$$\boxed{e^{z} = -2} = 2\,e^{i\pi} \times \underbrace{e^{2n\pi i}}_{=1}$$

$$= e^{\ln 2} \times e^{i\pi} \times e^{2n\pi i}$$

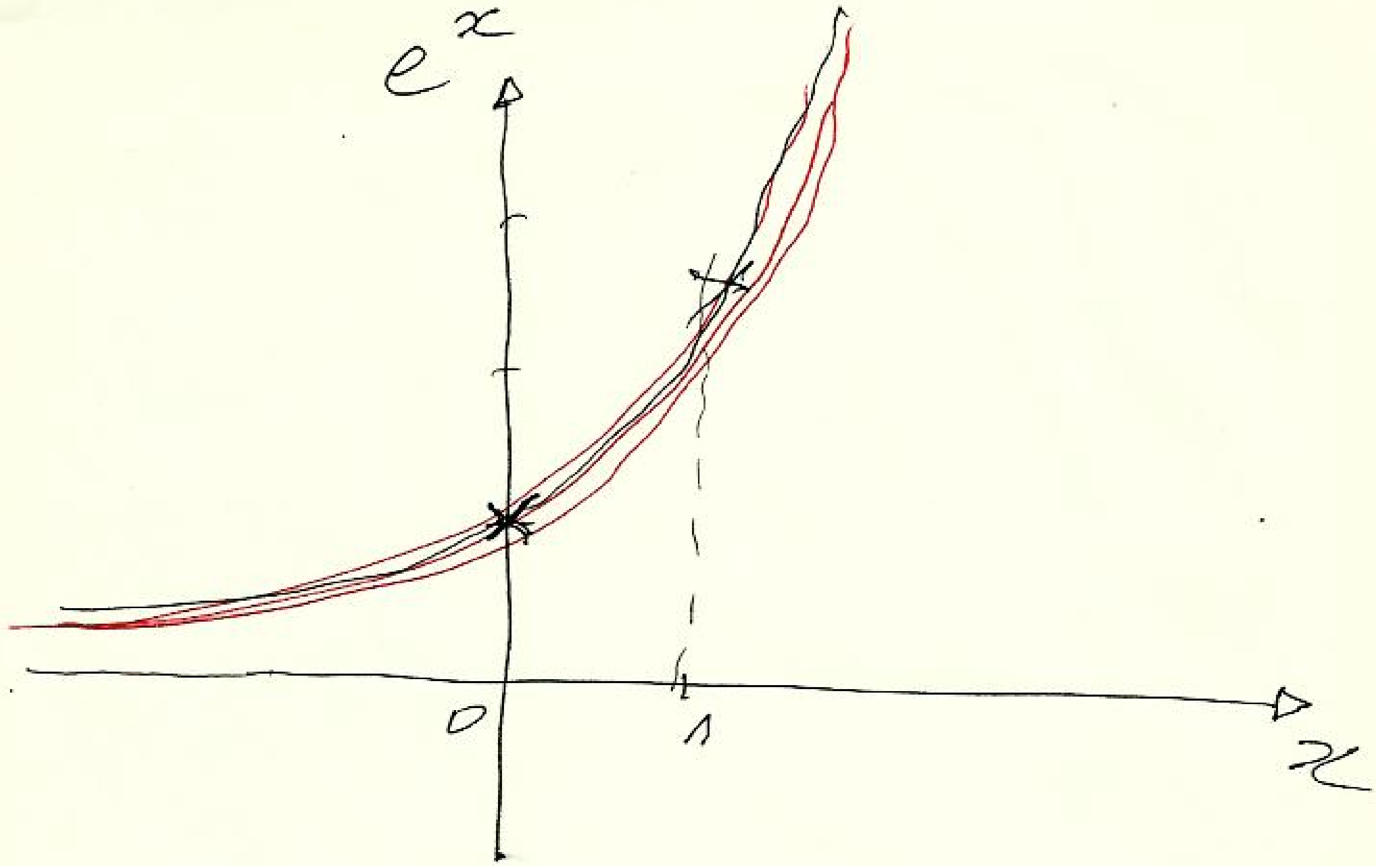
$$\boxed{e^{z} = e^{\ln 2 + i\pi(1+2n)}}$$

