Chapter 2

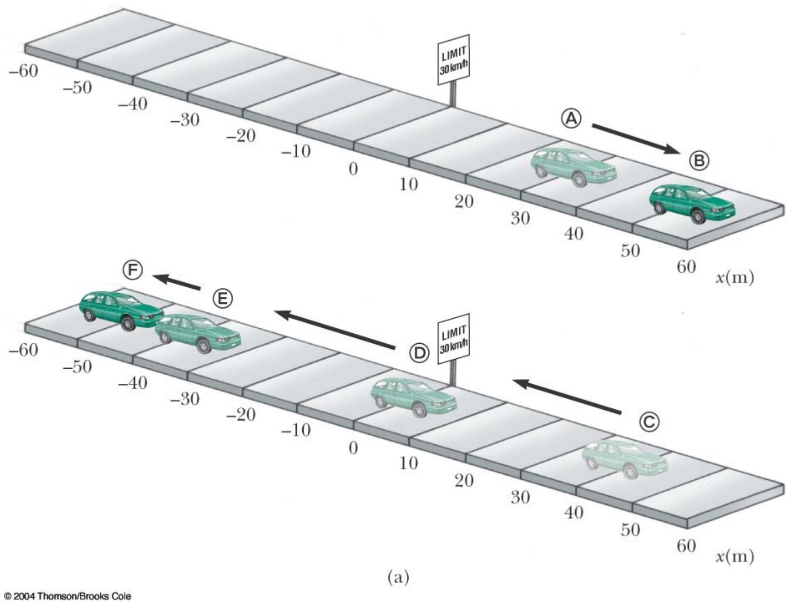
Motion in One Dimension

Kinematics

- Describes motion while ignoring the agents that caused the motion
- For now, will consider motion in one dimension
  - Along a straight line
- Will use the particle model
  - A particle is a point-like object, has mass but infinitesimal size

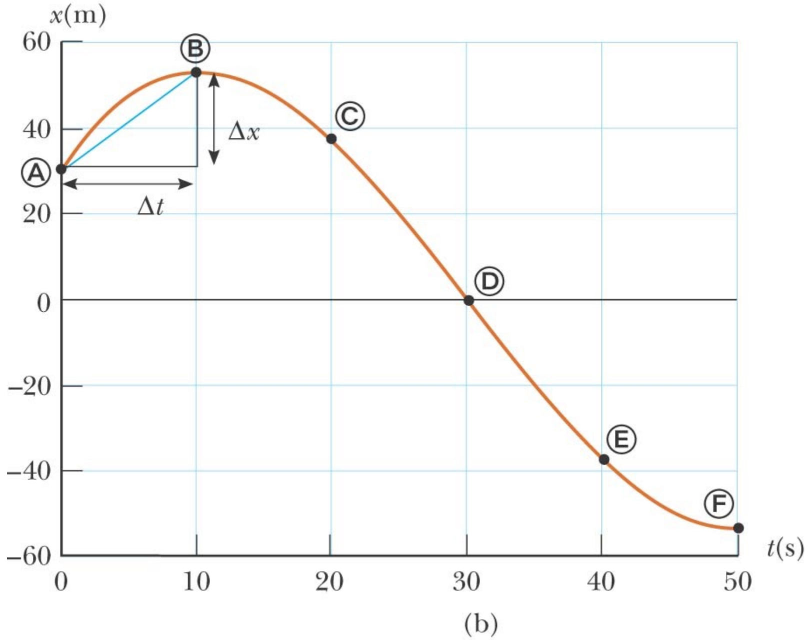
Position

- Defined in terms of a **frame of reference**
  - One dimensional, so generally the x- or y-axis
- The object's position is its location with respect to the frame of reference



Position-Time Graph

- The position-time graph shows the motion of the particle (car)
- The smooth curve is a guess as to what happened between the data points



Displacement

- Defined as the change in position during some time interval
  - Represented as Δx

