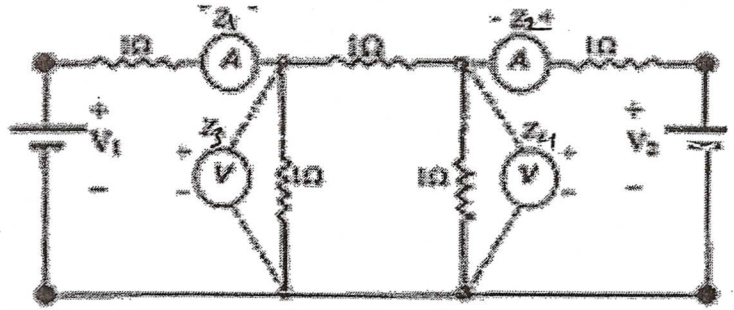


Consider the shown simple dc circuit:

The two voltage sources V_1 and V_2 are to be estimated based on the four meter readings $z_1 = 9.1$ A, $z_2 = 3$ A, $z_3 = 6.5$ V, and $z_4 = 5$ V. Meter variances are $\sigma_1^2 = \sigma_2^2 = 0.01$ and $\sigma_3^2 = \sigma_4^2 = 0.02$, respectively.



Calculate:

1. The coefficient matrix H of the system.
2. The estimated values of the voltage sources V_1 and V_2 .
3. The estimated values of the four measurements and estimated errors in the measurements.

$$\alpha = 0.01$$

4. check for Bad Data presence

Solution

$h = \begin{bmatrix} 0.625 & -0.125 \\ -0.125 & 0.625 \\ 0.375 & 0.125 \\ 0.125 & 0.375 \end{bmatrix}$	$w = \begin{bmatrix} 100 & 0 & 0 & 0 \\ 0 & 100 & 0 & 0 \\ 0 & 0 & 50 & 0 \\ 0 & 0 & 0 & 50 \end{bmatrix}$	$z = \begin{bmatrix} 9.1 \\ 3 \\ 6.5 \\ 5 \end{bmatrix}$	$\hat{x} = (h^T w h)^{-1} h^T w z$ $\begin{array}{l} V_1 = x_1 = 15.91053 \\ V_2 = x_2 = 7.8895 \end{array}$
$\hat{z} = h \hat{x} = \begin{bmatrix} 8.9579 \\ 2.9421 \\ 6.9526 \\ 4.9474 \end{bmatrix}$	$e = z - \hat{z} = \begin{bmatrix} 0.1421 \\ 0.0579 \\ -0.4526 \\ 0.0526 \end{bmatrix}$	$f = \sum e_i^2 / \sigma_i^2 = \sum w_i e_i^2 = 12.74$ <p>for $\chi^2_{k, \alpha}$ $k = \# \text{ of } Z - \# \text{ of } X = 4 - 2 = 2$ From Table :- $\chi^2_{2, 0.01} = 9.21$ $f > \chi^2$</p>	<div style="border: 1px solid black; padding: 5px; text-align: center;"> At least one bad measurement </div>

$i = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$R^1 = (i - h (h^T w h)^{-1} h^T w) w^{-1} =$	$\begin{bmatrix} 0.00193 & & & \\ & 0.00193 & & \\ & & 0.01614 & \\ & & & 0.01614 \end{bmatrix}$
$(\text{Standardized Error})_i = \frac{e_i}{\sqrt{R^1_{ii}}} =$	$\begin{bmatrix} 3.246 \\ 1.318 \\ -3.563 \\ 0.414 \end{bmatrix}$	<div style="border: 1px solid black; padding: 5px; text-align: center;"> z_3 is Rejected </div>

$h = \begin{bmatrix} 0.625 & -0.125 \\ -0.125 & 0.625 \\ 0.125 & 0.375 \end{bmatrix}$	$w = \begin{bmatrix} 100 & 0 & 0 \\ 0 & 100 & 0 \\ 0 & 0 & 50 \end{bmatrix}$	$z = \begin{bmatrix} 9.1 \\ 3 \\ 5 \end{bmatrix}$	$\hat{x} = (h^T w h)^{-1} h^T w z$ $\begin{array}{l} V_1 = x_1 = 16.1565 \\ V_2 = x_2 = 8.0174 \end{array}$
$\hat{z} = h \hat{x} = \begin{bmatrix} 9.0957 \\ 2.9913 \\ 5.0261 \end{bmatrix}$	$e = z - \hat{z} = \begin{bmatrix} 0.004348 \\ 0.008696 \\ -0.026087 \end{bmatrix}$	$f = \sum e_i^2 / \sigma_i^2 = \sum w_i e_i^2 = 0.04348$ <p>$\chi^2_{1, 0.01} = 6.64$ $f < \chi^2$</p>	<div style="border: 1px solid black; padding: 5px; text-align: center;"> No More Bad Data </div>