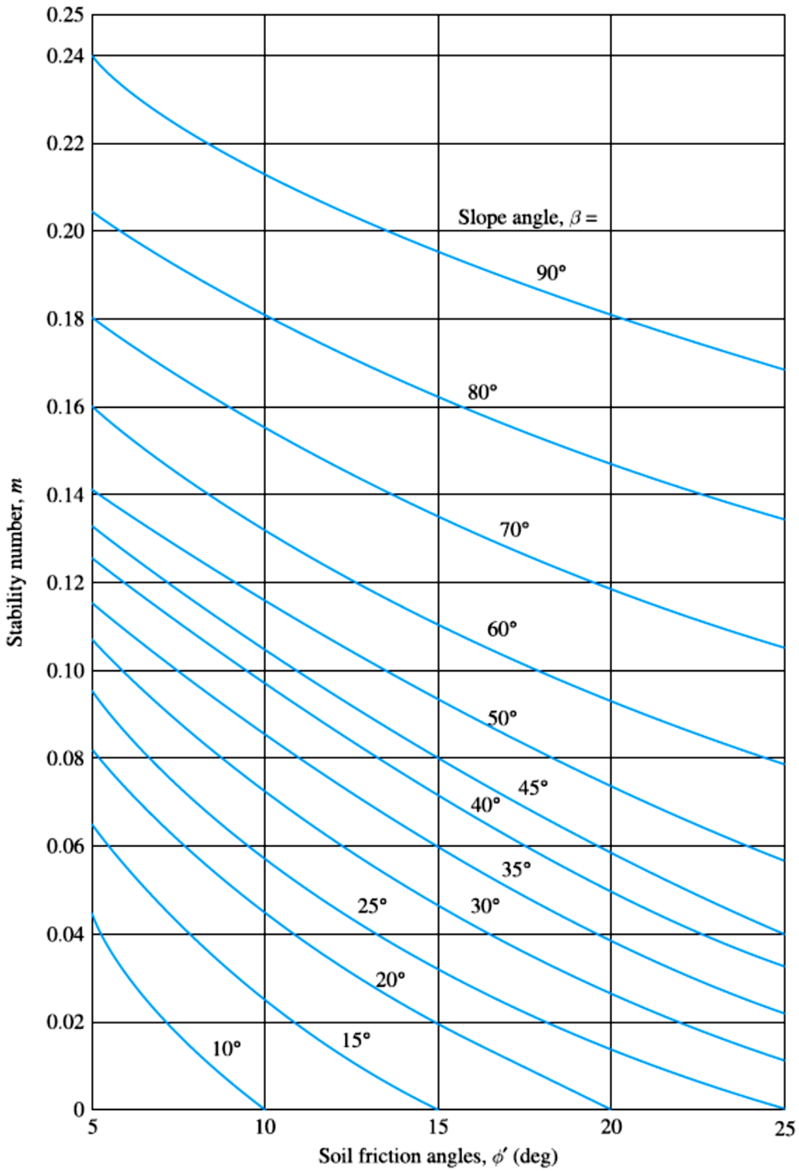


Friction Circle method



$$c'_d = \gamma H[f(\alpha, \beta, \theta, \phi')]$$

$$c_d = \gamma H m$$



Procedures of graphical solution

Given: H, β, γ, c', ϕ' Required: F_s

1. Assume ϕ_d *(Generally start with* ϕ'_d *) i.e. full friction is mobilized)*

2. Calculate
$$F_{\phi'} = \frac{\tan \phi'}{\tan \phi'_d}$$

3. With ϕ_d and β Use Chart to get m

4. Calculate
$$c_d = \gamma H m$$

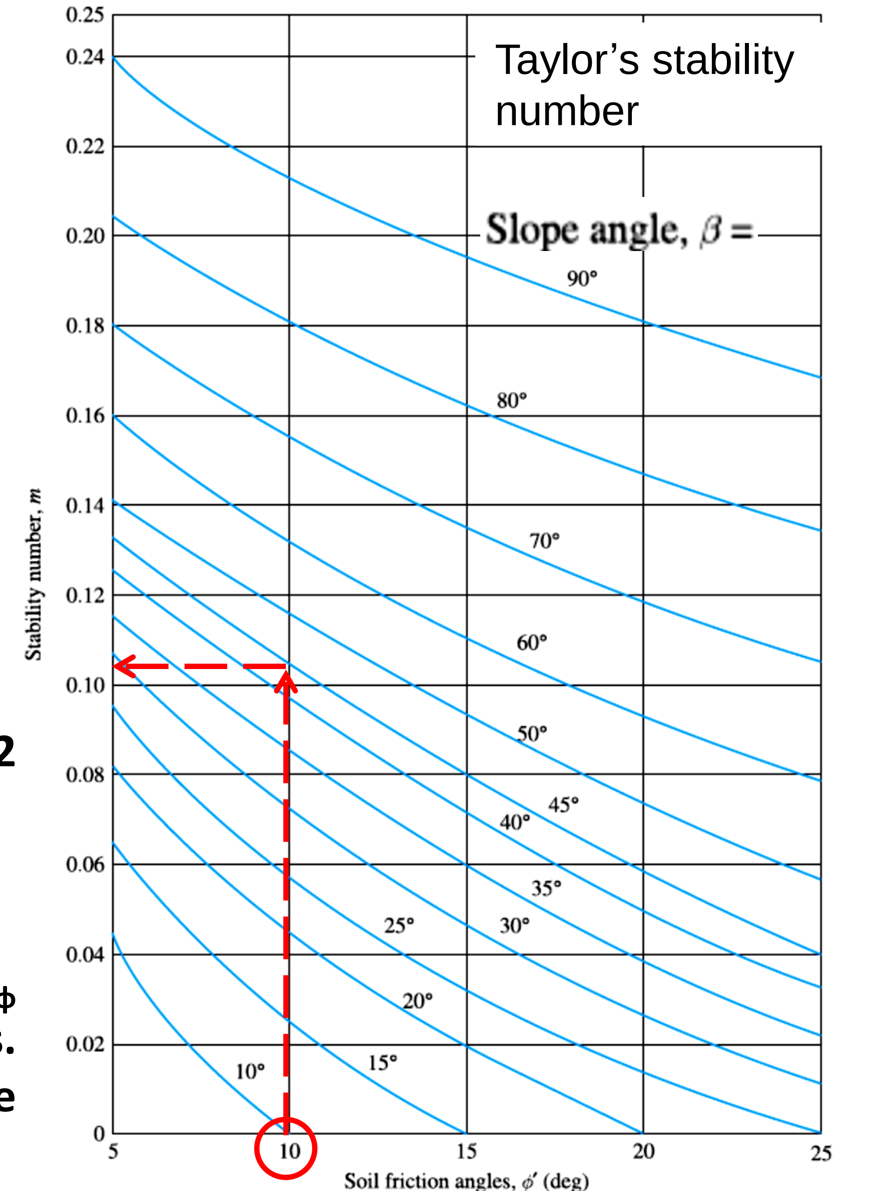
5. Calculate
$$F_c = \frac{c'}{c_d}$$

6. If $F_{c'} = F_{\phi'}$ The overall factor of safety
$$F_s = F_{c'} = F_{\phi'}$$

7. If $F_{c'} \neq F_{\phi'}$ reassume ϕ_d and repeat steps 2 through 5 until $F_{c'} = F_{\phi'}$

Or

Plot the calculated points on F_c versus F_ϕ coordinates and draw a curve through the points. [see next slide]. Then Draw a line through the origin that represents $F_s = F_c = F_\phi$



Procedures of graphical solution

Given: H, β, γ, c', ϕ'   **Required: F_s**

Calculations have shown that for $\phi > \sim 3°$, the critical circles are all *toe circles.*

Table 15.2 Determination of F_s by Frication Circle Method

ϕ'_d (1)	$F_{\phi'} = \dfrac{\tan\phi'}{\tan\phi'_d}$ (2)	m (3)	c'_d (4)	$F_{c'}$ (5)
$\phi'_{d(1)}$	$\dfrac{\tan\phi'}{\tan\phi'_{d(1)}}$	m_1	$m_1\gamma H = c'_{d(1)}$	$\dfrac{c'}{c'_{d(1)}} = F_{c'(1)}$
$\phi'_{d(2)}$	$\dfrac{\tan\phi'}{\tan\phi'_{d(2)}}$	m_2	$m_2\gamma H = c'_{d(2)}$	$\dfrac{c'}{c'_{d(2)}} = F_{c'(2)}$