    $\Delta x = x_f - x_i$
  - SI units are meters (m) Δx can be positive or negative
- Different than distance – the length of a path followed by a particle

Vectors and Scalars

- Vector quantities need both magnitude (size or numerical value) and direction to completely describe them
  - Will use + and – signs to indicate vector directions
- Scalar quantities are completely described by magnitude only

Average Velocity

- The **average velocity** is rate at which the displacement occurs

$$v_{average} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{\Delta t}$$

- The dimensions are length / time [L/T]
- The SI units are m/s
- Is also the slope of the line in the position – time graph

Average Speed

- Speed is a scalar quantity
  - same units as velocity
  - total distance / total time
- The average speed is not (necessarily) the magnitude of the average velocity

Instantaneous Velocity

- The limit of the average velocity as the time interval becomes infinitesimally short, or as the time interval approaches zero
- The instantaneous velocity indicates what is happening at every point of time

Instantaneous Velocity, equations

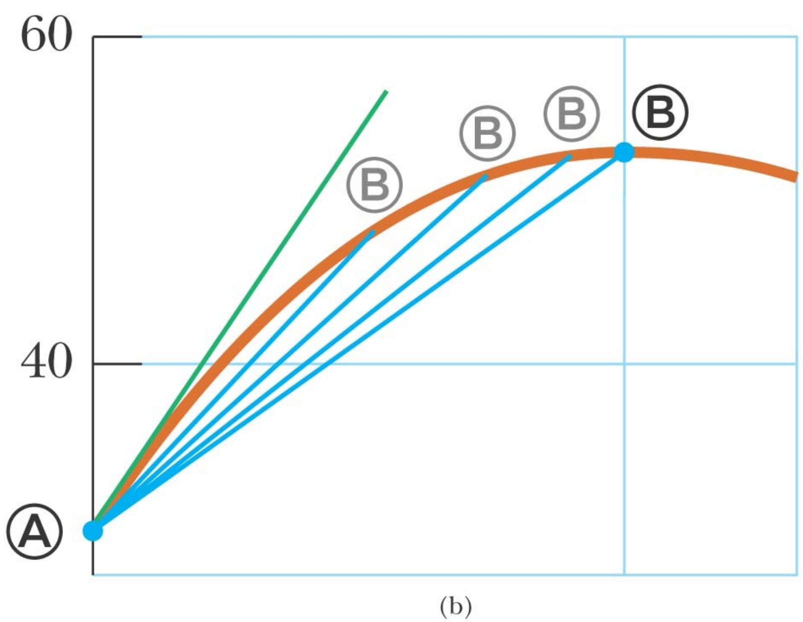
- The general equation for instantaneous velocity is

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

- The instantaneous velocity can be positive, negative, or zero

Instantaneous Velocity, graph

- The instantaneous velocity is the slope of the line tangent to the *x* vs. *t* curve
- This would be the green line
- The blue lines show that as Δt gets smaller, they approach the green line



Instantaneous Speed

- The instantaneous speed is the magnitude of the instantaneous velocity
- Remember that the average speed is <u>not</u> the magnitude of the average velocity

