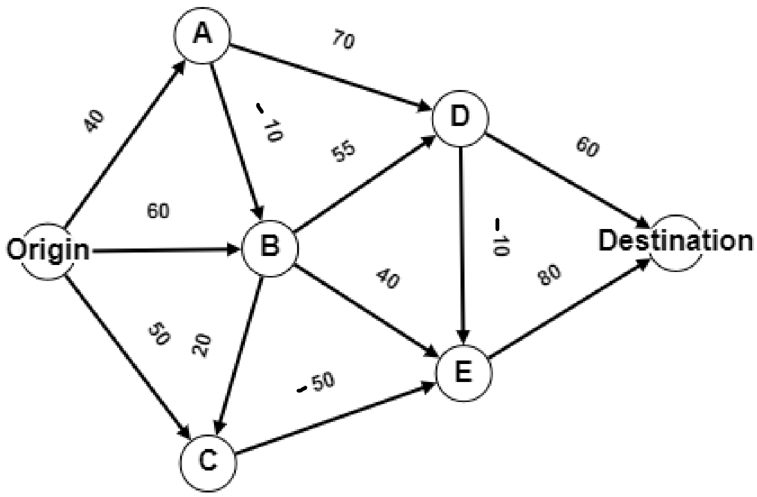
Network

Q.1: You need to take a trip by car to another town that you have never visited before. Therefore, you are studying a map to determine the shortest route to your destination. Depending on which route you choose, there are five other towns (call them A, B, C, D, E) that you might pass through on the way. The map shows the mileage along each road that directly connects two towns without any intervening towns.



Use the bellman's algorithm to find the **shortest path**

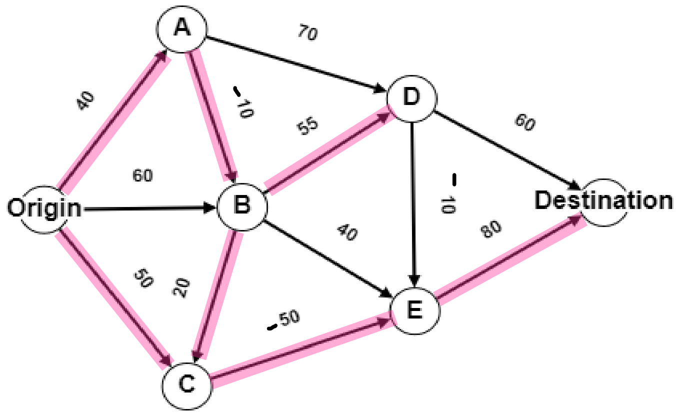
Answer :

Visited Node	The Set S	Existed Path	Calculated Wieght π	Minimum Weight	The Target Path
O	$S = \{O\}$		$\pi(O) = 0$	$\pi(O) = 0$	
A	$S = \{O, A\}$	$O \to A$	$\pi(A) = \pi(O) + w(O, A) = 0 + 40 = 40$	$\pi(A) = 40$	**O → A**
B	$S = \{O, A, B\}$	$O \to B$ $A \to B$	$\pi(B) = \min\{\pi(O) + w(O, B), \pi(A) + w(A, B)\}$ $= \min\{0 + 60, 40 + (-10)\} = \min\{60, 30\} = 30$	$\pi(B) = 30$	**A → B**
C,D	$S = \{O, A, B, C, D\}$	$O \to C$ $B \to C$ $A \to D$ $B \to D$	$\pi(C) = \min\{\pi(O) + w(O, C), \pi(B) + w(B, C)\}$ $= \min\{0 + 50, 30 + 20\} = 50$ $\pi(D) = \min\{\pi(A) + w(A, D), \pi(B) + w(B, D)\}$ $= \min\{40 + 70, 30 + 55\} = \min\{110, 85\} = 85$	$\pi(C) = 50$ $\pi(D) = 85$	**B → C** **O → C** **B → D**
E	$S =$ $\{O, A, B, C, D, E\}$	$B \to E$ $C \to E$ $D \to E$	$\pi(E) = \min\{\pi(B) + w(B, E), \pi(C) + w(C, E),$ $\pi(D) + w(D, E)\}$ $= \min\{30 + 40, 50 + (-50) + 85 + (-10)\}$ $= \min\{70, 0, 75\} = 0$	$\pi(E) = 0$	**C → E**
Des	$S =$ $\{O, A, B, C, D, E, Des\}$	$D \to Des$ $E \to Des$	$\pi(Des) = \min\{\pi(D) + w(D, Des),$ $\pi(E) + w(E, Des)\} = \min\{85 + 60, 0 + 80\}$ $= \min\{145, 80\} = 80$	$\pi(Des)$ $= 80$	**E → Des**

