

Probability & Statistics for Engineers

Chapter 1:

Introduction to Statistics & Data Analysis:

- Statistics:
- Collection of data
 - Displaying and organizing data
 - Descriptive statistical measures
 - Data Analysis and inferences

* Population whole set

N : population size

x_1, x_2, \dots, x_N

* Sample subset \equiv observations

n : sample size

x_1, x_2, \dots, x_n sample values

Statistics obtained from the sample are used to estimate (approximate) the parameters of the population.

* Scientific data

* Statistical Inference: (1) Estimation $\begin{cases} \rightarrow \text{point est.} \\ \rightarrow \text{interval est.} \end{cases}$
(2) Hypotheses Testing

* Measures of location (Central Tendency)

Sample mean $(\bar{x}) = \frac{x_1 + x_2 + \dots + x_n}{n} = \frac{\sum_{i=1}^n x_i}{n}$

note: $\sum_{i=1}^n (x_i - \bar{x}) = 0$

* Measures of variability (Dispersion)

Sample variance $S^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}$ \rightarrow degrees of freedom

Standard deviation $S = \sqrt{S^2}$

note: $S^2 = \frac{\sum x_i^2 - n(\bar{x})^2}{n-1}$

Probability : Chapter 2

* Sample space S [set of all possible outcomes of a random experiment]

* Event $A \subseteq S$

$\emptyset \subseteq S$ impossible event

$S \subseteq S$ sure event

$n(S)$ no. of elements in S

$n(A)$ no. of elements in A (occurrences)

→ Operations on events:

* Complement $A^c = \{x \in S : x \notin A\}$

* Intersection $A \cap B = AB = \{x \in S : x \in A \text{ and } x \in B\}$
(both A and B occur together)

- if $A \cap B = \emptyset \Rightarrow A$ and B are disjoint
(mutually exclusive)

* Union $A \cup B = \{x \in S : x \in A \text{ or } x \in B\}$
(At least one of A, B occurs)

→ Counting sample points:

Combination: $\binom{n}{r} = \frac{n!}{r!(n-r)!}$

note:

$$n! = n \cdot (n-1)!$$

• no. of ways of selecting r objects from n objects without regard to order.

• no. of combinations of n distinct objects taken r at a time.

• no. of different selections of r objects from n objects

• no. of ways of dividing a set of n objects into two sets (r & $(n-r)$)

$$\binom{n}{n} = 1, \binom{n}{0} = 1, \binom{n}{1} = n, \binom{n}{r} = \binom{n}{n-r}$$

→ Probability of an event:

• $0 \leq P(A) \leq 1$, $P(S) = 1$, $P(\emptyset) = 0$

$$P(A) = \frac{n(A)}{n(S)}$$

• $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

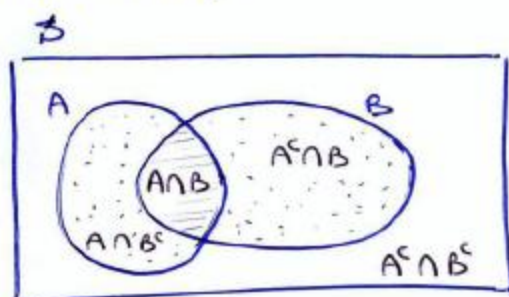
if A, B are disjoint $P(A \cap B) = 0$

$$P(A \cup B) = P(A) + P(B)$$

• $P(A^c) = 1 - P(A)$

• $P(A \cap B^c) = P(A) - P(A \cap B)$

• $P(A^c \cap B^c) = 1 - P(A \cup B)$



D' Morgan's:

$$A^c \cap B^c = (A \cup B)^c$$

$$A^c \cup B^c = (A \cap B)^c$$

* Conditional probability:

probability of occurring an event A , ^{if it's known} given that some event B has occurred - conditional prob. of A given B

$$P(A|B) = \frac{P(A \cap B)}{P(B)} , \quad P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$P(A \cap B) = P(A) P(B|A) = P(B) P(A|B)$$

Multiplicative rule:

$$P(A_1, A_2, A_3) = P(A_1) P(A_2|A_1) P(A_3|A_1, A_2)$$

Independent events: • A, B are independent events:

• if A_1, A_2, A_3 are independent: || if $P(A|B) = P(A)$ or

$$P(B|A) = P(B) \text{ or}$$

$$P(A_1, A_2, A_3) = P(A_1) P(A_2) P(A_3)$$

$$P(A \cap B) = P(A) P(B)$$

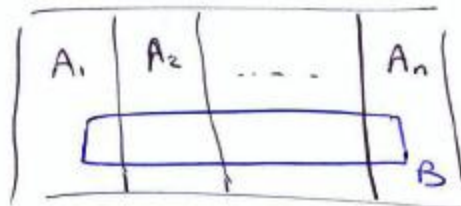
* Bayes' Rule

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Defn: Events A_1, A_2, \dots, A_n constitute a partition of the sample space S if:

$$- \bigcup_{i=1}^n A_i = S$$

$$- A_i \cap A_j = \emptyset \quad \forall i \neq j$$



Theorem: Total probability:

If A_1, A_2, \dots, A_n constitute a partition of S then for any event B :

$$P(B) = \sum_{k=1}^n P(A_k) P(B|A_k)$$

Bayes' rule:

$$P(A_i|B) = \frac{P(A_i) P(B|A_i)}{P(B)}$$