Chapter 3 Random variables and Probability distributions:

* Discrete random variables:

A discrete r. v. X takes the values $x_1, x_2, \dots, x_n$ with probabilities: $P(X=x_1)$, $P(X=x_2)$, $\dots$, $P(X=x_n)$

→ Probability distribution function: PDF

$$\boxed{f(x) = P(X=x)}$$

X	x_1	x_2	$\dots$	x_n
$f(x)$	$f(x_1)$	$f(x_2)$	$\dots$	$f(x_n)$

$f(x) \geq 0$

& $\sum_x f(x) = 1$

→ Cumulative distribution function: CDF

$$F(x) = P(X \leq x) = \sum_{t \leq x} f(t)$$

$$F(x) = \begin{cases} 0 & x < x_1 \\ f(x_1) & x_1 \leq x < x_2 \\ f(x_1)+f(x_2) & x_2 \leq x < x_3 \\ \vdots & \\ 1 & x \geq x_n \end{cases}$$

$$f(x_i) = F(x_i) - F(x_{i-1})$$

note:

$P(a < X \leq b) = F(b) - F(a)$

$P(a \leq X \leq b) = F(b) - F(a) + f(a)$

$P(a < X < b) = F(b) - F(a) - f(b)$

Mean : Expected value $\mu_X = E(X) = \sum x f(x)$

Variance : $\sigma_X^2 = Var(X) = E(X^2) - [E(X)]^2$

$$= \mu_{X^2} - \mu_X^2$$

where $E(X^2) = \sum x^2 f(x)$

* Continuous Random Variables:

Probability denisty function: PDF

$f(x) =$ any fun. of $x \quad \forall\ x_0 < x < x_1$ & $= 0$ elsewhere

$f(x) \geq 0$ & $\int_{-\infty}^{\infty} f(x)\, dx = 1$

note that: $f(x) \neq P(X = x)$ & $P(X = x) = 0 \quad \forall x$

Cumulative distribution function: CDF

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(t)\, dt$$

note: $P(a < X < b) \equiv P(a \leq X \leq b) = F(b) - F(a)$

Mean $\mu_X = E(X) = \int_{-\infty}^{\infty} x\, f(x)\, dx$

Variance $\sigma^2 = Var(X) = E(X^2) - [E(X)]^2$

$$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)\, dx$$

note that:

$$Var(X) = E(X^2) - [E(X)]^2$$

or $\sigma^2 = \mu_{X^2} - (\mu_X)^2$

then $\mu_{X^2} = \sigma^2 + \mu_X^2$

Chapter 4 Mathematical Expectation:

$$E(X) = \begin{cases} \sum_x x\, f(x) & X \text{ discrete r.v.} \\ \int_{-\infty}^{\infty} x\, f(x)\, dx & X \text{ continuous r.v.} \end{cases}$$

$$E(g(X)) = \begin{cases} \sum g(x)\, f(x) \\ \int_{-\infty}^{\infty} g(x)\, f(x)\, dx \end{cases}$$

$$E(aX \pm b) = a\, E(X) \pm b$$

$$Var(aX \pm b) = a^2\, Var(X)$$

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if X and Y independent random variables:

$$E(aX \pm bY) = a\, E(X) \pm b\, E(Y)$$

$$Var(aX \pm bY) = a^2\, Var(X) + b^2\, Var(Y)$$

Chebyshev's theorem:

$$\boxed{P(|X - \mu| < k\sigma) \geq 1 - \frac{1}{k^2}}$$

lower_bound

$$P(|X - \mu| < k\sigma) = P(-k\sigma < X - \mu < k\sigma)$$
$$= P(\mu - k\sigma < X < \mu + k\sigma)$$

Ex: $\boxed{P(\mu - 2\sigma < X < \mu + 2\sigma) \geq 0.75}$