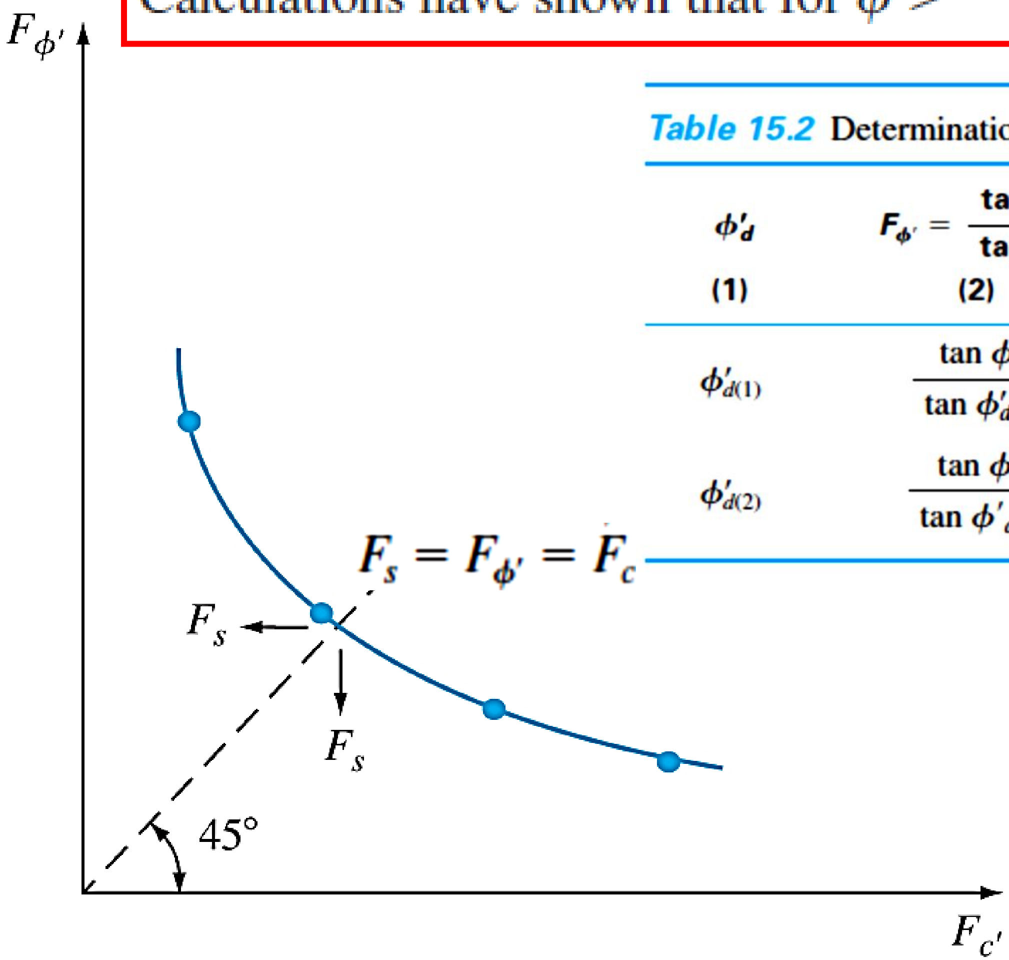


Figure 15.24 Plot of $F_{\phi'}$ versus $F_{c'}$ to determine F_s

Note: Similar to Culmann procedure for planar mechanism but here C_d is found based on m. In Culmann's method C_d is found from analytical equation.

Calculation of Critical Height

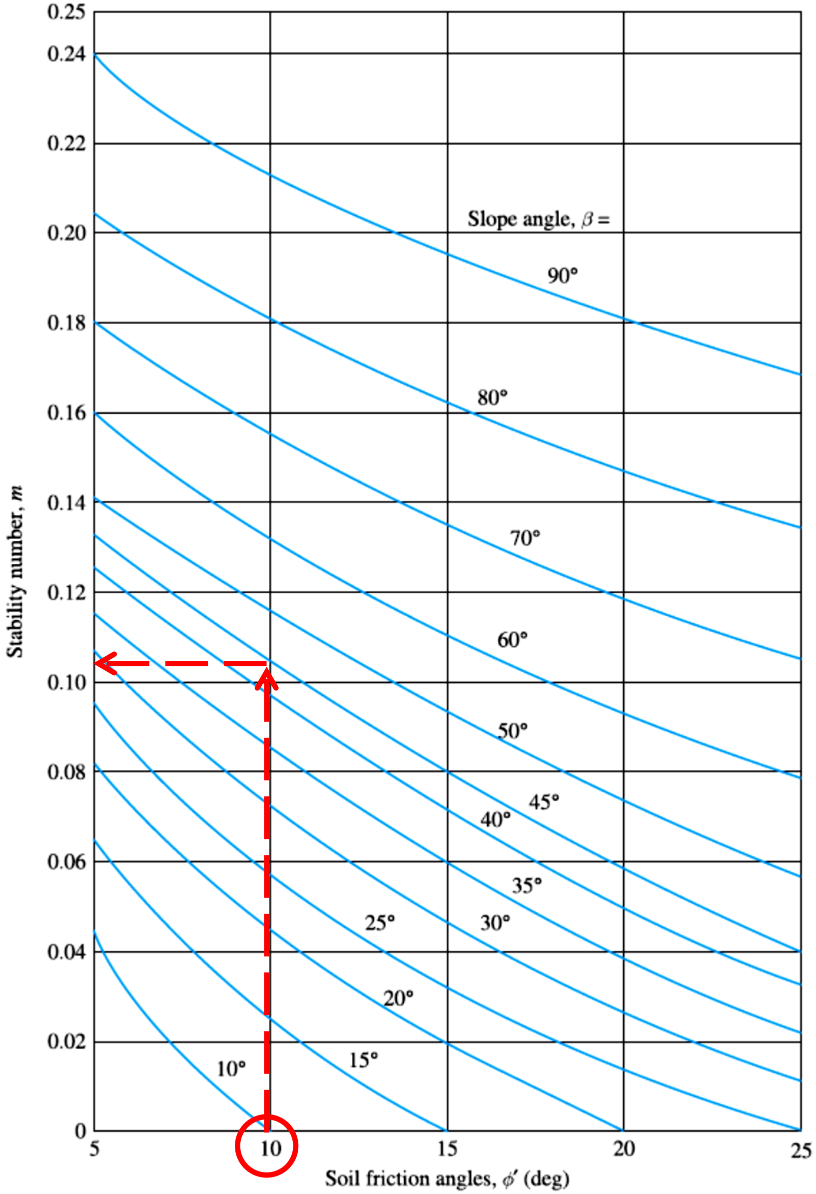
Given: β, γ, C', ϕ' **Required:** H_{cr}

