

- Moment of inertia about x- axis

$$I_x = \frac{b * h^3}{12} = \frac{500 * 400^3}{12} = 2.667 * 10^9 \text{ mm}^4$$

- Moment of inertia about Y- axis

$$I_y = \frac{b * h^3}{12} = \frac{400 * 500^3}{12} = 4.167 * 10^9 \text{ mm}^4$$

Example 1)

from previous example 2 in one way solid slab. Determine the bending moment acting on interior column CD in the second floor

$$L_{CA} = 9m, L_{CB} = 9m, \text{ height of columns} = 3.6m$$

$$W_u = 88.24 \text{ KN/m}, W_{DL} = 65.8 \text{ KN/m}$$

$$\text{beam dimension (400,700)}, \text{ column dimension (500,400)}$$

$$(FEM)_{CA} = +\frac{W_u * L_{CA}^2}{12} = \frac{88.24 * 9^2}{12} = 595.62 \text{ KN.m}$$

$$(FEM)_{CB} = +\frac{W_u * L_{CB}^2}{12} = \frac{65.8 * 9^2}{12} = 444.15 \text{ KN.m}$$

The unbalanced moment at the column joint C is:

$$M_C = (FEM)_{CA} + (FEM)_{CB} = 595.62 - 444.15 = 151.47 \text{ KN.m}$$

The moment in the top of column is given by:

$$M_{CD} = -M_C * \frac{\left[\frac{I}{L}\right]_{CD}}{\left[\frac{I}{L}\right]_{CG} + \left[\frac{I}{L}\right]_{CD} + \left[\frac{I}{L}\right]_{CA} + \left[\frac{I}{L}\right]_{CB}}$$

$$\left[\frac{I}{L}\right]_{CD} = \left[\frac{I}{L}\right]_{DC} = \left[\frac{I}{L}\right]_{CG} = \left[\frac{I}{L}\right]_{DH} = \left[\frac{500 * 400^3}{12}\right] = 740.74 * 10^3 \frac{\text{mm}^4}{\text{mm}}$$

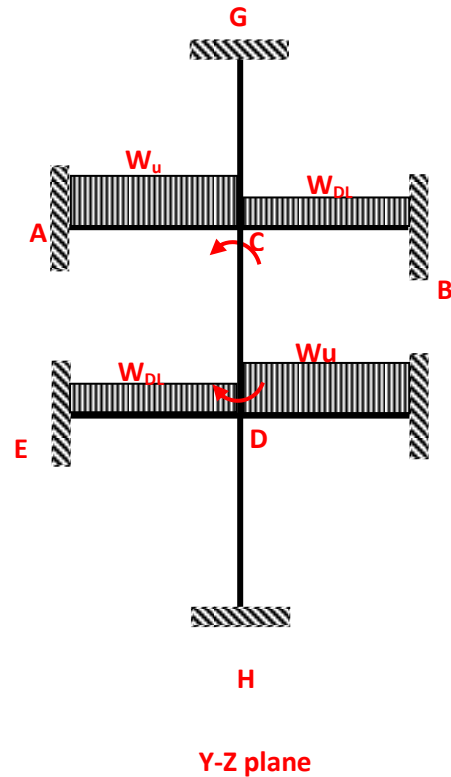
$$\left[\frac{I}{L}\right]_{CA} = \left[\frac{I}{L}\right]_{CB} = \left[\frac{I}{L}\right]_{DE} = \left[\frac{I}{L}\right]_{DF} = \left[\frac{400 * 700^3}{12}\right] = 1270.37 * 10^3 \frac{\text{mm}^4}{\text{mm}}$$

$$M_{CD} = -151.47 * \frac{740.74 * 10^3}{740.74 * 10^3 + 740.74 * 10^3 + 1270.37 * 10^3 + 1270.37 * 10^3}$$

$$M_{CD} = -151.47 * 0.184 = -27.9 \text{ KN.m}$$

$$(FEM)_{DF} = -\frac{W_u * L_{DF}^2}{12} = \frac{88.24 * 9^2}{12} = -595.62 \text{ KN.m}$$

$$(FEM)_{DE} = +\frac{W_{DL} * L_{DE}^2}{12} = \frac{65.8 * 9^2}{12} = +444.15 \text{ KN.m}$$