Average Acceleration

- Acceleration is the rate of change of the velocity

$$\bar{a}_x = \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{\Delta t}$$

- Dimensions are L/T^2
- SI units are m/s^2

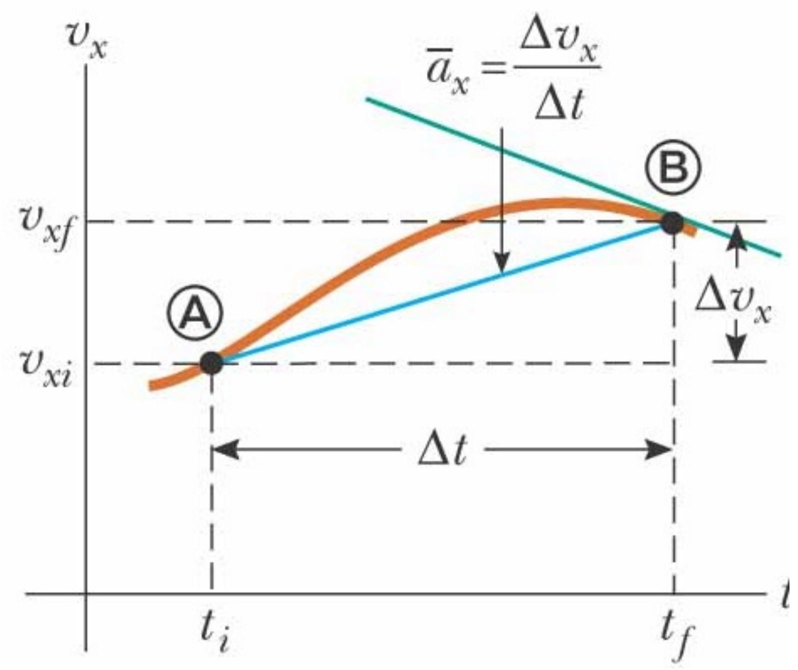
Instantaneous Acceleration

- The instantaneous acceleration is the limit of the average acceleration as Δt approaches 0

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} = \frac{d^2 x}{dt^2}$$

Instantaneous Acceleration -- graph

- The slope of the velocity vs. time graph is the acceleration
- The green line represents the instantaneous acceleration
- The blue line is the average acceleration



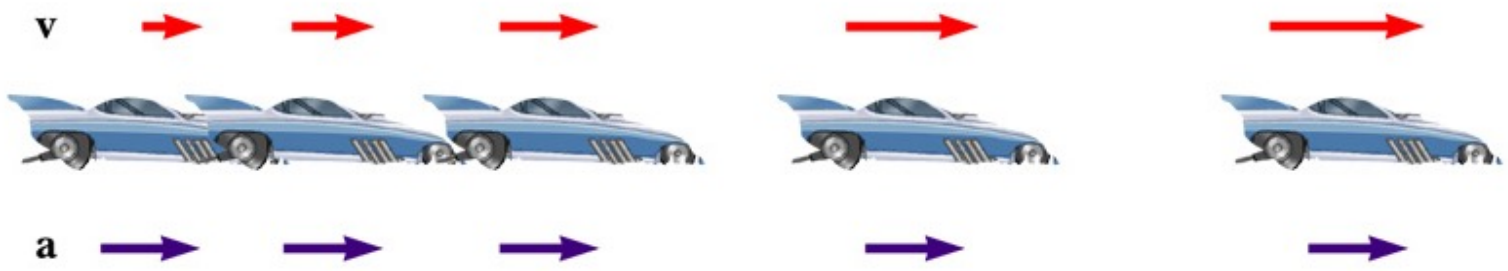
Acceleration and Velocity, 1

- When an object’s velocity and acceleration are in the same direction, the object is speeding up
- When an object’s velocity and acceleration are in the opposite direction, the object is slowing down

Acceleration and Velocity, 2

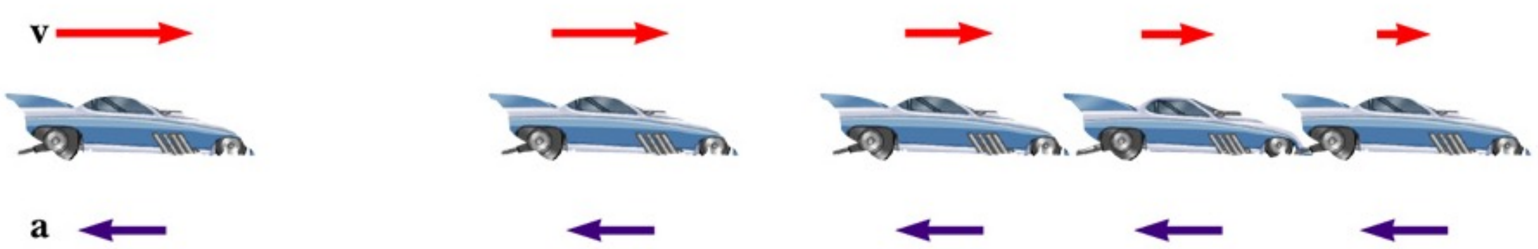
- The car is moving with constant positive velocity (shown by red arrows maintaining the same size)
- Acceleration equals zero