H_{cr} means that $F_{c'} = F_{\phi'} = F_s = 1.0$

1. For the given β and ϕ', use Chart to get m.

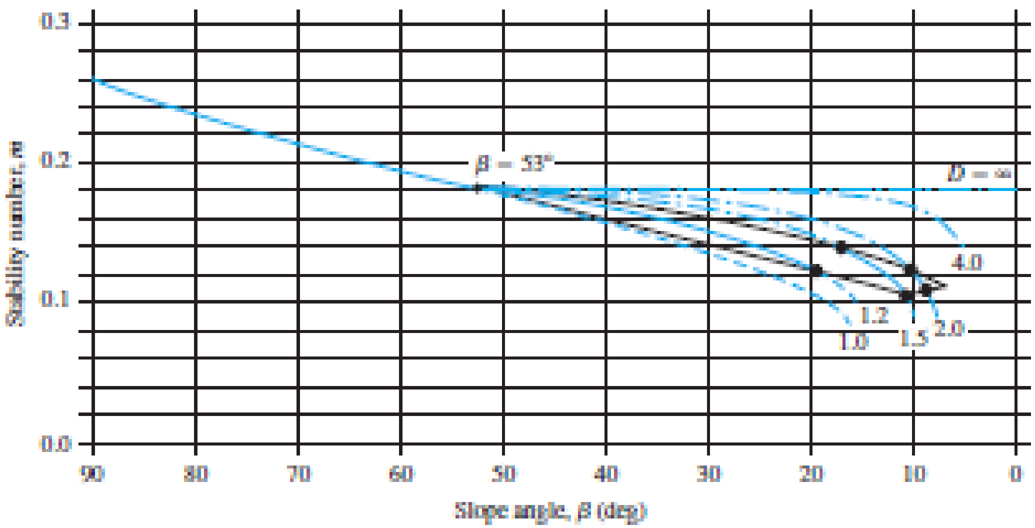
2. Calculate

$$H_{cr} = \frac{c'}{\gamma\, m}$$

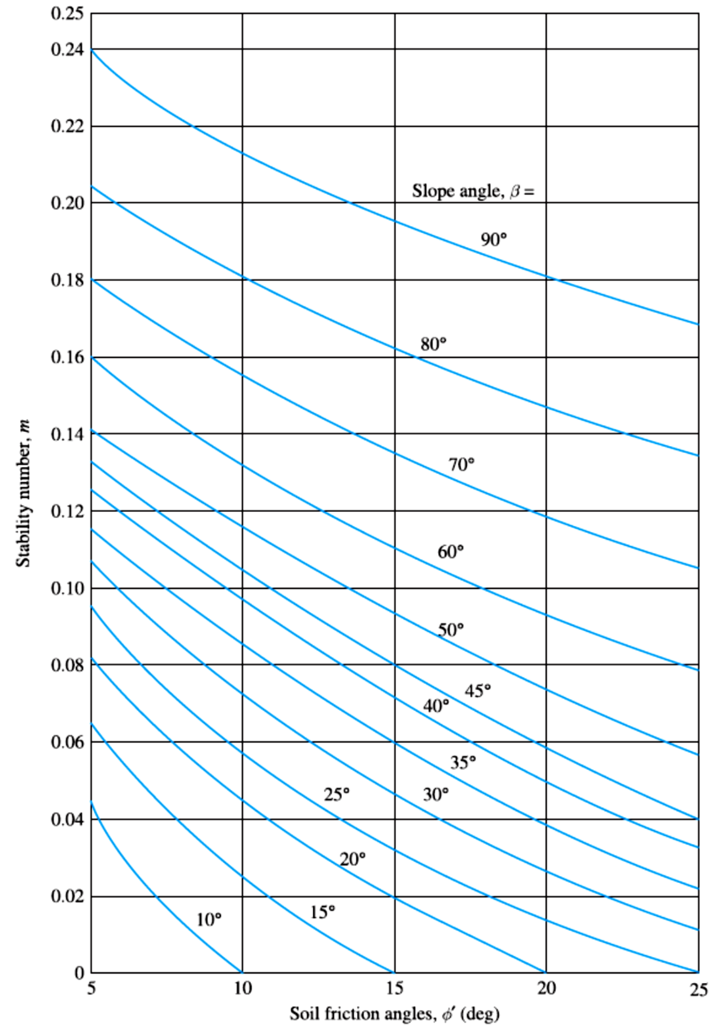


SUMMARY

Mass Procedure – Rotational mechanism need only the use of Taylor's chart.



[φ = 0]



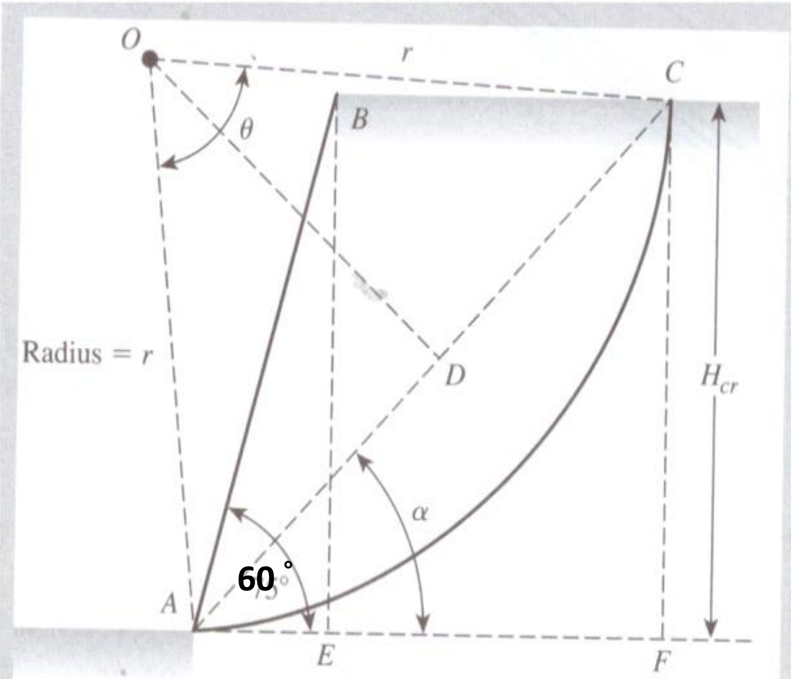
[C– φ]

EXAMPLE

- **Example**
- **Given:** c_u = 40 kN/m² & γ = 17.5 kN/m³
- **Required:**

1. **Max. Depth**
2. **Radius r when F_s=1**
3. **Distance BC**

- **β = 60 ° > 53 ° from Fig.15.13 m= 0.195**



$$H_{cr} = \frac{c_u}{\gamma\,\text{m}} = \frac{40}{17.5*0.195} = 11.72\,m$$

$$r = \frac{\overline{DC}}{\sin\frac{\theta}{2}} \qquad \overline{DC} = \frac{\overline{AC}}{2} = \frac{\left(\frac{H_{cr}}{\sin\alpha}\right)}{2}$$

$$From\ \ \text{Fig.15.14}\ \ \text{for}\ \ \beta = 60^{\text{o}}\ \alpha = 35^{\text{o}}\ and\ \theta = 72.5^{\text{o}}$$

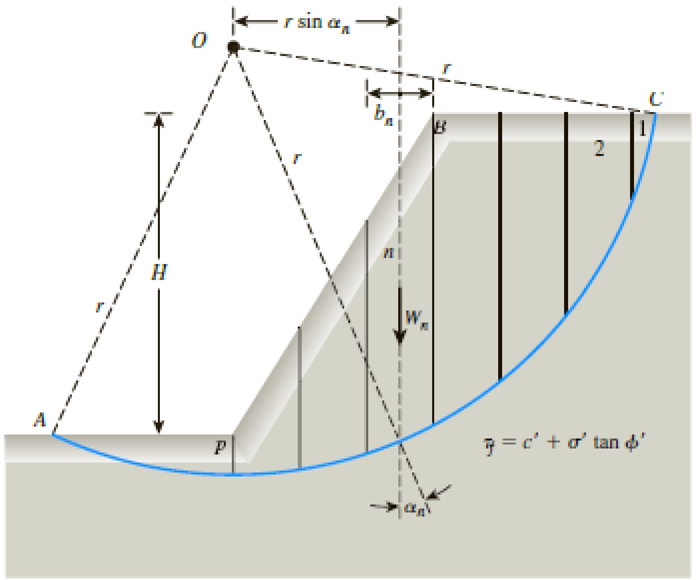
$$r = \frac{H_{cr}}{2\sin\alpha\sin\frac{\theta}{2}} = \frac{11.72}{2(\sin 35)(\sin 36.25)} = 17.28\,\text{m}$$

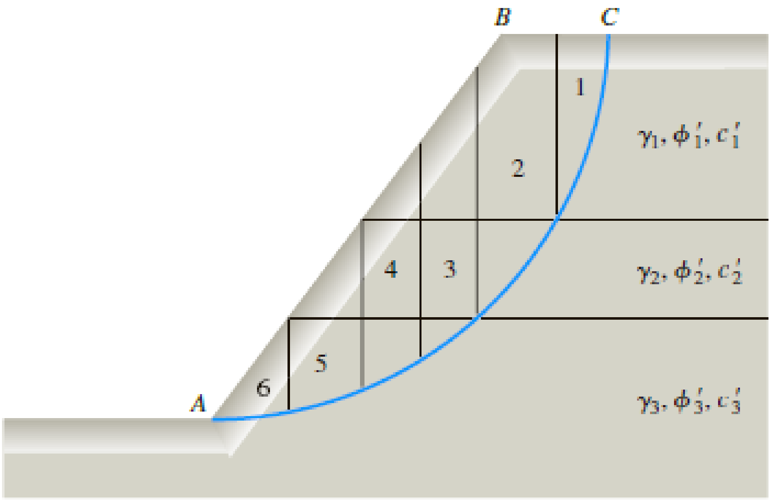
$$\overline{\text{BC}} = \overline{\text{EF}} = \overline{\text{AF}} - \overline{\text{AE}} = H_{cr}\left(\cot\alpha - \cot 60\right) = 9.97\,\text{m}$$

METHOD OF SLICES

Method of Slices

- **Method of Slices**
- **Non-homogenous soils (mass procedure is not accurate)**
- **Soil mass is divided into several vertical Parallel slices**
- **The width of each slice need not be the same**
- **It is sometimes called the Swedish method**