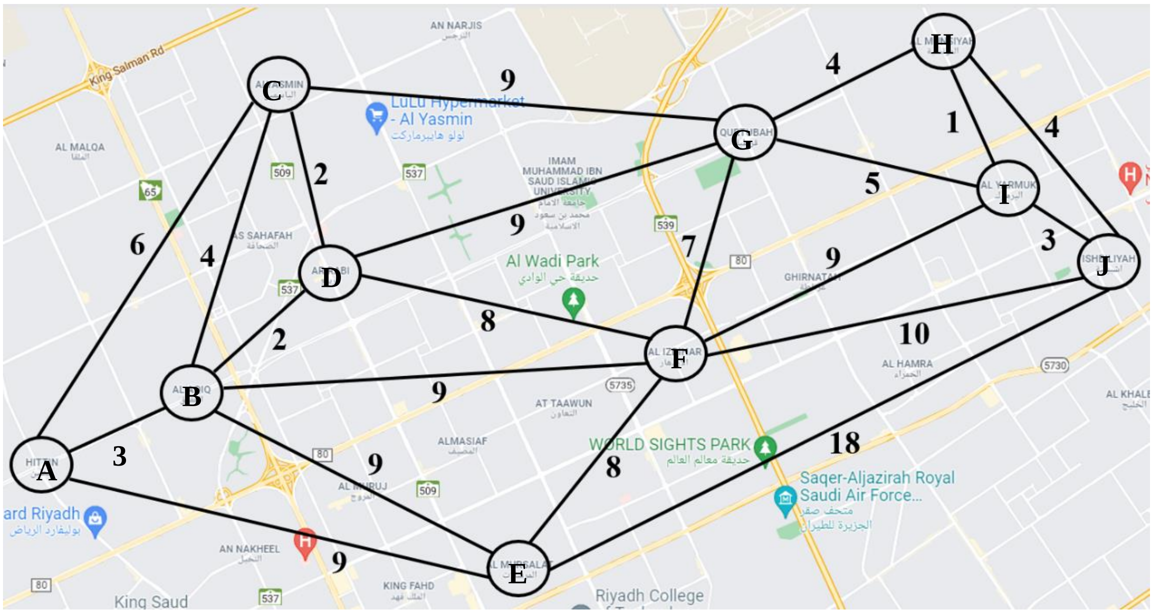
There are two shortest path solutions:

$\boldsymbol{Origin} \rightarrow \boldsymbol{A} \rightarrow \boldsymbol{B} \rightarrow \boldsymbol{C} \rightarrow \boldsymbol{E} \rightarrow \boldsymbol{Destination}$ with $\pi(\text{Des}) = 40 - 10 + 20 - 50 + 80 = 80$

$\boldsymbol{Origin} \rightarrow \boldsymbol{C} \rightarrow \boldsymbol{E} \rightarrow \boldsymbol{Destination}$ with $\pi(\text{Des}) = 50 - 50 + 80 = 80.$



Q.2: STC Company will soon begin laying cable in new neighborhoods in Riyadh represented as $\{A, \ldots, J\}$. The cable should connect the new neighborhoods with each other. Some of the paths might be more expensive, because they are longer. So, construct a **minimum spanning tree** that would be the one with the lowest total cost, representing the least expensive path for laying the cable. The network is visualized on the map of Riyadh city with nodes and paths, representing the neighborhoods and roads respectively. The weights represent the distances along each path that connects two neighborhoods in 1000 meters.



Answer : Using Kruskal's algorithm, that is, finding a set of links $(n - 1$ links for n nodes$)$ that provides a path between each pair of nodes with shortest total length. We can start with any node on the network and apply the algorithm, and it will result the same minimum spanning tree regardless of the starting node. Arbitrarily, I will start with the node A.

Iteration	Connected Nodes	Closest Unconnected Node	Arc
1	A	B	(A, B)
2	A, B	D	(B, D)
3	A, B, D	C	(D, C)
4	A, B, D, C	F	(D, F)
5	A, B, D, C, F	E	(F, E)
6	A, B, D, C, F, E	G	(F, G)
7	A, B, D. C, F, E, G	H	(G, H)
8	A, B, D, C, F, E, G, H	I	(H, I)
9	A, B, D, C, F, E, G, H, I	J	(I, J)