Acceleration and Velocity, 3



- Velocity and acceleration are in the same direction
- Acceleration is uniform (blue arrows maintain the same length)
- Velocity is increasing (red arrows are getting longer)
- This shows positive acceleration and positive velocity

Acceleration and Velocity, 4



- Acceleration and velocity are in opposite directions
- Acceleration is uniform (blue arrows maintain the same length)
- Velocity is decreasing (red arrows are getting shorter)
- Positive velocity and negative acceleration

Kinematic Equations -- summary

Table 2.2

Kinematic Equations for Motion of a Particle Under Constant Acceleration	
Equation	**Information Given by Equation**
$v_{xf} = v_{xi} + a_x t$	Velocity as a function of time
$x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})t$	Position as a function of velocity and time
$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$	Position as a function of time
${v_{xf}}^2 = {v_{xi}}^2 + 2a_x(x_f - x_i)$	Velocity as a function of position

Note: Motion is along the x axis.

Kinematic Equations

- The kinematic equations may be used to solve any problem involving one-dimensional motion with a constant acceleration
- You may need to use two of the equations to solve one problem
- Many times there is more than one way to solve a problem

Kinematic Equations, specific

- For constant *a*, $v_{xf} = v_{xi} + a_x t$
- Can determine an object's velocity at any time *t* when we know its initial velocity and its acceleration
- Does not give any information about displacement

Kinematic Equations, specific

- For constant acceleration,

$$\overline{v}_x = \frac{v_{xi} + v_{xf}}{2}$$

- The average velocity can be expressed as the arithmetic mean of the initial and final velocities

Kinematic Equations, specific

- For constant acceleration,

$$x_f = x_i + \overline{v}\,t = x_i + \frac{1}{2}\left(v_{xi} + v_{fx}\right)t$$

- This gives you the position of the particle in terms of time and velocities
- Doesn't give you the acceleration

Kinematic Equations, specific

- For constant acceleration,

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

- Gives final position in terms of velocity and acceleration
- Doesn't tell you about final velocity

Kinematic Equations, specific

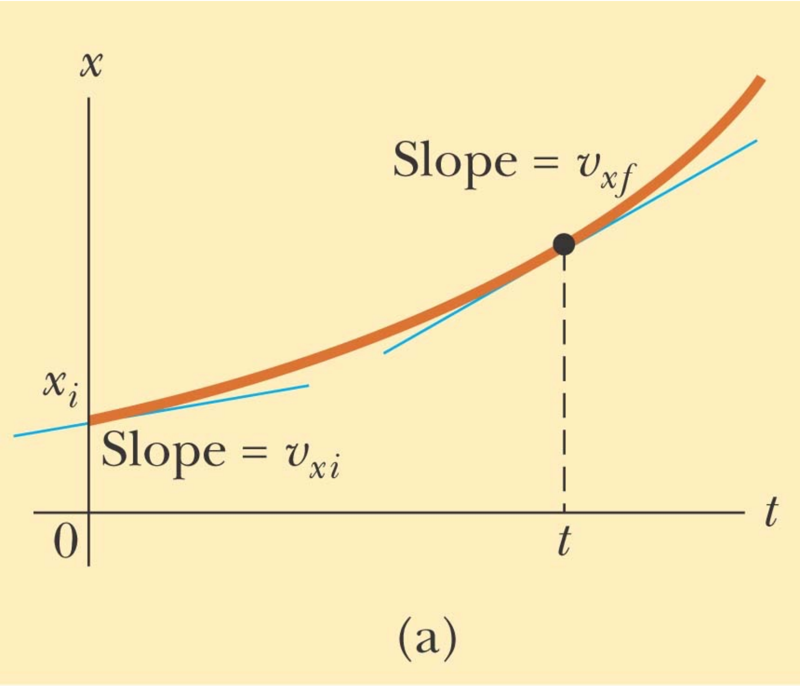
- For constant *a*,

$$v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right)$$

- Gives final velocity in terms of acceleration and displacement
- Does not give any information about the time