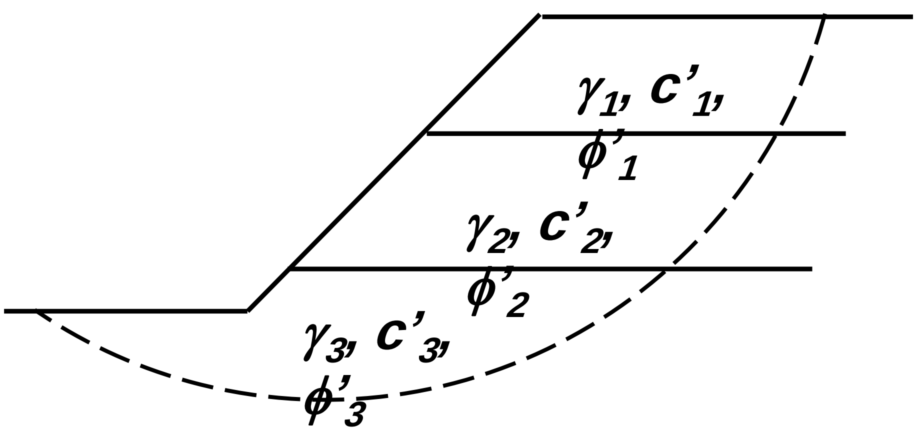


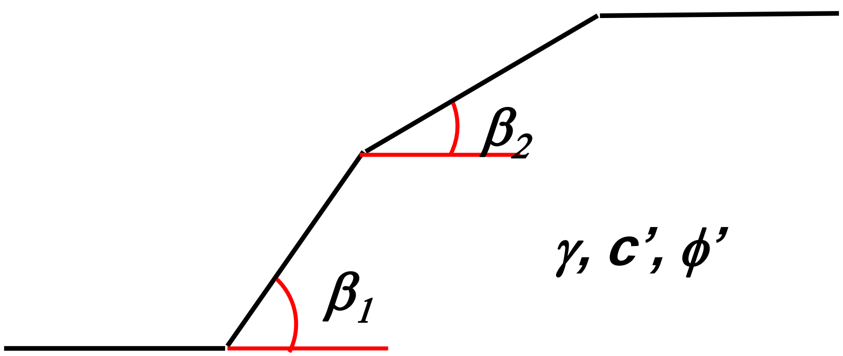
Method of Slices

- **It is a general method that can be used for analyzing irregular slopes in non-homogeneous slopes in which the values of c' and ϕ' are not constant.**

- **Because the SWEDISH GEOTECHNCIAL COMMISION used this method extensively, it is sometimes referred to as the SWEDISH Method.**

- **In mass procedure only the moment equilibrium is satisfied. Here attempt is made to satisfy force equilibrium.**

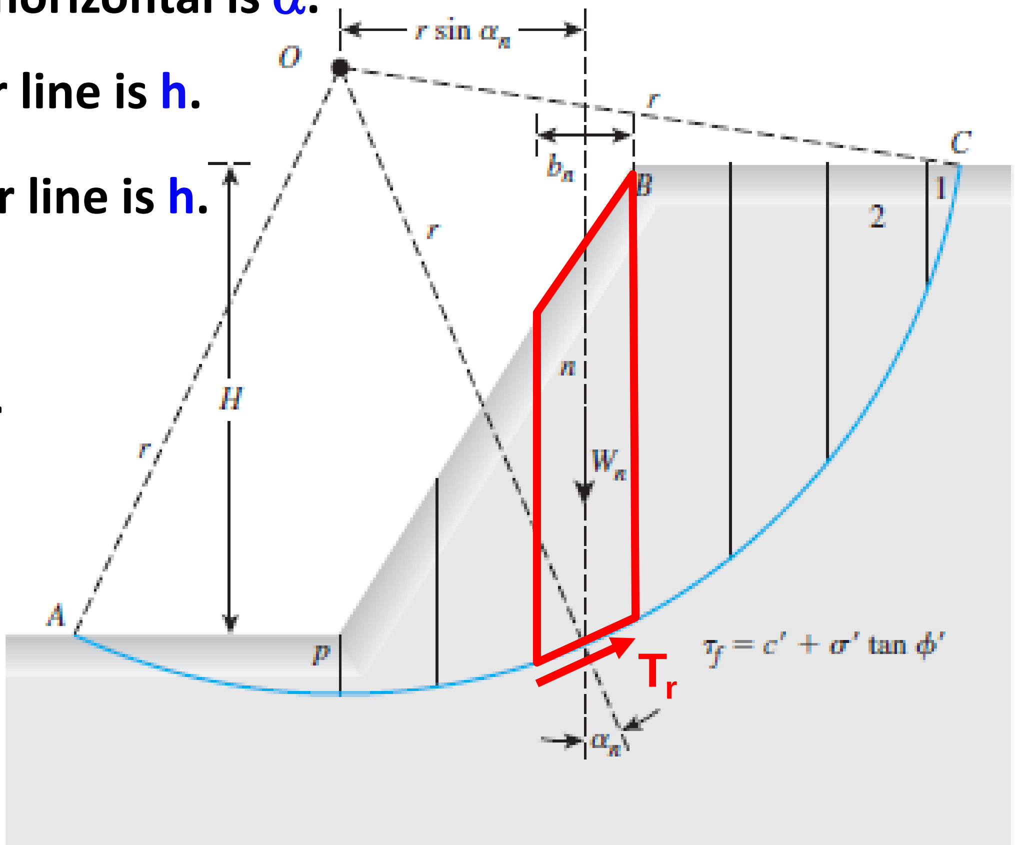




Non-homogeneous Slope **Irregular Slope**

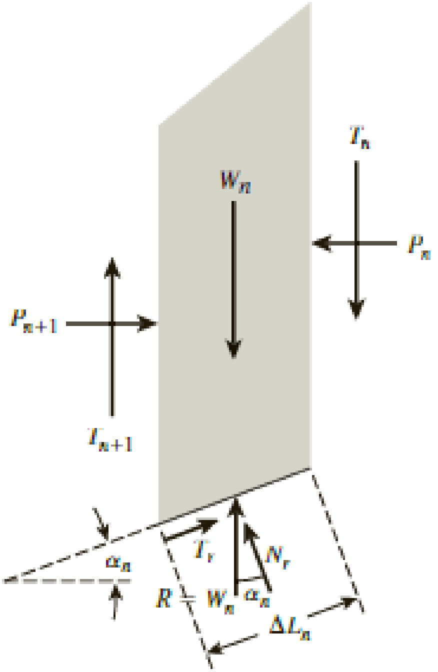
Method of Slices

- The soil mass above the trial slip surface is divided into several vertical parallel slices. The width of the slices need not to be the same (better to have it equal).
- The accuracy of calculation increases if the number of slices is increased.
- The base of each slice is assumed to be a straight line.
- The inclination of the base to the horizontal is α.
- The height measured in the center line is h.
- The height measured in the center line is h.
- The procedure requires that a series of trial circles are chosen and analyzed in the quest for the circle with the minimum factor of safety.



Method of Slices

- **Forces acting on each slice**
- **Total weight $w_i = \gamma hb$**
- **Total normal force at the base $N_r = \sigma * L$**
- **Shear force at the base $T_r = \tau * L$**
- **Total normal forces on the sides, P_n and P_{n+1}**
- **Shear forces on the sides, T_n and T_{n+1}**
- **5 unknowns T_r ,P_n ,P_{n+1} ,T_n ,T_{n+1}**
- **3 equations $\Sigma F_x=0$, $\Sigma F_y=0$,$\Sigma M=0$**
- **System is statically indeterminate**
- **Assumptions must be made to solve the problem**
- **Different assumptions yield different methods**
- **Two Methods:**
- **Ordinary Method of Slices (Fellenius Method)**
- **Bishop's Simplified Method of Slices**



Method of Slices

For the whole sliding mass

$$\Sigma M_o = 0$$

$$\Sigma W * r * \sin\alpha - \Sigma T * r = 0$$

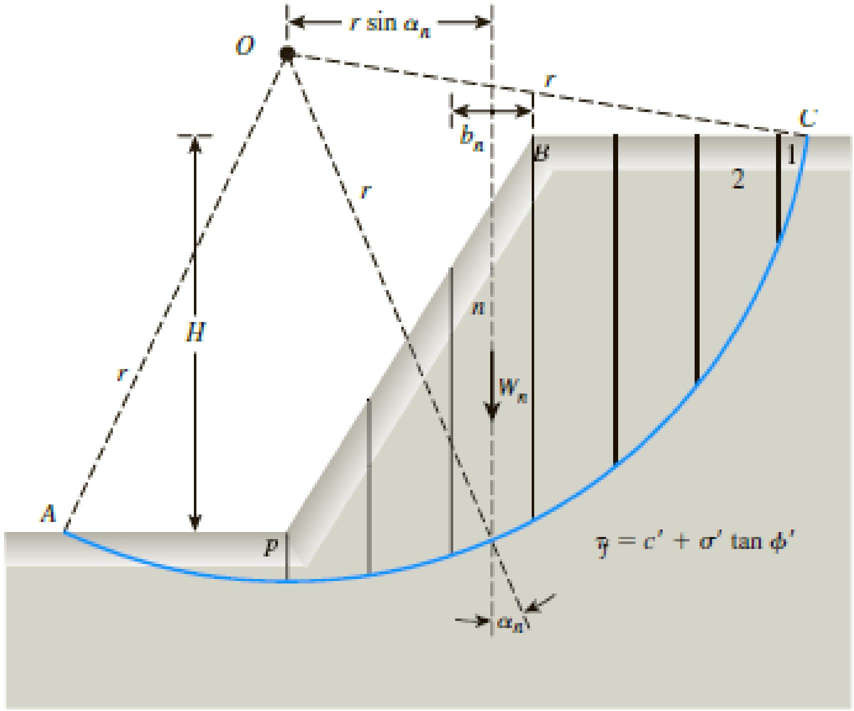
$$\Sigma W * \sin\alpha = \Sigma T$$

$$T = \tau_d * l = \frac{\tau_f}{F_s} * l$$

$$\Sigma W * \sin\alpha = \frac{\tau_f}{F_s} * l$$

$$Fs = \frac{\Sigma \tau_f * l}{\Sigma W * \sin\alpha}$$

$$Fs = \frac{\Sigma(c * l + \sigma_n * \tan\phi * l)}{\Sigma W * \sin\alpha}$$



EXAMPLE 15.10

Example 15.10

Figure 15.29 shows a 10-m high slope in saturated clay. Given: the saturated unit weight of soil $\gamma = 19\ kN/m^3$ and the undrained shear strength $c_u = 70\ kN/m^2$. Determine the factor of safety F_s using the method of slices for the trial circle shown.

