Solution

EE 211: Computational Techniques in Electrical Engineering

Midterm 1 (1st Semester, 1433/34, 2012/2013) *Name:* ______________________

Wednesday, October 10, 2012

Total Marks=40 Student-ID: ______________________

Total Time= 90 Minutes

Problem#1 [10 Marks]

(a) [6] Determine the linear and quadratic Taylor's polynomial for the function $f(x) = (1 + x)^{\frac{1}{3}}$ at a=1

(b) [4] Using the degree 1 and degree 2 Taylor's polynomial from **part(a)** to calculate $(5)^{\frac{1}{3}}$.

Problem # 2 [10 Marks]

Given $f(x) = \frac{1}{1-x}$, $x \neq 1$

(a) [5] Obtain an n^{th} degree Taylor polynomial for f(x) about $a = 0$

(b) [5] For the above function write the formula for the error term

Problem # 3 [10 Marks]

The half power beamwidth of antenna can be evaluated by finding the root of the function $f(x) = \cos(x)\cos(3x) - 0.5$. Use **Bisection Method** to find the root of f(x). Use an error tolerance of ε=0.01, and the initial interval [.2 0.4]. Note the argument of the trigonometric functions is in radians.

Problem # 4 [10 Marks]

Use the **Newton's method** to find the root for $f(x) = x - \exp(-x)$. Use an error tolerance of $\varepsilon = 1\times 10^{-6}$, and initial guess $x_0=1.0$.

P # 1 (a) $f(x) = (1+x)^{\frac{1}{3}}$ at $x = a = 1$

$f(x) = (1+x)^{1/3}$ $\quad f(1) = (2)^{\frac{1}{3}} = 1.26$

$f'(x) = \frac{1}{3}(1+x)^{-2/3}$ $\quad f'(1) = \frac{1}{3}(2)^{-\frac{2}{3}} = 0.21$

$f''(x) = -\frac{2}{9}(1+x)^{-\frac{5}{3}}$ $\quad f''(1) = -\frac{2}{9}(2)^{-\frac{5}{3}} = -0.07$

Linear, Degree $n = 1$

$P_1(x) = f(a) + (x-a)f'(a)$ $\quad a = 1$

$P_1(x) = 1.26 + (x-1)\,0.21$

Quadratic, Degree $n = 2$

$P_2(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2!}f''(a)$

$P_2(x) = 1.26 + 0.21(x-1) - 0.035(x-1)^2$

(b) $(5)^{\frac{1}{3}} = (1+x)^{\frac{1}{3}} = (1+4)^{\frac{1}{3}}$

use $x = 4$

$P_1(4) = 1.26 + (4-1)\,0.21 = 1.89$

$P_2(4) = 1.26 + 0.21(4-1) - 0.035(4-1)^2$

$P_2(4) = 1.575$

P # 2 $\quad f(x) = \dfrac{1}{1-x}$

(a) $\quad f(x) = \dfrac{1}{1-x} = (1-x)^{-1} \qquad a = 0 \qquad f(0) = 1$

$f'(x) = (-1)(1-x)^{-2}(-1) = (1-x)^{-2} \qquad f'(0) = 1$

$f''(x) = (-2)(1-x)^{-3}(-1) = 2(1-x)^{-3} \qquad f''(0) = 2 = 2!$

$f'''(x) = 2(-3)(1-x)^{-4}(-1) = 6(1-x)^{-4} \qquad f'''(0) = 6 = 3!$

$f^{4}(x) = (6)(-4)(1-x)^{-5}(-1) = 24(1-x)^{-5} \qquad f^{4}(0) = 24 = 4!$

$\vdots$

General $\quad f^{n}(x) = n!\,(1-x)^{-(n+1)}$

$f^{n}(0) = n!$

$$P_n(x) = 1 + x + x^2 + \cdots + x^n$$

$$= \sum_{j=0}^{n} \frac{(x-a)^j}{j!} f^{(j)}(a)$$

$$= \sum_{j=0}^{n} \frac{x^j}{j!} f^{j}(0)$$

$$P_n(x) = 1 + \frac{x}{1!} + \frac{x^2}{2!} \times 2! + \cdots + \frac{x^n}{n!} \times n!$$

$$P_n(x) = 1 + x + x^2 + \cdots + x^n$$

P # 2 b

$$R_n(x) = \frac{(x-a)^{n+1}}{(n+1)!} f^{n+1}(c) \qquad a \leq c \leq x$$

$$R_n(x) = \frac{x^{n+1}}{(n+1)!} f^{n+1}(c) \qquad 0 \leq c \leq x$$

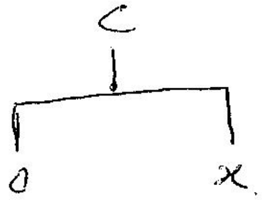
$$f^{n}(x) = n!\,(1-x)^{-(n+1)}$$

$$f^{n+1}(x) = (n+1)!\,(1-x)^{-(n+2)}$$

$$f^{n+1}(c) = (n+1)!\,(1-c)^{-(n+2)}$$

$$R_n(x) = x^{n+1}(1-c)^{-(n+2)}$$

$$R_n(x) = \frac{x^{n+1}}{(1-c)^{n+2}}$$



P#3 $f(x) = \cos(x)\cos(3x) - 0.5$, $\mathcal{E} = 0.01$, $[0.2 \quad 0.4]$

n	a	b	c	f(c)	b-c
1	0.2	0.4	0.3	0.09385	0.1
2	0.3	0.4	0.35	−0.0326	0.05
3	0.3	0.35	0.325	+0.03179	0.025
4	0.325	0.35	0.3375	-1.43×10^{-4}	0.0125
5	0.325	0.3375	$\underbrace{0.33125}_{\text{Final Root}}$		6.25×10^{-3}
	$n \geq \dfrac{\ln\left[\frac{0.2}{0.01}\right]}{\ln[2]} = 4.32$				
	$\boxed{n \geq 5}$				

P#4 Newton's Method : $f(x) = x - e^{-x}$
$x_0 = 1.0$, $\mathcal{E} = 1\times10^{-6}$

n	x_n	$f(x_n)$	$x_n - x_{n-1}$
0	1.0	0.63212	—
1	0.5378828	−0.04610049	−0.4621172
2	0.566987	-2.4495×10^{-4}	0.0291042
3	0.5671433	-6.93×10^{-9}	1.563×10^{-4}
4	$\underbrace{0.56714329}_{\text{Root}}$	-1.11×10^{-16}	-1×10^{-8}

$$f(x) = x - e^{-x}$$

$$f'(x) = 1 + e^{-x}$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \qquad n = 0, 1, 2, \ldots$$

$$x_{n+1} = x_n - \frac{(x_n - e^{-x_n})}{1 + e^{-x_n}}$$