The unbalanced moment at the column joint D is:

$$M_D = (FEM)_{DF} + (FEM)_{DE} = -595.62 + 444.15 = -151.47 \text{ KN.m}$$

The moment in the bottom of column is given by:

$$M_{DC} = -M_C * \frac{\left[\frac{I}{L}\right]_{DC}}{\left[\frac{I}{L}\right]_{DC} + \left[\frac{I}{L}\right]_{DH} + \left[\frac{I}{L}\right]_{DE} + \left[\frac{I}{L}\right]_{DF}}$$

$$\left[\frac{I}{L}\right]_{CD} = \left[\frac{I}{L}\right]_{DC} = \left[\frac{I}{L}\right]_{CG} = \left[\frac{I}{L}\right]_{DH} = \left[\frac{500 * 400^3}{12}\right] = 740.74 * 10^3 \frac{\text{mm}^4}{\text{mm}}$$

$$\left[\frac{I}{L}\right]_{CA} = \left[\frac{I}{L}\right]_{CB} = \left[\frac{I}{L}\right]_{DE} = \left[\frac{I}{L}\right]_{DF} = \left[\frac{400 * 700^3}{12}\right] = 1270.37 * 10^3 \frac{\text{mm}^4}{\text{mm}}$$

$$M_{CD} = -(-151.47) * \frac{740.74 * 10^3}{740.74 * 10^3 + 740.74 * 10^3 + 1270.37 * 10^3 + 1270.37 * 10^3}$$

$$M_{CD} = 151.47 * 0.184 = 27.9 \text{ KN.m}$$

$$L_{CA} = 4.4\text{m} , L_{CB} = 4.4\text{m} , \text{height of columns} = 3.6 \text{ m}$$

$$W_u = 24.22 \text{ KN/m} , W_{DL} = 24.22 \text{ KN/m}$$

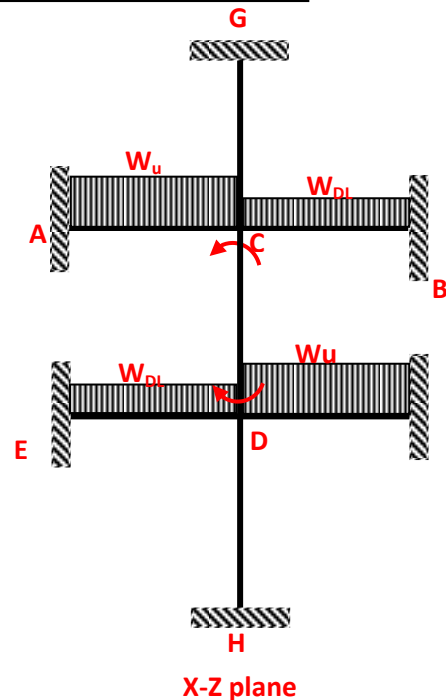
$$\text{beam dimension (400,700)} , \text{column dimension (500,400)}$$

$$(FEM)_{CA} = + \frac{W_u * L_{CA}^2}{12} = \frac{24.22 * 4.4^2}{12} = 39.075 \text{ KN.m}$$

$$(FEM)_{CB} = + \frac{W_u * L_{CB}^2}{12} = \frac{24.22 * 4.4^2}{12} = 39.075 \text{ KN.m}$$

The unbalanced moment at the column joint C is:

$$M_C = (FEM)_{CA} + (FEM)_{CB} = 39.075 - 39.075 = 0 \text{ KN.m}$$



Due to equality of loading (dead load and ultimate load) over the beams the unbalance moment at the column in joint C in X-Z plane is zero that means there is no moment acting in the column CD in the top and the bottom because is similar geometry to the top

Example 2)

from previous **example 2** in one way solid slab. Determine the bending moment acting on the corner column CD in the second floor .

$$L_{CA} = 9m , L_{CB} = 9m , \text{height of columns} = 3.6 m$$

$$W_u = 88.24 \text{ KN/m} , W_{DL} = 65.8 \text{ KN/m}$$

$$\text{beam dimension (400,700)} , \text{ column dimension (500,400)}$$

$$(FEM)_{CA} = + \frac{W_u * L_{CA}^2}{12} = \frac{88.24 * 4.4^2}{12} = 595.62 \text{ KN.m}$$