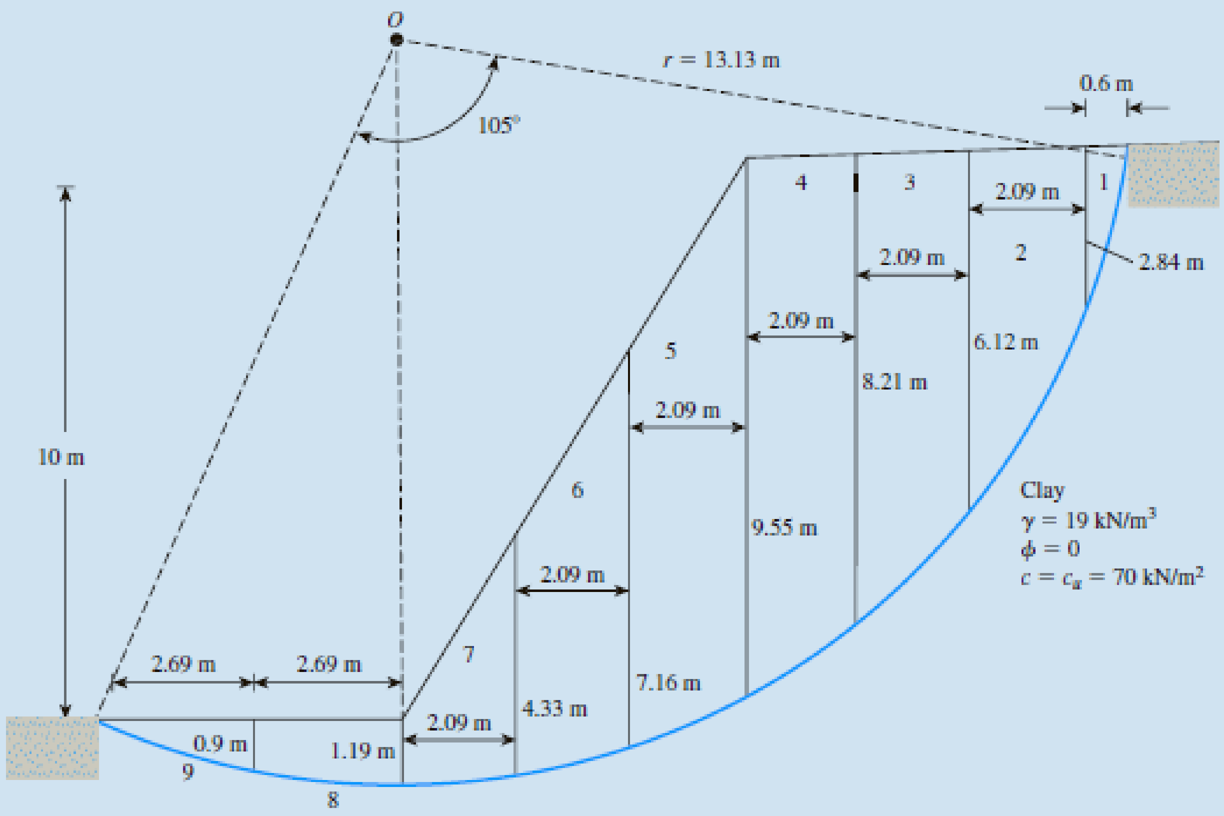


Figure 15.29

EXAMPLE 15.10

Solution

The trial wedge has been divided into nine slices. The following table gives the calculations for the driving moment M_d about O [also see Eq. (15.43)].

Slice no.	Weight (kN/m)	Moment arm (m)	M_d (kN-m/m)
1	$\frac{1}{2}(2.84)(0.6)(19) = 16.188$	$(6)(2.09) + (0.2) = 12.74$	**206.24**
2	$\frac{1}{2}(2.84 + 6.12)(2.09)(19) = 177.9$	$(5)(2.09) + \frac{2.09}{2} = 11.495$	**2044.96**
3	$\frac{1}{2}(6.12 + 8.21)(2.09)(19) = 284.52$	$(4)(2.09) + \frac{2.09}{2} = 9.405$	**2675.91**
4	$\frac{1}{2}(8.21 + 9.55)(2.09)(19) = 352.62$	$(3)(2.09) + \frac{2.09}{2} = 7.315$	**2579.41**
5	$\frac{1}{2}(9.55 + 7.16)(2.09)(19) = 331.78$	$(2)(2.09) + \frac{2.09}{2} = 5.225$	**1733.55**
6	$\frac{1}{2}(7.16 + 4.33)(2.09)(19) = 228.13$	$2.09 + \frac{2.09}{2} = 1.045$	**715.19**
7	$\frac{1}{2}(4.33 + 1.19)(2.09)(19) = 109.6$	$\frac{2.09}{2} = 1.045$	**114.53**
8	$\frac{1}{2}(1.19 + 0.9)(2.69)(19) = 53.41$	$-\frac{2.69}{2} = -1.345$	**−71.84**
9	$\frac{1}{2}(0.9 + 0)(2.69)(19) = 23.0$	$-\left(2.69 + \frac{2.69}{2}\right) = -4.035$	**−92.81**
			Σ9905.14 kN-m/m

From Eq. (15.44),

$$\text{Resisting moment, } M_R = c_u r^2 \theta = (70)(13.13)^2\left(\frac{105}{180}\right) \times \pi = 22{,}115.4 \text{ kN-m/m}$$

So,

$$F_s = \frac{M_R}{M_d} = \frac{22{,}115.4}{9{,}905.14} = \mathbf{2.23}$$

Method of Slices

$$Fs = \frac{\Sigma(c*l + \sigma_n * \tan\phi * l)}{\Sigma W * \sin\alpha}$$

$$\Sigma\sigma_n * l = \Sigma N$$

$$Fs = \frac{\Sigma c*l + \tan\phi * \Sigma N}{\Sigma W * \sin\alpha}$$

Equation is exact but approximations are introduced in finding the value of force N

Two Methods:

- Ordinary Method of Slices
- Bishop's Simplified Method of Slices

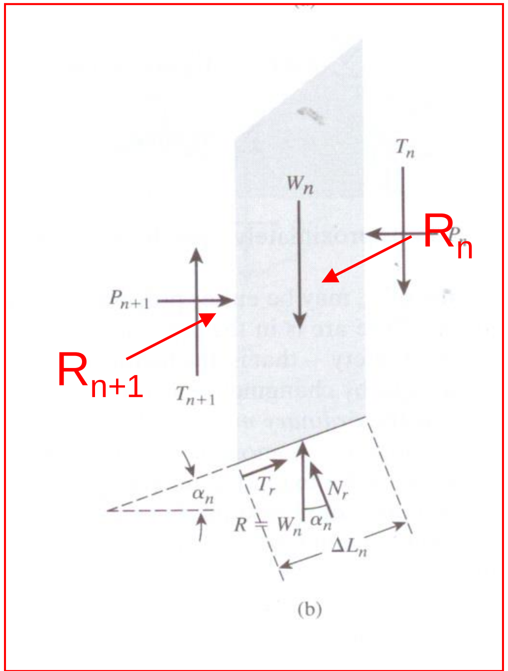
Ordinary Method of Slices

Ordinary Method of Slices

Fellenius' Method

Assumption

- For each slice, the resultant of the interslice forces is zero.
- The resultants of P_n and T_n are equal to the resultants of P_{n+1} and T_{n+1}, also their lines of actions coincide.



