April22 **Grading scheme of midterm106**

Part 1

a) $T_4 = \frac{4-0}{2.4}\left(0 + 2\sqrt{2} + 8\sqrt{5} + 18\sqrt{10} + 16\sqrt{17}\right) \approx 71.803825 \quad (\mathbf{1.5}) + (\mathbf{0.5})$

b) $\sum_1^n (2k + \alpha) = n(n+1) + n\alpha = n^2$ $(\mathbf{1.5})$ $so\ \alpha = -1 \quad (\mathbf{0,5})$

c)

$\frac{1}{4}\int_0^4 (2 + 3x^2)dx = 18 = 2 + 3z^2$ $(\mathbf{2})$ $so\ z = \frac{4}{\sqrt{3}}$ $(\mathbf{1})$ $(half\ a\ mark\ if\ z = \pm\frac{4}{\sqrt{3}})$

d) $lny = x^2 \ln\left(2 + \sqrt{x}\right) so\ y' = \left(2xln\left(2 + \sqrt{x}\right) + \frac{x^2}{4\sqrt{x}+2x}\right) y \quad (\mathbf{0,5}) + (\mathbf{1.5})$

e) $\int \frac{e^{5x}}{e^{10}+16} dx = \frac{1}{5}\int \frac{du}{u^2+16} = \frac{1}{20}\tan^{-1}\left(\frac{e^{5x}}{4}\right) + C \quad (\mathbf{2}) + (\mathbf{1})$

f) $\int \frac{dx}{x\sqrt{x^6-4}} = \frac{1}{3}\int \frac{du}{u\sqrt{u^2-4}} = \frac{1}{6}\sec^{-1}(\frac{|x^3|}{2}) + C \quad (\mathbf{2}) + (\mathbf{1})$

g) $\int \frac{dx}{x\sqrt{9+(lnx)^2}} = \int \frac{du}{\sqrt{9+u^2}} = \sinh^{-1}((lnx)/3) + C \quad (\mathbf{2}) + (\mathbf{1})$

Part2

a) Find $\int x\tan^{-1} x dx = \frac{x^2}{2}\tan^{-1} x - \frac{1}{2}\int \frac{x^2}{1+x^2} dx \quad (\mathbf{1.5})$

$= \frac{x^2}{2}\tan^{-1} x - \frac{1}{2}(x - \tan^{-1} x) + C \quad (\mathbf{1.5})$

b) $\int (sinx)^5 (cosx)^4 dx = -\int (1-u^2)^2 u^4 du \quad (\mathbf{1.5})$

$= -\frac{(sinx)^9}{9} + \frac{2}{7}(sinx)^7 - \frac{1}{5}(sinx)^5 + C$ $(\mathbf{1.5})$

cosx cosx cosx