The unbalanced moment at the column joint C is:

$$M_C = (FEM)_{CA} = 595.62 \text{ KN.m}$$

The moment in the top of column is given by:

$$M_{CD} = -M_C * \frac{\left[\frac{I}{L}\right]_{CD}}{\left[\frac{I}{L}\right]_{CG} + \left[\frac{I}{L}\right]_{CD} + \left[\frac{I}{L}\right]_{CA}}$$

$$\left[\frac{I}{L}\right]_{CD} = \left[\frac{I}{L}\right]_{DC} = \left[\frac{I}{L}\right]_{CG} = \left[\frac{I}{L}\right]_{DH} = \left[\frac{\frac{500 * 400^3}{12}}{3600} \right] = 740.74 * 10^3 \frac{mm^4}{mm}$$

$$\left[\frac{I}{L}\right]_{CA} = \left[\frac{I}{L}\right]_{DE} = \left[\frac{\frac{400 * 700^3}{12}}{9000} \right] = 1270.37 * 10^3 \frac{mm^4}{mm}$$

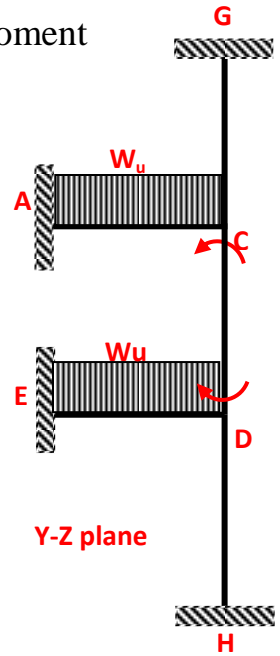
$$M_{CD} = -595.62 * \frac{740.74 * 10^3}{740.74 * 10^3 + 740.74 * 10^3 + 1270.37 * 10^3}$$

$$M_{CD} = -595.62 * 0.2692 = -160.341 \text{ KN.m}$$

$$(FEM)_{DE} = + \frac{W_{DL} * L_{DE}^2}{12} = \frac{88.24 * 4.4^2}{12} = 595.62 \text{ KN.m}$$

The unbalanced moment at the column joint D is:

$$M_D = (FEM)_{DE} = 595.62 \text{ KN.m}$$



The moment in the bottom of column is given by:

$$M_{DC} = -M_C * \frac{\left[\frac{I}{L}\right]_{DC}}{\left[\frac{I}{L}\right]_{DC} + \left[\frac{I}{L}\right]_{DH} + \left[\frac{I}{L}\right]_{DE}}$$

$$\left[\frac{I}{L}\right]_{CD} = \left[\frac{I}{L}\right]_{DC} = \left[\frac{I}{L}\right]_{CG} = \left[\frac{I}{L}\right]_{DH} = \left[\frac{500 * 400^3}{12}\right] = 740.74 * 10^3 \frac{mm^4}{mm}$$

$$\left[\frac{I}{L}\right]_{CA} = \left[\frac{I}{L}\right]_{CB} = \left[\frac{I}{L}\right]_{DE} = \left[\frac{400 * 700^3}{12}\right] = 1270.37 * 10^3 \frac{mm^4}{mm}$$

$$M_{CD} = -595.62 * \frac{740.74 * 10^3}{740.74 * 10^3 + 740.74 * 10^3 + 1270.37 * 10^3}$$

$$M_{CD} = -595.62 * 0.2692 = -160.341 \text{ KN.m}$$

$L_{CA} = 4.4\text{m}$, $L_{CB} = 4.4\text{m}$, height of columns = 3.6 m

$W_u = 24.22 \text{ KN/m}$, $W_{DL} = 24.22 \text{ KN/m}$

beam dimension (400,700) , column dimension (500,400)

$$(FEM)_{CA} = + \frac{W_u * L_{CA}^2}{12} = \frac{24.22 * 4.4^2}{12} = 39.075 \text{ KN.m}$$

