

PHYS 502
Final Exam
Tuesday 21st May 2013

Instructor: Dr. V. Lempesis

Student Name:

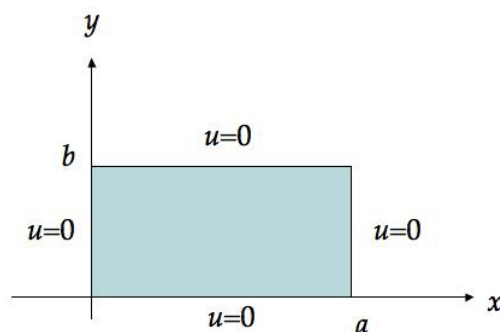
Student ID Number:.....

Student Grade:/40

Please answer two out of four of the following questions

1. An orthogonal membrane of length a and width b has all its sides fixed. The membrane is given an initial displacement $f(x,y)$. The points of the membrane have a zero initial velocity. The membrane is let free to vibrate. Find the displacement $u(x,y,t)$ of the points of the membrane and the frequencies of the relevant normal modes.

You are given: $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2}$.



(20 marks)

2. A one-dimensional beam has a length $L=2$. The coefficient of thermal conductivity of the beam is $\sigma=4$. The initial temperature of the beam is $u(x,0)=30^\circ\text{C}$. The two ends of the beam are kept at constant temperatures $u(0,t)=5^\circ\text{C}$ and $u(L,t)=35^\circ\text{C}$. Find the temperature distribution $u(x,t)$ in the beam. You are given:

$\frac{\partial u}{\partial t} = \sigma \frac{\partial^2 u}{\partial x^2}$. (Hint: This problem has non-homogeneous boundary conditions. You must solve the corresponding problem with homogeneous conditions and then find the solution for the non-homogeneous conditions case)

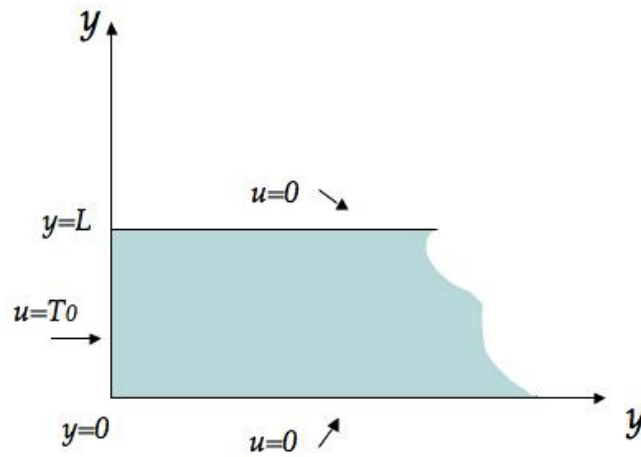
(20 marks)

3. A circular disc of radius $a=1/2$ has its surface insulated. The upper half of the disk has a constant temperature $T_1=5^\circ\text{C}$ at its rim

and the lower has a constant temperature $T_2 = 25^\circ\text{C}$ at its rim. Find the steady state temperature of the disk.

(20 marks)

4. A plate of infinite length and width L has its parallel sides at zero temperature and the left side at temperature T_0 as shown in figure below. Find the steady state temperature of the plate.



(20 marks)