Ordinary Method of Slices

$$\Sigma F_n = 0 \ \ (to\, stay\, away\ \ from\, T_r\ )$$

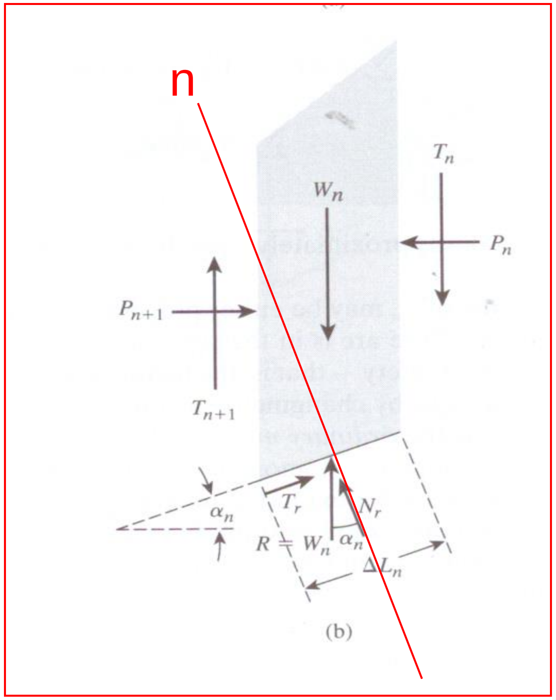
$$N_r = W_n * cos\,\alpha_n$$

$$F_s = \frac{\Sigma(c * \Delta l_n + W_n * cos\,\alpha_n\, tan\phi)}{\Sigma W_n * sin\alpha_n}$$

$$For\ undrained\ condition:$$

$$c = c_u \qquad \phi = 0$$

$$F_s = \frac{c_u l}{\Sigma W_n * sin\alpha_n}$$

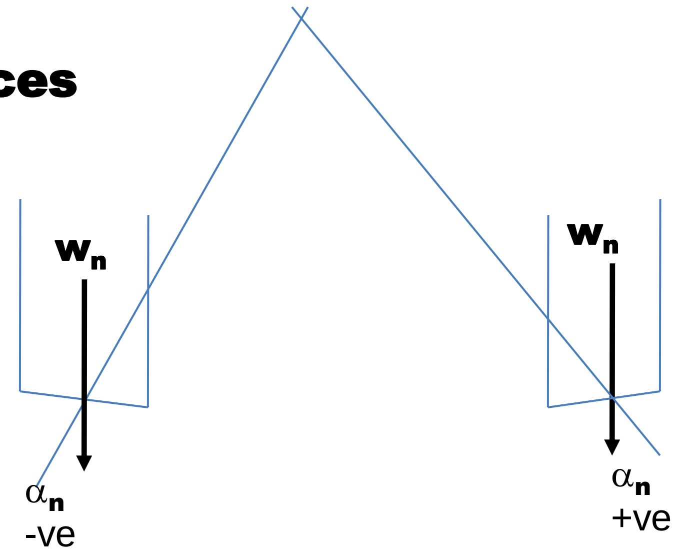


Ordinary Method of Slices

Steps for Ordinary Method of Slices

- **Draw the slope to a scale**
- **Divide the sliding wedge to various slices**
- **Calculate w_n and α_n for each slice**
- **α_n is taken at the middle of the slice**
- **Calculate the terms in the equation**

$$F_s = \frac{\Sigma(c^*\Delta l_n + W_n^* \cos\alpha_n \tan\phi)}{\Sigma W_n^* \sin\alpha_n}$$



- **Fill the following table**

Slice#	w_n	α_n	$\sin\alpha_n$	$\cos\alpha_n$	Δl_n	$w_n \sin\alpha_n$	$w_n \cos\alpha_n$

EXAMPLE 15.11

Example 15.11

Figure 15.30 shows a slope which has similar dimensions as in Figure 15.29 (Example 15.10). For the soil, given: $\gamma = 19\ \text{kN/m}^3$, $\phi' = 20°$, and $c' = 20\ \text{kN/m}^2$. Determine F_s using the ordinary method of slices.

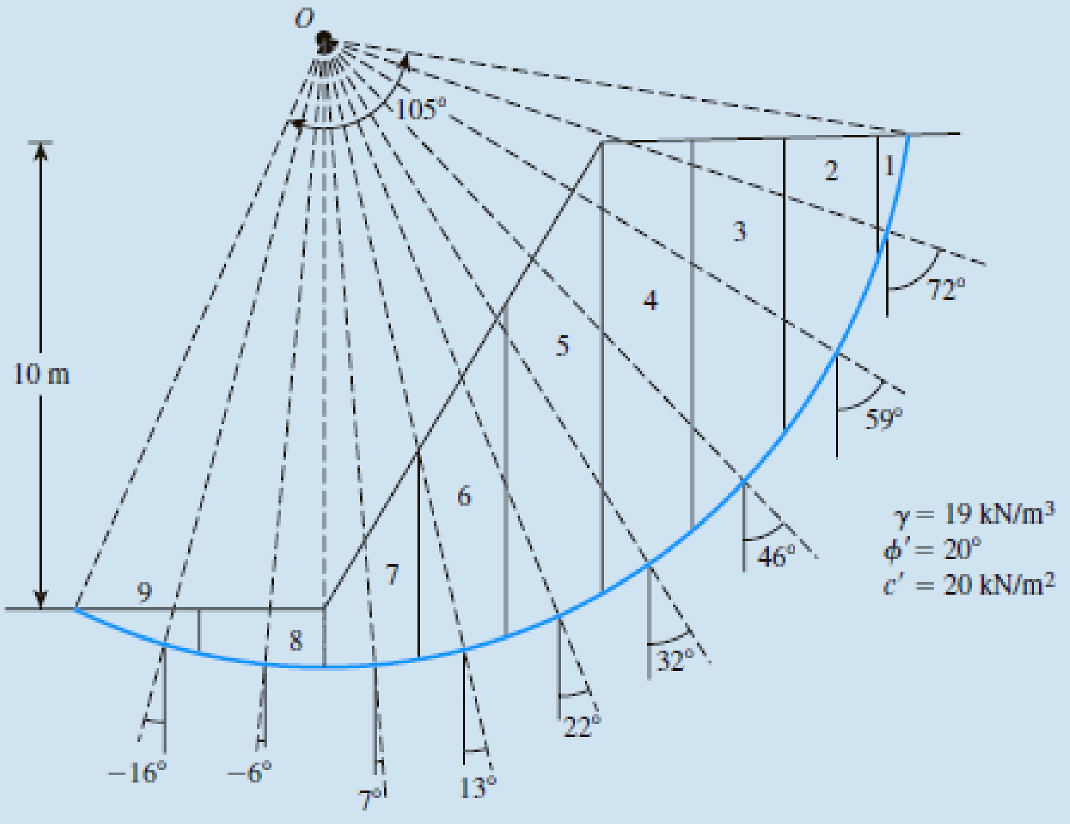


Figure 15.30 (*Note*: the width and height of each slice is same as in Figure 15.29)

EXAMPLE 15.11

Solution

Since the magnitude of γ and the dimension slices are the same in Figures 15.29 and 15.30, the weight W_n for each slice will be the same as in Example 15.10. Now the following table can be prepared.

Slice no. (1)	W_n (kN/m) (2)	α_n (deg) (3)	$\sin \alpha_n$ (4)	$\cos \alpha_n$ (5)	ΔL_n (m) (6)	$W_n \sin \alpha_n$ (kN/m) (7)	$W_n \cos \alpha_n$ (kN/m) (8)
1	16.188	72	0.951	0.309	1.942	15.395	5.00
2	177.9	59	0.788	0.515	4.058	140.185	91.62
3	284.52	46	0.719	0.695	3.007	204.57	197.74
4	352.62	32	0.530	0.848	2.465	186.89	299.02
5	331.78	22	0.375	0.927	2.255	124.42	307.56
6	228.13	13	0.225	0.974	2.146	51.33	222.2
7	109.6	7	0.122	0.993	2.105	13.37	108.83
8	53.41	−6	−0.105	0.995	2.704	−5.61	53.14
9	23.0	−16	−0.276	0.961	2.799	−6.35	22.10
					$\Sigma \approx$ 23.48 m	$\Sigma \approx$ 724.2 kN/m	Σ1307.21 kN/m