Sol: $z = \ln 2 + (2n+1)\pi i \; ; \; n \in \mathbb{Z}$

$$\mathbb{Z} = \{ \ldots, -2, -1, 0, 1, 2, \ldots \}$$

relative numbers

$$z = \partial_z \log$$



$$e^0 = 1 ; \quad e^1 = e = 2.7$$

$$e^{z} = e^{z+2n\pi i}$$

$$\left(e^{2\pi i}\right) = \underbrace{\cos 2\pi}_{1} + i \underbrace{\sin 2\pi}_{0} = 1$$

$$= e^{2n\pi i}$$

$$\text{Log}\, ab = \text{Log}\, a + \text{Log}\, b$$

$$\ln a^2 = 2 \ln a$$

$$\ln(a^n) = n \ln a$$

$$\ln\left(e^z\right) = z \underbrace{\ln e}_{=1} = z$$

$$\ln e^{i\theta} = i\theta$$

$$\text{Log}\, z = ? = \text{Log}(r \times e^{i\theta})$$

$$\text{Log}(a \times b) = \text{Log}\, a \pm \text{Log}\, b$$

$$\text{Log}\, z = \text{Log}\, r e^{i\theta} = \text{Log}\, r + \text{Log}\, e^{i\theta}$$

$$= (\text{Log}\, r) + i\theta$$

$$\boxed{\text{Log}\, z = \ln|z| + i\,\text{Arg}(z)}$$

Ex: $\text{Log}(-1) = ?$

$$\text{Log}\, e^{\pi i} = \pi i$$

$$\boxed{\text{Log}(-1) = i\pi}$$

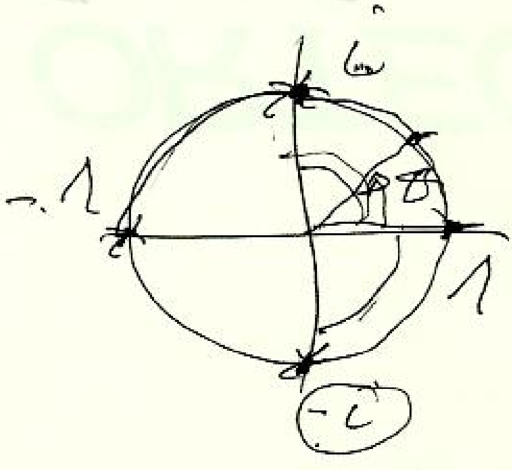
$$\mathrm{Log}(-e\,i) = \mathrm{Log}(e \times (-i))$$

$$= \mathrm{Log}(e^1 \times e^{-i\frac{\pi}{2}})$$

$$= \mathrm{Log}(e^{1 - i\frac{\pi}{2}})$$

$$= 1 - i\frac{\pi}{2}$$

$$\cancel{-i = e^{-i\frac{\pi}{2}}}$$



$$e^0 = 1$$

$$e^{i\frac{\pi}{2}} = i \longrightarrow \frac{1}{e^{i\frac{\pi}{2}}} = \frac{1}{i}$$

