Computing Canonical Cover

Dr. Bassam Hammo

Canonical Cover

- A *canonical cover* for F is a set of dependencies F_c such that
  - F logically implies all dependencies in F_c, and
  - F_c logically implies all dependencies in F, and
  - No functional dependency in F_c contains an extraneous attribute, and
  - Each left side of functional dependency in F_c is unique
- Intuitively, a canonical cover of F is a "minimal" set of functional dependencies equivalent to F, having no redundant dependencies or redundant parts of dependencies

Extraneous Attributes

- Consider *F*, and a functional dependency, $A \rightarrow B$.
- "Extraneous": Are there any attributes in *A or B* that can be safely removed ?
  - Without changing the constraints implied by *F*

Testing if an Attribute is Extraneous

- Consider a set F of functional dependencies and the functional dependency $\alpha \rightarrow \beta$ in F.
- To test if attribute $A \in \alpha$ is extraneous in α
  1. compute $(\{\alpha\} - A)^+$ using the dependencies in F
  2. check that $(\{\alpha\} - A)^+$ contains A; if it does, A is extraneous
- To test if attribute $A \in \beta$ is extraneous in β
  1. compute α^+ using only the dependencies in
     $$F' = (F - \{\alpha \rightarrow \beta\}) \cup \{\alpha \rightarrow (\beta - A)\},$$
  2. check that α^+ contains A; if it does, A is extraneous

Computing Canonical Cover

$R = \{A,B,C,D,E,F,G,H\}$

$F = \{AC \rightarrow G,\ D \rightarrow EG,\ BC \rightarrow D,\ CG \rightarrow BD,\ ACD \rightarrow B,\ CE \rightarrow AG\}$

Find the canonical cover of F.

1. **Simplify all RHS (Decomposition)**
2. **For all FDs on LHS find a redundant (extraneous) attribute**
3. **Eliminate all redundant FDs**
4. **Apply Union if needed**
5. **The result is Fc**

Computing Canonical Cover

R = {A,B,C,D,E,F,G,H}

F = {AC→G, D→EG, BC→D, CG→BD, ACD→B, CE→AG}

Find the canonical cover of F:

AC → G

D → E

D → G

BC → D

CG → B

CG → D

ACD → B

CE → A

CE → G

Computing Canonical Cover

R = {A,B,C,D,E,F,G,H}

F = {AC→G, D→EG, BC→D, CG→BD, ACD→B, CE→AG}

Find the canonical cover of F:

AC → G

D → E √

D → G √

BC → D

CG → B

CG → D

ACD → B

CE → A

CE → G

Find the extraneous attribute in this FD:

D?

(AC)+ → ACG**B**, so we got B; D is extraneous and can be safely eliminated.

Rewrite the new FD as AC → B

Computing Canonical Cover

R = {A,B,C,D,E,F,G,H}

F = {AC→G, D→EG, BC→D, CG→BD, ACD→B, CE→AG}

Find the canonical cover of F:

AC → G

D → E √

D → G √

BC → D

CG → B

CG → D

AC → B

CE → A

CE → G

Find the extraneous attribute in this FD:

A? C?
A+ → A, so can't get G; C is not extraneous
C+ → C, so can't get G; A is not extraneous

Keep this FD as is

Computing Canonical Cover

R = {A,B,C,D,E,F,G,H}

F = {AC→G, D→EG, BC→D, CG→BD, ACD→B, CE→AG}

Find the canonical cover of F:

AC → G

D → E √

D → G √

BC → D

CG → B

CG → D

AC → B

CE → A

CE → G

Find the extraneous attribute in this FD:

B? C?
B+ → B, so can't get D; C is not extraneous
C+ → C, so can't get D; B is not extraneous

Keep this FD as is

Computing Canonical Cover

R = {A,B,C,D,E,F,G,H}

F = {AC→G, D→EG, BC→D, CG→BD, ACD→B, CE→AG}

Find the canonical cover of F:

AC → G

D → E √

D → G √

BC → D

CG → B

CG → D

AC → B

CE → A

CE → G

Find the extraneous attribute in this FD:

G? C?
C+ → C, so can't get B; G is not extraneous
G+ → G, so can't get B; C is not extraneous

Keep this FD as is

Computing Canonical Cover

R = {A,B,C,D,E,F,G,H}

F = {AC→G, D→EG, BC→D, CG→BD, ACD→B, CE→AG}

Find the canonical cover of F:

AC → G

D → E √

D → G √

BC → D

CG → B

CG → D

AC → B

CE → A

CE → G

Find the extraneous attribute in this FD:

G? C?
C+ → C, so can't get D; G is not extraneous
G+ → G, so can't get D; C is not extraneous