Note: $\Delta L_n = b_n/\cos\alpha_n$

$$F_s = \frac{(\Sigma \text{Col.6})(c') + (\Sigma \text{Col.8})(\tan \phi')}{(\Sigma \text{Col.7})}$$

$$= \frac{(23.48)(20) + (1307.21)(\tan 20)}{724.2}$$

$$= \mathbf{1.305}$$

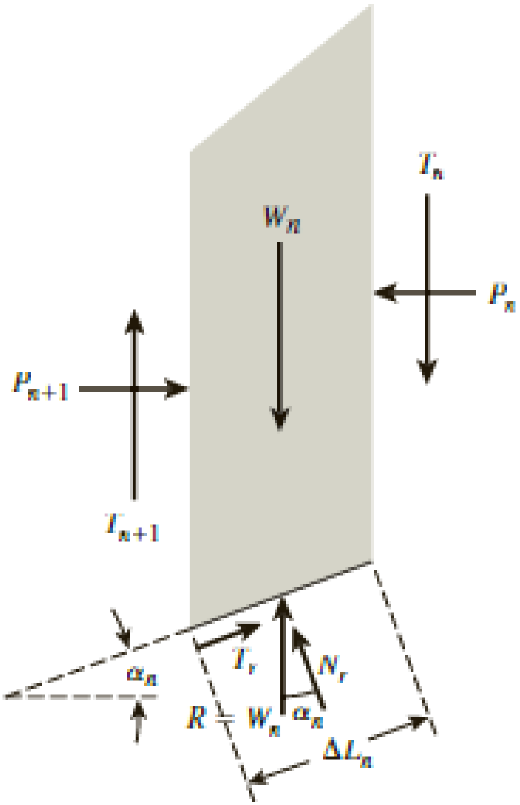
Bishop's Simplified Method of Slices

Bishop's Simplified Method of Slices

Assumption

For each slice, the resultant of the interslice forces is Horizontal.

i.e. $T_n = T_{n+1}$



Bishop's Simplified Method of Slices

$$\Sigma F_y = 0 \quad (\text{to stay away from } P_n \text{ and } P_{n+1})$$

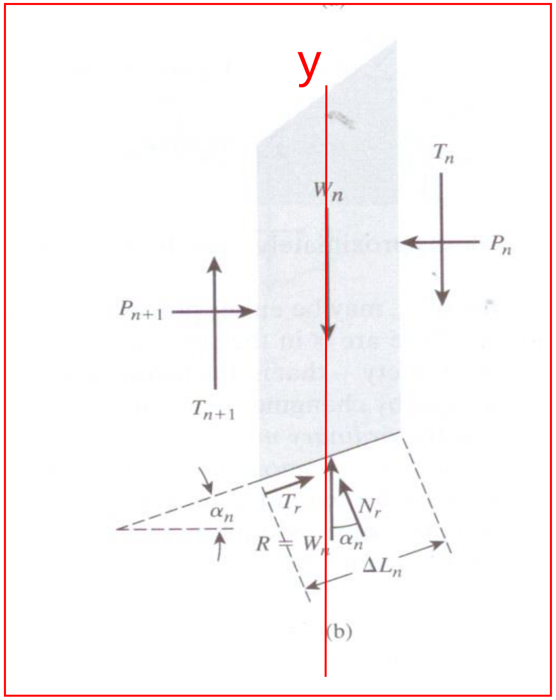
$$W_n = N_r * \cos\alpha_n + T_r * \sin\alpha_n$$

$$T_r = \tau_d * \Delta l_n = \left(\frac{c + \sigma_n \tan\phi}{F_s}\right)\Delta l_n$$

$$T_r = \frac{c\Delta l_n}{F_s} + \frac{\sigma_n \Delta l_n \tan\phi}{F_s}$$

$$T_r = \frac{c\Delta l_n}{F_s} + \frac{N_r \tan\phi}{F_s}$$

$$W_n = N_r * \cos\alpha_n + \frac{c\Delta l_n}{F_s}\sin\alpha_n + \frac{N_r \tan\phi}{F_s}\sin\alpha_n$$



Bishop's Simplified Method of Slices

$$N_r = \frac{W_n - \dfrac{c\Delta l_n}{F_s}\sin\alpha_n}{\cos\alpha_n + \dfrac{\tan\phi\sin\alpha_n}{F_s}}$$

$$F_s = \frac{\Sigma c l_n + \tan\phi\left[\dfrac{W_n - \dfrac{c\Delta l_n}{F_s}\sin\alpha_n}{\cos\alpha_n + \dfrac{\tan\phi\sin\alpha_n}{F_s}}\right]}{\cos\alpha_n + \dfrac{\tan\phi\sin\alpha_n}{F_s}} \qquad but \quad l_n = \frac{b_n}{\cos\alpha_n}$$

$$F_s = \frac{1}{\Sigma W_n \sin\alpha_n}\sum\frac{cb_n + W_n\tan\phi}{\cos\alpha_n + \dfrac{\tan\phi\sin\alpha_n}{F_s}}$$

Trail and error procedure

Bishop's Simplified Method of Slices

Steps for Bishop's Simplified Method of Slices

- **Draw the slope to a scale**
- **Divide the sliding wedge to various slices**
- **Calculate w_n and α_n for each slice**
- **α_n is taken at the middle of the slice**
- **Calculate the terms in the equation**

$$F_s = \frac{1}{\Sigma W_n \sin\alpha_n} \sum \frac{cb_n + W_n \tan\phi}{\cos\alpha_n + \dfrac{\tan\phi \sin\alpha_n}{F_s}}$$

- **Fill the following table**

Slice#	w_n	α_n	$\sin\alpha_n$	$\cos\alpha_n$	b_n	$w_n \sin\alpha_n$

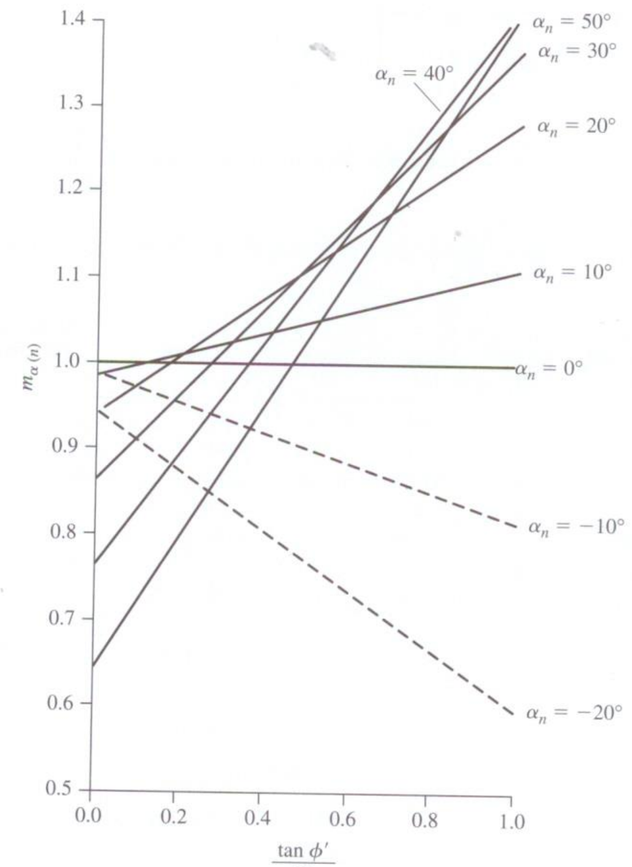
- **Assume F_s and plug it in the right-hand term of the equation then calculate F_s**
- **Repeat the previous step until the assumed F_s = the calculated F_s.**

Bishop’s Simplified Method of Slices

$$m_{\alpha(n)} = \cos\alpha_n + \frac{\tan\phi \sin\alpha_n}{F_s}$$

$$F_s = \sum \frac{(cb_n + W_n \tan\phi)\dfrac{1}{m_{\alpha(n)}}}{\Sigma W_n \sin\alpha_n}$$

$$F_s = \frac{1}{\sum W_n \sin\alpha_n} \sum \left(\frac{c' b_n + W_n \tan\phi'}{\cos\alpha_n + \dfrac{\sin\alpha_n \tan\phi'}{F_s}} \right)$$