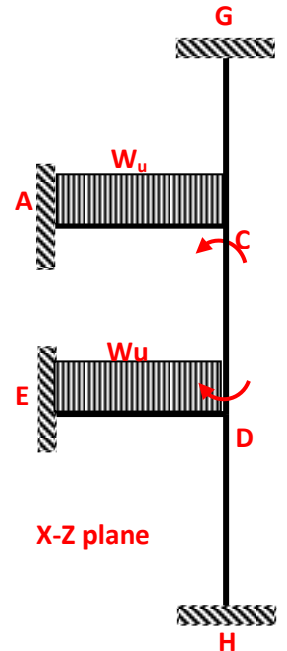
The unbalanced moment at the column joint C is:

$$M_C = (FEM)_{CA} = 39.075 = 39.075 \text{ KN.m}$$

The moment in the top of column is given by:

$$M_{CD} = -M_C * \frac{\left[\frac{I}{L}\right]_{CD}}{\left[\frac{I}{L}\right]_{CG} + \left[\frac{I}{L}\right]_{CD} + \left[\frac{I}{L}\right]_{CA}}$$

$$\left[\frac{I}{L}\right]_{CD} = \left[\frac{I}{L}\right]_{DC} = \left[\frac{I}{L}\right]_{CG} = \left[\frac{I}{L}\right]_{DH} = \left[\frac{400 * 500^3}{12}\right] = 1157.407 * 10^3 \frac{mm^4}{mm}$$



$$\left[\frac{I}{L}\right]_{CA} = \left[\frac{I}{L}\right]_{DE} = \left[\frac{\frac{400 * 700^3}{12}}{9000}\right] = 1270.37 * 10^3 \frac{mm^4}{mm}$$

$$M_{CD} = -39.075 * \frac{1157.407 * 10^3}{1157.407 * 10^3 + 1157.407 * 10^3 + 1270.37 * 10^3}$$

$$M_{CD} = -39.075 * 0.3228 = -12.613 \text{ KN.m}$$

$$(FEM)_{DE} = + \frac{W_{DL} * L_{DE}^2}{12} = \frac{88.24 * 4.4^2}{12} = 595.62 \text{ KN.m}$$

The unbalanced moment at the column joint D is:

$$M_D = (FEM)_{DE} = 595.62 \text{ KN.m}$$

The moment in the bottom of column is given by:

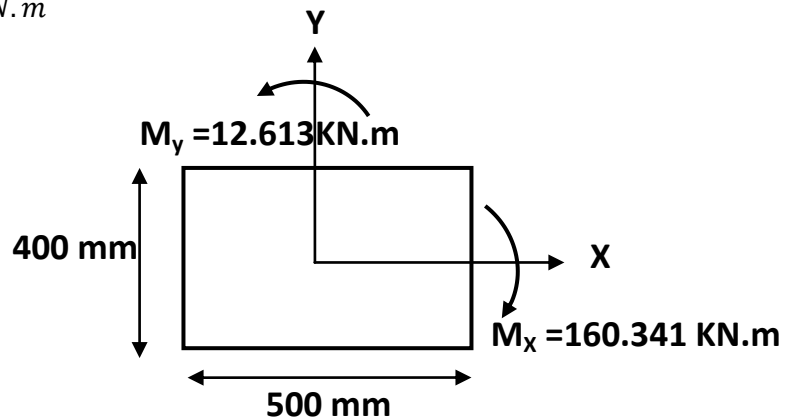
$$M_{DC} = -M_C * \frac{\left[\frac{I}{L}\right]_{DC}}{\left[\frac{I}{L}\right]_{DC} + \left[\frac{I}{L}\right]_{DH} + \left[\frac{I}{L}\right]_{DE}}$$

$$\left[\frac{I}{L}\right]_{CD} = \left[\frac{I}{L}\right]_{DC} = \left[\frac{I}{L}\right]_{CG} = \left[\frac{I}{L}\right]_{DH} = \left[\frac{\frac{400 * 500^3}{12}}{3600}\right] = 1157.407 * 10^3 \frac{mm^4}{mm}$$

$$\left[\frac{I}{L}\right]_{CA} = \left[\frac{I}{L}\right]_{CB} = \left[\frac{I}{L}\right]_{DE} = \left[\frac{\frac{400 * 700^3}{12}}{9000}\right] = 1270.37 * 10^3 \frac{mm^4}{mm}$$