$$e^{i\pi} = -1$$

$$e^{-i\frac{\pi}{2}} = -i \Leftarrow e^{-i\frac{\pi}{2}} = -i$$

Sound $\geqq$ please

$$\cos z = \frac{e^{iz}+e^{-iz}}{2}, \quad \sin z = \frac{e^{-}-e^{-}}{2i}$$

Hyperbolic funct°

$$\cosh z = \frac{e^{z}+e^{-z}}{2}, \quad \sinh z = \frac{e^{z}-e^{-z}}{2}$$

$$(\sinh z)' = \frac{e^{z}+e^{-z}}{2} = \cosh(z)$$

$$(\cosh z)' = \frac{e^{z}-e^{-z}}{2} = \sinh(z)$$

$$e^{iz} = \cos z + i\sin z$$

$$e^{-iz} = \cos z - i\sin z$$

$$\cos z = \frac{e^{iz}+e^{-iz}}{2} \; ; \; \sin z = \frac{e^{iz}-e^{-iz}}{2i}$$

Ex: Solve $\cos z = 2$ is possible in $\mathbb{C}$

$$e^{iz} + e^{-iz} = 4$$

$e^{iz} = X$; ~~So~~ $e^{-iz} = \frac{1}{e^{+iz}} = \frac{1}{X}$

$$X + \frac{1}{X} = 4$$

$$e^{iz} = X$$

$$\boxed{X^2 - 4X + 1 = 0}$$

$$(X-2)^2 - 3 = 0$$

$$(x-2)^2 = (\sqrt{3})^2$$

$$x = 2 \pm \sqrt{3}$$

$$\boxed{\cos z = 2}$$

$$e^{iz} = x \Longrightarrow iz = \mathrm{Log}\, x$$

$$x^2 - 4x + 1 = 0 \qquad z = -i\,\mathrm{Log}(2 \pm \sqrt{3})$$

$$x = 2 \pm \sqrt{3}$$

$$z_1 = -i \ln(2+\sqrt{3})$$

$$z_2 = -i \ln(\underbrace{2-\sqrt{3}}_{>0})$$

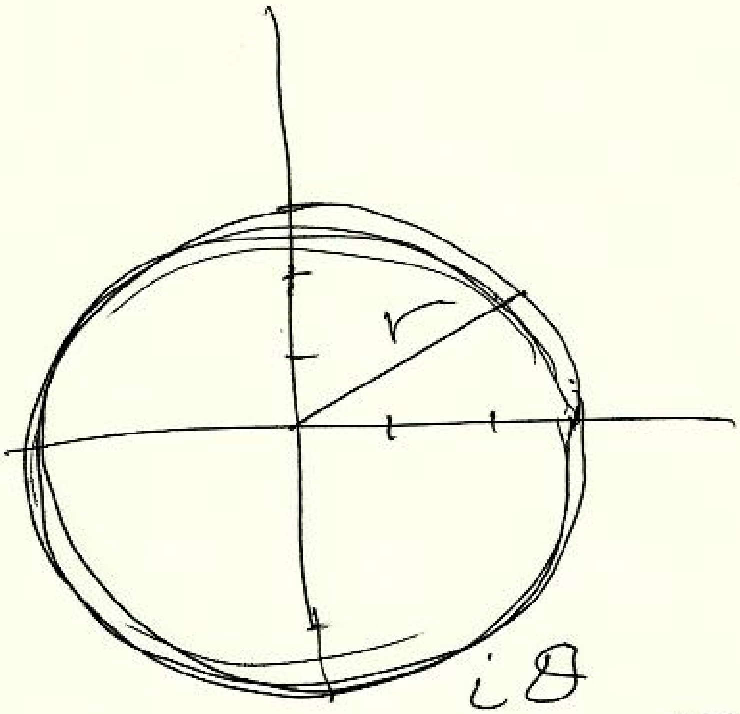
. . . . .

$$\cos z = -2$$

$$\cos z = 3$$

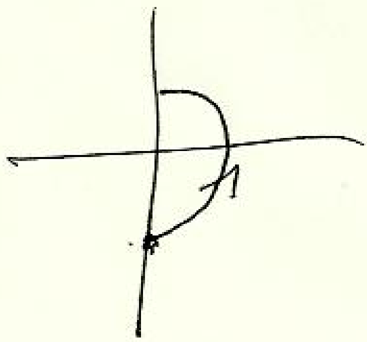
$$\sin z = 2$$

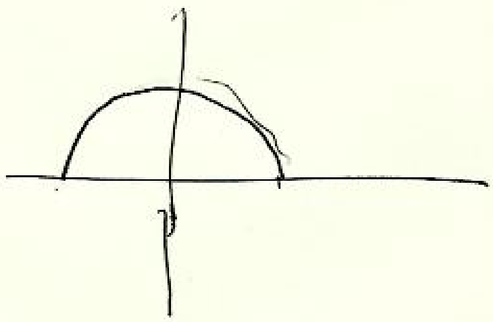
. . . . .



$$z = 3e^{i\theta} = r e^{i\theta}$$

$$0 \leq \theta \leq 2\pi$$





$$-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2} \quad \Big\{ \quad 0 \leq \theta \leq \pi$$

$(\sin z)' = \cos z$

$(\cos z)' = -\sin z$

$(\sinh z)' = \cosh(z)$

$(\cosh(z))' = \sinh(z)$

$\boxed{\cancel{\sin^2 z + \cos^2 z = 1}}$

$\cosh^2(z) - \sinh^2(z) = 1$

prove:

$$\left(\frac{e^z + e^{-z}}{2}\right)^2 - \left(\frac{e^z - e^{-z}}{2}\right)^2$$

$$= \frac{1}{4}\left[(e^z + e^{-z})^2 - (e^z - e^{-z})^2\right]$$

$$= \frac{1}{4}\left[\cancel{e^{2z}} + \cancel{e^{-2z}} + 2 - \cancel{e^{2z}} + \cancel{e^{-2z}} + 2\right] = 1$$

$$\boxed{\begin{aligned} \cos^2 z + \sin^2 z &= 1 \\ \cosh^2 z - \sinh^2 z &= 1 \end{aligned}}$$

$$\lim_{z \to \infty} \frac{i - 2z}{iz + 1} = \lim_{z \to \infty} \frac{-2z}{iz} = -\frac{2}{i}$$

$$= -\frac{2}{i} \times \underbrace{\frac{i}{i}}_{1} = -\frac{2i}{-1} = 2i$$

$$\frac{1}{i} = -i$$

So

$$-\frac{2}{i} = -2 \times \left(\frac{1}{i}\right) = -2(-i) = 2i$$