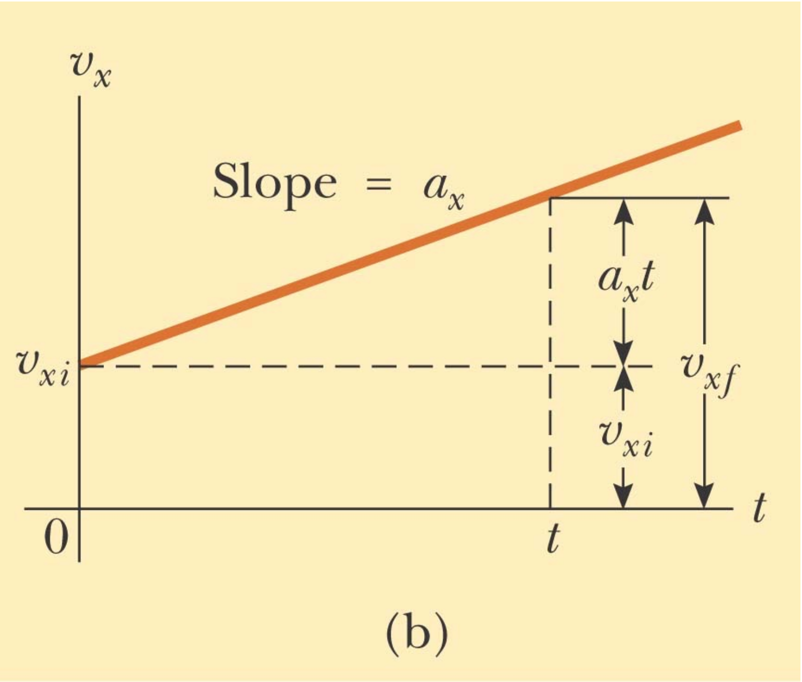
Graphical Look at Motion – displacement – time curve

- The slope of the curve is the velocity
- The curved line indicates the velocity is changing
  - Therefore, there is an acceleration



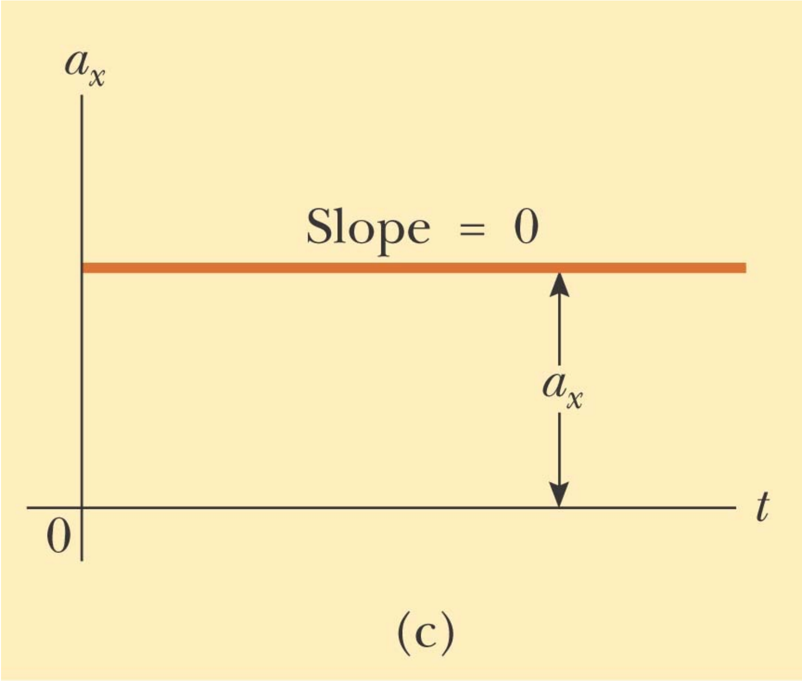
Graphical Look at Motion – velocity – time curve