$$M_{CD} = -39.075 * \frac{1157.407 * 10^3}{1157.407 * 10^3 + 1157.407 * 10^3 + 1270.37 * 10^3}$$

$$M_{CD} = -39.075 * 0.3228 = -12.613 \text{ KN.m}$$



Type of column:-**1- Ties column :**

$$S = \min[16 d_b, 48 d_s, \min(b, h)]$$

d_b = main bar diameter

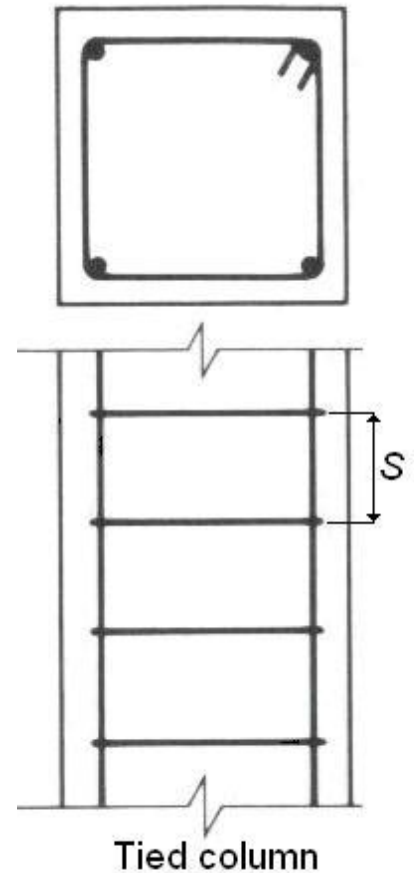
d_s = tie (stirrup) diameter

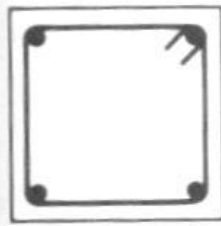
(b, h) = section dimensions

if $d_b = 32$ or less $\Rightarrow d_s \leq 10 \text{ mm}$

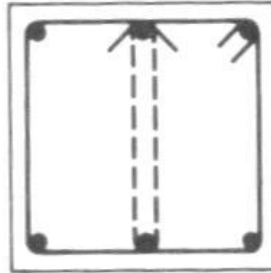
if $d_b > 32 \Rightarrow d_s \leq 12 \text{ mm}$

- Maximum distance between untied bar and tied one is 150 mm
- Minimum number of bars are 4 for rectangular or circular ties
- The maximum angle in a tie is 135°
- First tie at a distance of half spacing above slab and above footing
- Last tie at a distance of half spacing below lowest reinforcement bar of slab