Fig
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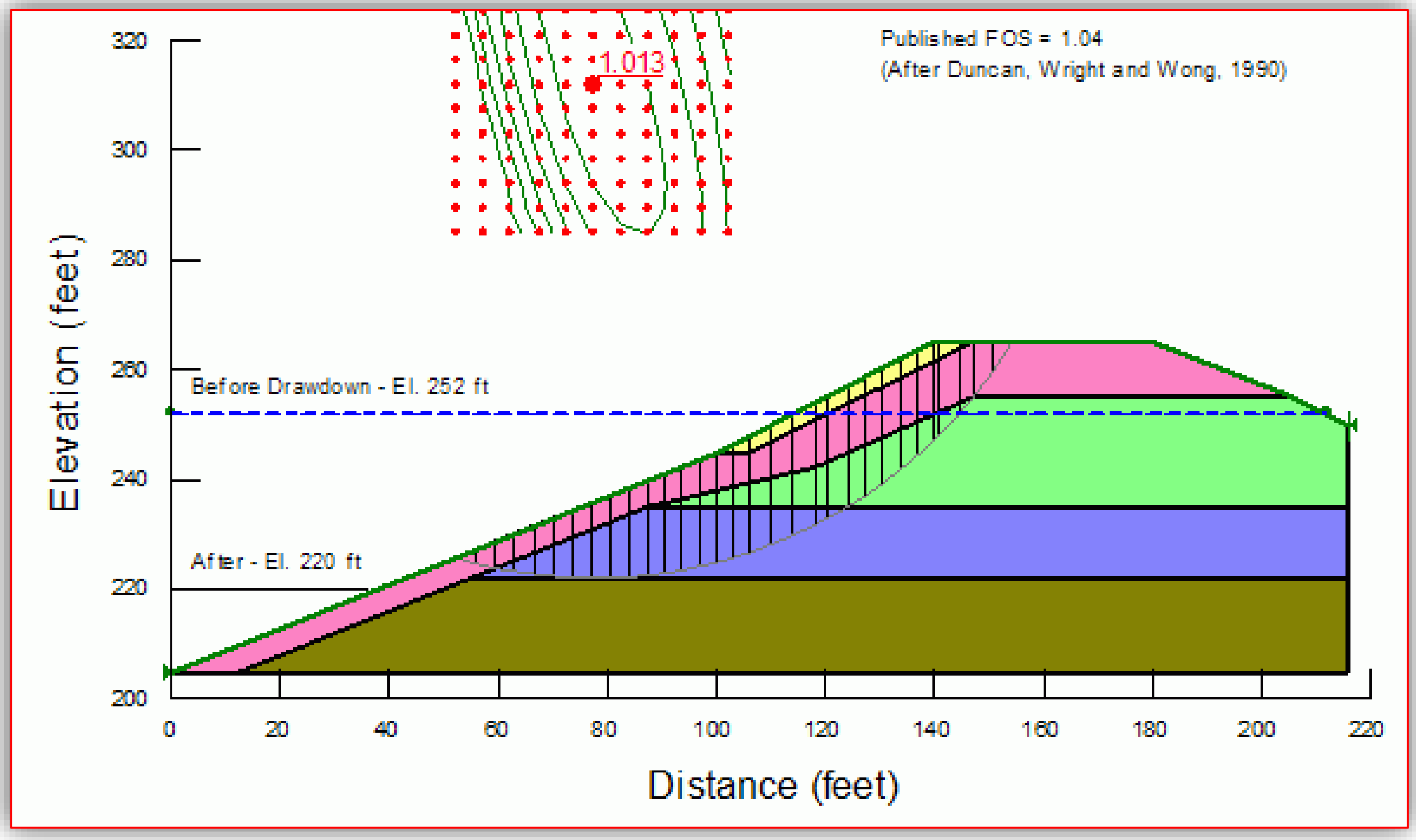
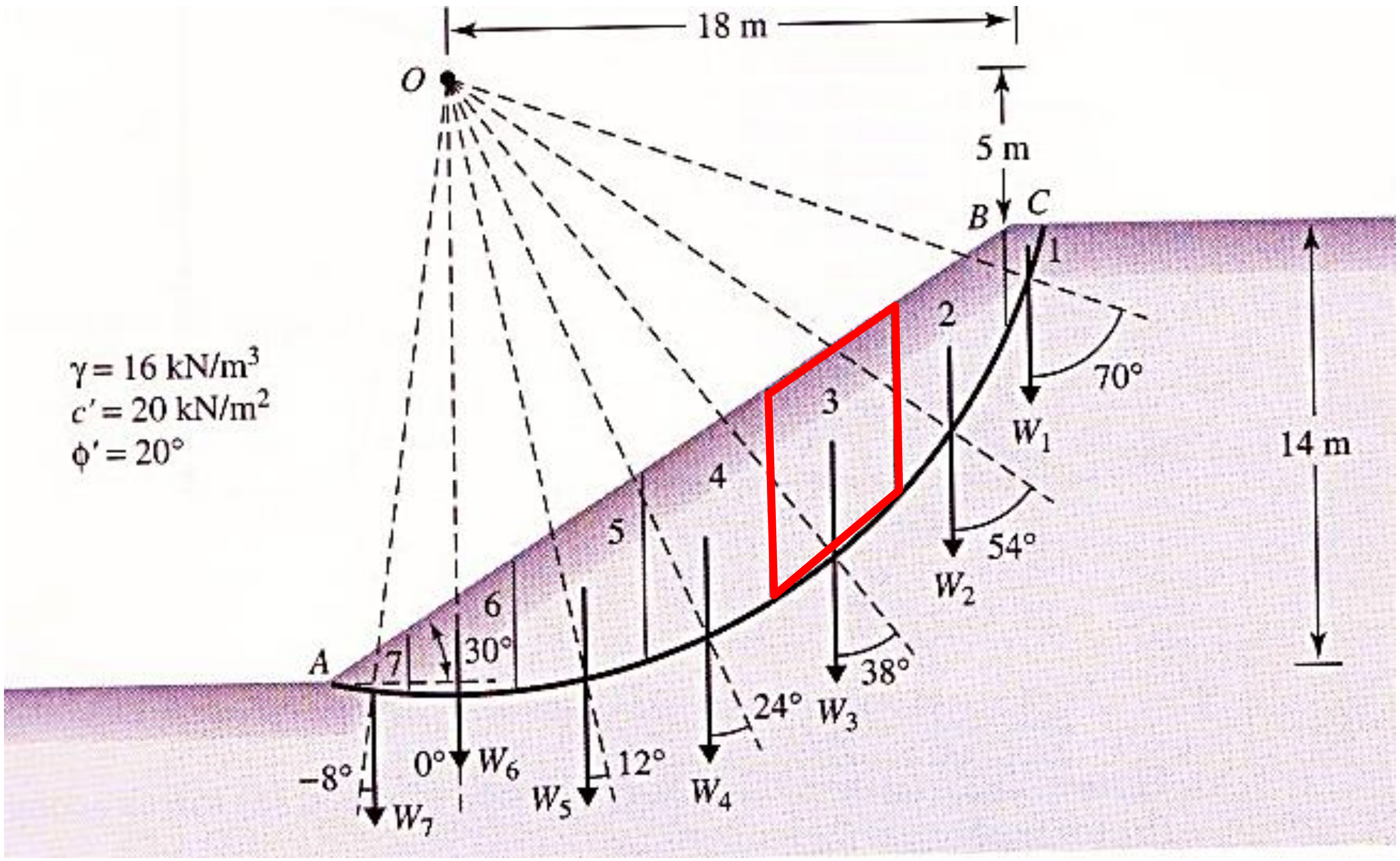
Bishop's Simplified Method of Slices

- **Example of specialized software:**
  - **Geo-Slope,**
  - **Geo5,**
  - **SVSlope**
  - **Many others**



Final Exam

Determine the safety factor for the given trial rupture surface shown in Figure 3. Use Bishop's simplified method of slices with first trial factor of safety F_s = 1.8 and make <u>only one iteration</u>. The following table can be prepared; however, only needed cells can be generated "filled".



SOLUTION

$F_s = 1.8$

Table 1. "Fill only necessary cell for this particular problem"

Slice No. (1)	Width b_n (m) (2)	Height h_l (m) (3)	Height h_2 (m) (4)	Area A (m^2) (5)	Weight W_n (kN/m) (6)	$\alpha_{(n)}$ (7)	$m_{\alpha(n)}$ (8)	$W_n \sin\alpha$ (kN/m) (9)	$c' b_n + \frac{w_n \tan\emptyset'}{m_{\alpha(n)}}$
1					22.4	70			
2					294.4	54			
3					?	38			
4					435.2	24			
5					390	12			
6					268.8	0.0			
7					66.58	-8			
								$\sum$	$\sum$

$$F_s = \frac{\sum_{n=1}^{n=p} (c'b_n + W_n \tan\phi') \frac{1}{m_{\alpha(n)}}}{\sum_{n=1}^{n=p} W_n \sin\alpha_n}$$

$$m_{\alpha(n)} = \cos\alpha_n + \frac{\tan\phi' \sin\alpha_n}{F_s}$$

Remarks on Method of Slices

- Bishop's simplified method is probably the most widely used (but it has to be incorporated into computer programs).
- It yields satisfactory results in most cases.
- The F_s determined by this method is an underestimate (conservative) but the error is unlikely to exceed 7% and in most cases is less than 2%.
- The ordinary method of slices is presented in this chapter as a learning tool only. It is used rarely now because it is too conservative.
- The Bishop Simplified Method yields factors of safety which are higher than those obtained with the Ordinary Method of Slices.
- The two methods do not lead to the same critical circle.
- Analyses by more refined methods involving consideration of the forces acting on the sides of slices show that the Simplified Bishop Method yields answers for factors of safety which are very close to the correct answer.

Remarks on Method of Slices

Two Methods:

Ordinary Method of Slices

- **Underestimate F_s (too conservative)**
- **Error compared to accurate methods (5-20%)**
- **Rarely used**

Bishop's Simplified Method of Slices

- **The most widely used method**
- **Yields satisfactory results when applying computer program**

The end