- The slope gives the acceleration
- The straight line indicates a constant acceleration



Graphical Look at Motion – acceleration – time curve

- The zero slope indicates a constant acceleration



Freely Falling Objects

- A ***freely falling object*** is any object moving freely under the influence of gravity alone.
- It does not depend upon the initial motion of the object
  - Dropped – released from rest
  - Thrown downward
  - Thrown upward

Acceleration of Freely Falling Object

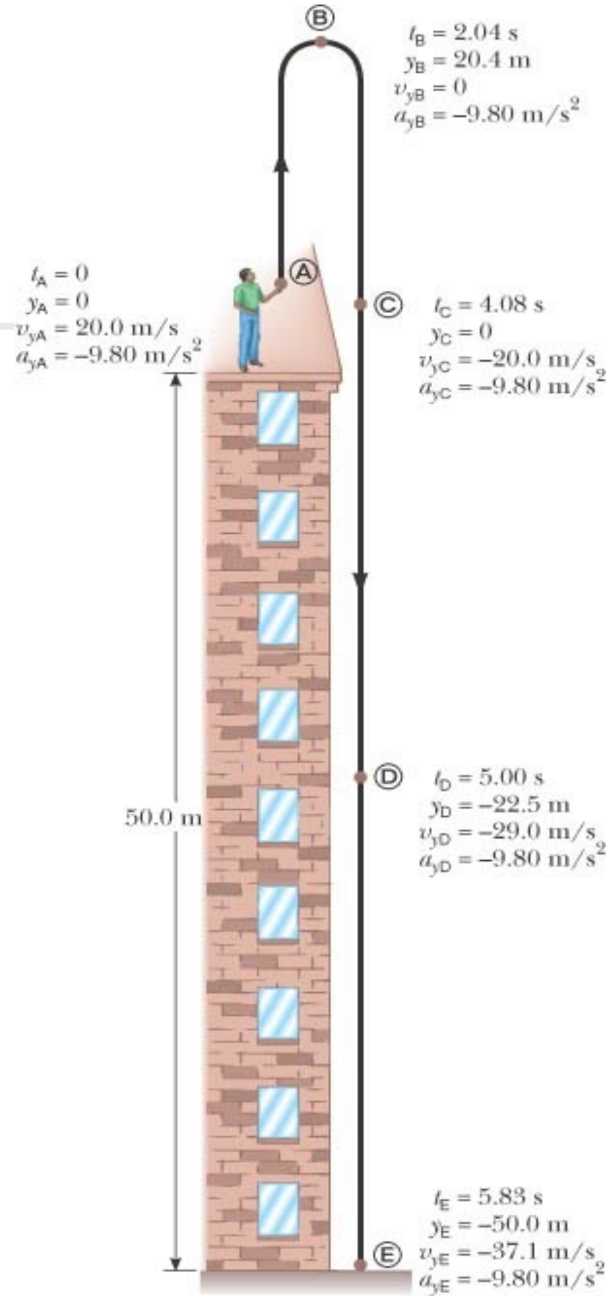
- The acceleration of an object in free fall is directed downward, regardless of the initial motion
- The magnitude of free fall acceleration is g = 9.80 m/s^2
  - g decreases with increasing altitude
  - g varies with latitude
  - 9.80 m/s^2 is the average at the Earth's surface

Acceleration of Free Fall, cont.