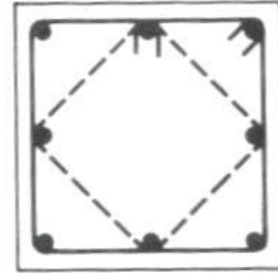




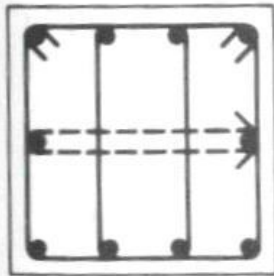
4 bars



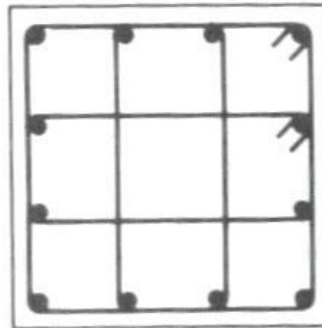
6 bars



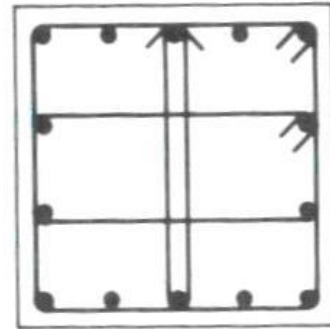
8 bars



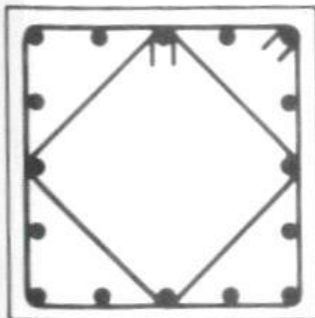
10 bars



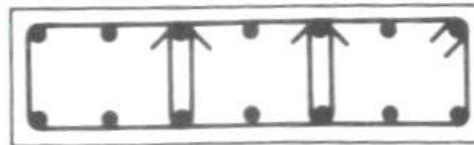
12 bars



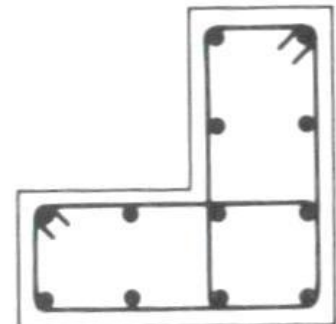
14 bars



16 bars



Wall column



Corner column

2-spiral column:

$$S = \min[16 d_b, 48 d_s, \min(b, h)]$$

d_b = main bar diameter

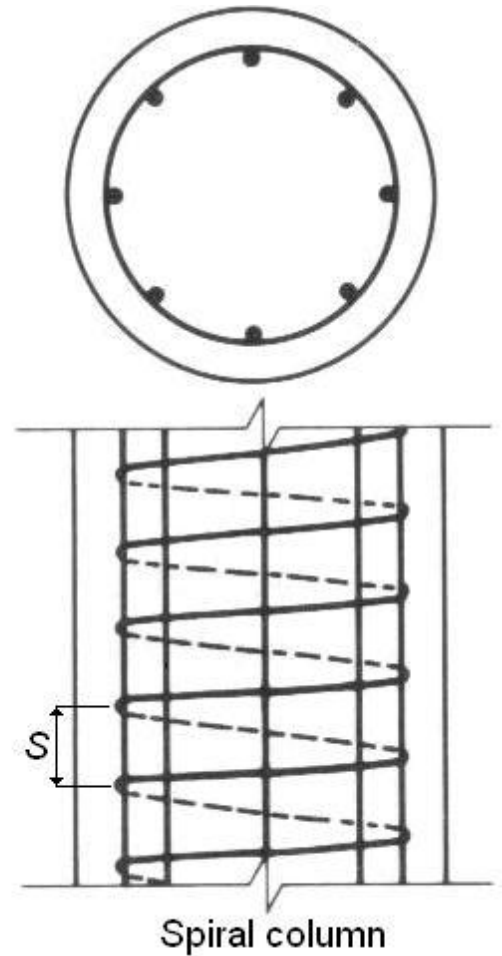
d_s = tie (stirrup) diameter

(b, h) = section dimensions

if $d_b = 32$ or less $\Rightarrow d_s \leq 10 \text{ mm}$

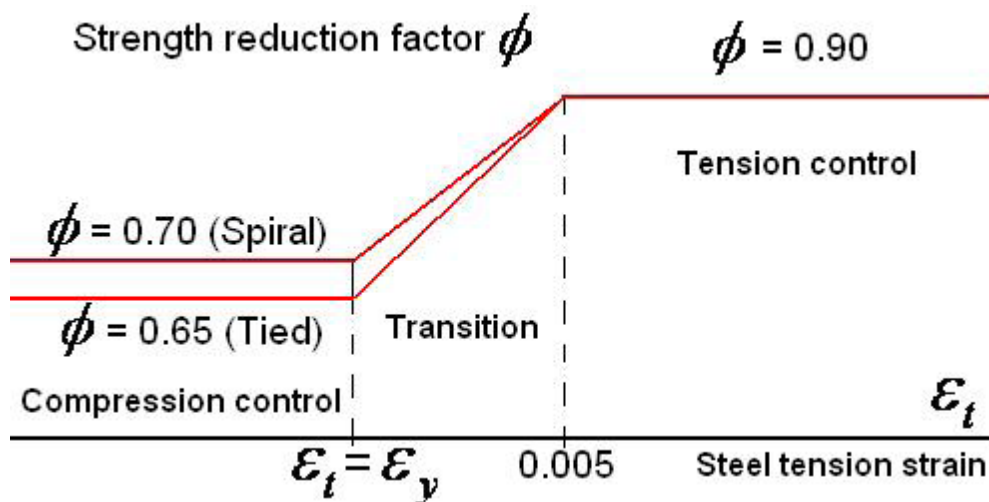
if $d_b > 32 \Rightarrow d_s \leq 12 \text{ mm}$