Keep this FD as is

Computing Canonical Cover

R = {A,B,C,D,E,F,G,H}

F = {AC→G, D→EG, BC→D, CG→BD, ACD→B, CE→AG}

Find the canonical cover of F:

AC → G

D → E √

D → G √

BC → D

CG → B

CG → D

AC → B

CE → A

CE → G

If we continue we will not find any extraneous attribute on LHS of any FD. So we are done with step #2

Computing Canonical Cover

R = {A,B,C,D,E,F,G,H}

F = {AC→G, D→EG, BC→D, CG→BD, ACD→B, CE→AG}

Find the canonical cover of F:

AC → G

D → E √

D → G √

BC → D

CG → B

CG → D

AC → B

CE → A

CE → G

Find the redundant FDs:

(AC)+ → ACBDE**G ; so we got G from other FDs**

Remove the entire FD from the list.

Computing Canonical Cover

R = {A,B,C,D,E,F,G,H}

F = {AC→G, D→EG, BC→D, CG→BD, ACD→B, CE→AG}

Find the canonical cover of F:

- ~~AC → G~~

D → E √

D → G √

BC → D

CG → B

CG → D

AC → B

CE → A

CE → G

Find the redundant FDs:

(CG)+ → CGDEA**B ; so we got B from other FDs**

Remove the entire FD from the list.

Computing Canonical Cover

R = {A,B,C,D,E,F,G,H}

F = {AC→G, D→EG, BC→D, CG→BD, ACD→B, CE→AG}

Find the canonical cover of F:

- ~~AC → G~~
- D → E √
- D → G √
- BC → D
- ~~CG → B~~
- CG → D
- AC → B
- CE → A
- CE → G

Find the redundant FDs:

(CE)+ → CEGD **; so we could not get A from other FDs**

Keep this FD in the list.

Computing Canonical Cover

R = {A,B,C,D,E,F,G,H}

F = {AC→G, D→EG, BC→D, CG→BD, ACD→B, CE→AG}

Find the canonical cover of F:

- ~~AC → G~~
- D → E √
- D → G √
- BC → D
- ~~CG → B~~
- CG → D
- AC → B
- CE → A
- CE → G

Find the redundant FDs:

(CE)+ → CEABD**G ; so we got G from other FDs**

Remove this FD from the list.

Computing Canonical Cover

R = {A,B,C,D,E,F,G,H}

F = {AC→G, D→EG, BC→D, CG→BD, ACD→B, CE→AG}

Find the canonical cover of F:

- ~~**AC → G**~~
- D → E √
- D → G √
- BC → D
- ~~CG → B~~
- CG → D
- AC → B
- CE → A
- ~~CE → G~~

Find the redundant FDs:

(CE)+ → CEABD**G ; so we got G from other FDs**

Remove this FD from the list.

End of step# 3

Computing Canonical Cover

R = {A,B,C,D,E,F,G,H}

F = {AC→G, D→EG, BC→D, CG→BD, ACD→B, CE→AG}

Find the canonical cover of F:

D → E

D → G

BC → D

CG → B

AC → B

CE → A

Apply union (if any) on the remaining Fds

D → EG

The result is the canonical cover (Fc) of F

End of step# 4

Computing Canonical Cover

$R = \{A,B,C,D,E,F,G,H\}$

$F = \{AC \rightarrow G, D \rightarrow EG, BC \rightarrow D, CG \rightarrow BD, ACD \rightarrow B, CE \rightarrow AG\}$

Find the canonical cover of F:

$F_C = \{AC \rightarrow B, D \rightarrow EG, BC \rightarrow D, CG \rightarrow B, CE \rightarrow A\}$

$F_C = \{\mathbf{AC \rightarrow B, D \rightarrow EG, BC \rightarrow D, CG \rightarrow D, CE \rightarrow A}\}$

* Different order of considering the extraneous attributes can result in different F_C

Example2: Computing a Canonical Cover

- $R = (A, B, C)$
  $$F = \{A \rightarrow BC \\ B \rightarrow C \\ A \rightarrow B \\ AB \rightarrow C\}$$

- The canonical cover is:

Example3: Computing a Canonical Cover

- Given $F = \{A \rightarrow C, AB \rightarrow C\}$
  - B is extraneous in $AB \rightarrow C$ because $\{A \rightarrow C, AB \rightarrow C\}$ is equivalent to $\{A \rightarrow C, A \rightarrow C\} = \{A \rightarrow C\}$
- Given $F = \{A \rightarrow C, AB \rightarrow CD\}$
  - C is extraneous in $AB \rightarrow CD$ because $\{A \rightarrow C, AB \rightarrow CD\}$ is equivalent to $\{A \rightarrow C, AB \rightarrow D\}$