- We will neglect air resistance
- Free fall motion is constantly accelerated motion in one dimension
- Let upward be positive
- Use the kinematic equations with $a_y = g = -9.80 \text{ m/s}^2$

Free Fall Example

- Initial velocity at A is upward (+) and acceleration is *g* (-9.8 m/s²)
- At B, the velocity is 0 and the acceleration is *g* (-9.8 m/s²)
- At C, the velocity has the same magnitude as at A, but is in the opposite direction
- The displacement is –50.0 m (it ends up 50.0 m below its starting point)

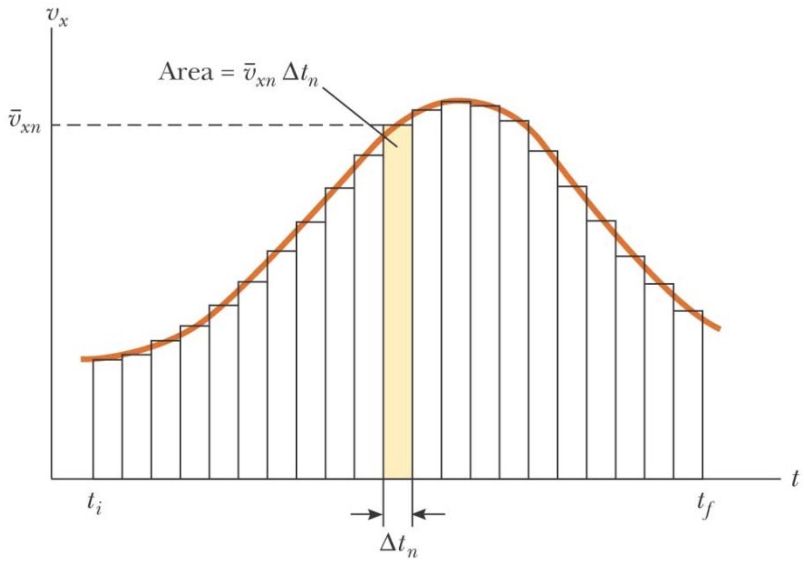


Motion Equations from Calculus

- Displacement equals the area under the velocity – time curve

$$\lim_{\Delta t_n \to 0} \sum_n v_{xn} \Delta t_n = \int_{t_i}^{t_f} v_x(t)\,dt$$

- The limit of the sum is a definite integral



Kinematic Equations – General Calculus Form

$$a_x = \frac{dv_x}{dt}$$

$$v_{xf} - v_{xi} = \int_0^t a_x dt$$

$$v_x = \frac{dx}{dt}$$

$$x_f - x_i = \int_0^t v_x dt$$

Kinematic Equations – Calculus Form with Constant Acceleration

- The integration form of $v_f - v_i$ gives

$$v_{xf} - v_{xi} = a_x t$$

- The integration form of $x_f - x_i$ gives

$$x_f - x_i = v_{xi} t + \frac{1}{2} a_x t^2$$

General Problem Solving Strategy

- Conceptualize
- Categorize
- Analyze
- Finalize

Problem Solving – Conceptualize

- Think about and understand the situation
- Make a quick drawing of the situation
- Gather the numerical information
  - Include algebraic meanings of phrases
- Focus on the expected result
  - Think about units
- Think about what a reasonable answer should be

Problem Solving – Categorize

- Simplify the problem
  - Can you ignore air resistance?
  - Model objects as particles
- Classify the type of problem
- Try to identify similar problems you have already solved

Problem Solving – Analyze

- Select the relevant equation(s) to apply
- Solve for the unknown variable
- Substitute appropriate numbers
- Calculate the results
  - Include units
- Round the result to the appropriate number of significant figures

Problem Solving – Finalize

- Check your result
  - Does it have the correct units?
  - Does it agree with your conceptualized ideas?
- Look at limiting situations to be sure the results are reasonable
- Compare the result with those of similar problems

Problem Solving – Some Final Ideas

- When solving complex problems, you may need to identify sub-problems and apply the problem-solving strategy to each sub-part
- These steps can be a guide for solving problems in this course