- Minimum number of bars are 6 for spirals



- Strength reduction factor for columns:-

$$\phi P_n \geq P_u \quad , \quad \phi M_n \geq M_u$$



- **Longitudinal reinforcement:-**

$$\rho = \frac{A_s}{A_g}$$

$$\rho = (1 - 8)\% \text{ of gross area } A_g \text{ of section}$$

the most economical tied column section generally involves ρ (1 – 2) percent

- **Strength of columns in axial compression:-**

$$\phi P_{n,max} = 0.85 * 0.65 * [0.85 f'_c (A_g - A_{st}) + f_y * A_{st}] \Rightarrow \text{for Spiral column}$$

$$\phi P_{n,max} = 0.80 * 0.65 * [0.85 f'_c (A_g - A_{st}) + f_y * A_{st}] \Rightarrow \text{for Ties column}$$

- **Column tension strength:-**

$$\phi P_{n,Tension} = -0.9 * f_y * A_g$$

- **Concrete shear strength for columns**

$$\phi V_c = 0.75 * \left[1 + \frac{P_u}{14 * A_g} \right] \frac{\sqrt{f'_c}}{6} * b_w * d * 10^{-3}$$

if $0.5\phi V_c < V_u$ shear reinforcement is required

- **Design of concrete section:-**

The minimum gross section of column is

1- Tied column:-

$$A_g \geq \frac{P_u}{0.4(f'_c + \rho * f_y)}$$

2- Spiral column:-

$$A_g \geq \frac{P_u}{0.5(f'_c + \rho * f_y)}$$

Example 3)

From previous example 1 .Design a tied interior column CD in the second floor.

$$P_u = 900.728 \text{ kn} , Lu = 3.6 \text{ m} , f_c' = 35 \text{ MPa} , f_y = 420 \text{ MPa} , \rho = 1\%$$

Estimate of own weight of column =(2 - 5)% of reaction of beams

$$P_u = (2 * 900.728) + \left(\frac{2 * 900.728}{100} \right) = 1819.471 \text{ kn}$$

$$A_g \geq \frac{P_u}{0.4(f_c' + \rho * f_y)} = \frac{1819.471 * 10^3}{0.4(30 + 0.01 * 420)} = 133002.27 \text{ mm}^2$$

- For square section $b=h \geq \sqrt{133002.2} = 364.7 \text{ mm}$

Use (b , h)=(370mm , 370mm)

- For rectangular section $b= 400 \text{ mm}$ then $h \geq \frac{133002.27}{400} = 332.5 \text{ mm}$

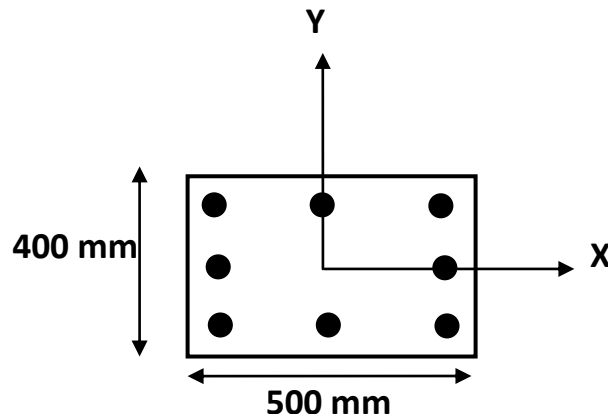
Column size (400,500) is adequate because actual area is greater than required area .

Select reinforcement:-

$$\rho = \frac{A_s}{A_g}$$

$$A_s = \rho * A_g = 0.01 * (400 * 500) = 2000 \text{ mm}^2$$

$$\text{use } d_b = 20 \text{ mm} \Rightarrow n = \frac{2000}{314} = 6.37 \text{ use even number} = 8$$



Check maximum compression capacity:

$$\phi P_{n,max} = 0.80 * 0.65 * [0.85 * 30 * (200000 - 2512) + 420 * 2512] * 10^{-3} = P_u = 1$$

$$\phi P_{n,max} = 3167.272 \text{ KN} > P_u = 1819.471 \text{ kn}$$