Department of Statistics
& Operations Research
College of Science, King Saud University

STAT 324

Final Examination

First Semester

1433 – 1434 H

Student Name:			
Student No.:		Group No. :	
Attendance No.:		Teacher's Name:	

- Mobile Telephones are not allowed in the classrooms.
- Time allowed is **3** hours.
- Answer all questions.
- Choose the nearest number to your answer.
- WARNING: Do not copy answers from your neighbors. They have different question forms.
- For each question, put the code of the correct answer in the following table beneath the question number.

Questions 1

Assume that the average weight of a 6th grader is 80 pounds, with a standard deviation of 20 pounds. Suppose you draw a random sample of 25 students.

1)	The probability that the average weight of the sampled students is more than 75 pounds is:							
	(A)	0.1056	(B)	0.8944	(C)	0.4013	(D)	0.5967
2)	The average of the 25 students is called a							
	(A)	statistic	(B)	parameter	(C)	sample	(D)	population

Questions 2

A certain machine makes electrical resistors that have an average resistance of 100 ohms and a standard deviation of 36 ohms. If a random sample of size, 36 resistors are drawn from the product of this machine, then:

3)	The probability that the average resistance of the 36 resistors will be between 95 and 105 ohms is:							
	(A)	0.7647	(B)	0.6174	(C)	0.8432	(D)	0.5934

Question 3

A sample of size 100 is taken from a population having a proportion $p_1 = 0.8$. Another independent sample of size 400 is taken from a population having a proportion $p_2 = 0.5$. Let $\hat{p}_1$ and $\hat{p}_2$ be the two samples proportions, then

4)	The sampling distribution for the difference in sample proportions $\hat{p}_1 - \hat{p}_2$ has a mean equals:							
	(A)	0.8	(B)	1.3	(C)	0.3	(D)	0.0
5)	The sampling distribution for the difference in sample proportions $\hat{p}_1 - \hat{p}_2$ has a standard deviation equals:							
	(A)	0.015	(B)	0.047	(C)	0.0022	(D)	0.1239
6)	$P(\hat{p}_1 - \hat{p}_2 < 0.3) =$							
	(A)	0.500	(B)	0.442	(C)	0.993	(D)	0.016

Questions 4

A random sample of size 36 is taken from a normal population (first population) having a mean of 100 and a standard deviation of 6. A second random sample of size 25 is taken from a different normal population (second population) having a mean of 98 and a standard deviation of 5. Assume that these two samples are independent.

7)	The probability that the sample mean of the first population will exceed the sample mean of the second population by at least 4 is:							
	(A)	0.9832	(B)	0.9207	(C)	0.0793	(D)	0.0013
8)	The probability that the difference between the two sample means will be less than 2 is:							
	(A)	0.099	(B)	0.949	(C)	0.849	(D)	0.500

Questions 5

The average life of a manufacturer's battery is 5 years, with a standard deviation of 1 year. Assuming the life of the battery follows approximately a normal distribution. So,

9)	The probability that the mean life of a random sample of size 16 of such batteries will be between 4.5 and 5.5 years is:							
	(A)	0.1039	(B)	0.9544	(C)	0.2135	(D)	0.7865

10)	If $P(\overline{X} > k) = 0.1492$ where $\overline{X}$ represents the sample mean for a random sample of size 16 to be selected from the production lot, then the numerical value of k is:							
	(A)	5.260	(B)	6.500	(C)	5.647	(D)	4.653

Question 6

Suppose that 25% of the male students and 20% of the female students in a certain university smoke cigarettes. A random sample of 50 male students is taken. Another random sample of 100 female students is independently taken from this university. Let $\hat{p}_1$ and $\hat{p}_2$ be the proportions of smokers in the two samples, respectively, then:

11)	$E(\hat{p}_1 - \hat{p}_2) = \mu_{\hat{p}_1 - \hat{p}_2}$, the mean of $\hat{p}_1 - \hat{p}_2$. is equal to:							
	(A)	0.20	(B)	0.25	(C)	0.45	(D)	0.05
12)	$Var(\hat{p}_1 - \hat{p}_2) = \sigma^2_{\hat{p}_1 - \hat{p}_2}$, the variance of $\hat{p}_1 - \hat{p}_2$ is equal to:							
	(A)	0.5535	(B)	0.5054	(C)	0.00535	(D)	0.0352
13)	An approximate distribution of $\hat{p}_1 - \hat{p}_2$ is:							
	(A)	Binomial Distribution	(B)	Normal Distribution	(C)	Exponential Distribution	(D)	Poisson Distribution
14)	$P(0.05 < \hat{p}_1 - \hat{p}_2 < 0.08) =$							
	(A)	0.5871	(B)	0.7422	(C)	0.1591	(D)	0.2578

Question 7

A study was made by a taxi company to decide whether the use of new tires (N) instead of the present tires (P) improves fuel economy. 6 cars were equipped with tires (N) and driven over a prescribed test course. Without changing drivers and cares, test course was also made with tires (P). The gasoline consumption, in kilometers per liter (km/L), was recorded as follows (assume the population to be normally distributed with unknown equal's variances):

Car	1	2	3	4	5	6
Type (N)	4.5	4.8	6.6	7.0	6.7	4.6
Type (P)	3.9	4.9	6.2	6.5	6.8	4.1

$\bar{X} = 5.7$
$\bar{X} = 5.4$

15)	A 95% confidence interval for the true mean gasoline brand (N) consumption:			
	(A)	$2.642 \le \mu_N \le 4.930$	(B)	$5.221 \le \mu_N \le 9.727$
	(C)	$6.154 \le \mu_N \le 6.938$	(D)	$4.462 \le \mu_N \le 6.938$
16)	A 99% confidence interval for the difference between the true mean gasoline brand (N) consumption and the true mean gasoline brand (P) consumption ($\mu_N - \mu_P$) is:			
	(A)	$-1.939 \le \mu_N - \mu_P \le 0.539$	(B)	$-2.939 \le \mu_N - \mu_P \le 1.539$
	(C)	$-1.937 \le \mu_N - \mu_P \le 2.537$	(D)	$0.939 \le \mu_N - \mu_P \le 1.539$

Question 8

A random sample of 10 bicycle owners shows that in a small village a bicycle is driven on average 7.575 km in one hour with a standard deviation of 1.724 km. If μ is the average distance bicycles is driven in the village in one hour, assuming that the population follows a normal distribution, then:

17)	The point estimate for μ is:							
	(A)	0.9772	(B)	7.575	(C)	1.233	(D)	0.5793
18)	The maximum error of estimating μ with a 95% confident is:							
	(A)	1.233	(B)	0.573	(C)	0.977	(D)	2.332

19)	The lower bound of the 95% confidence interval for estimating μ is:							
	(A)	0.972	(B)	12.332	(C)	0.573	(D)	6.342

Questions 9

A study conducted by Saudi airline showed that a random sample of nine of its passengers at the King Khalid airport, took an average of 24.1 minutes to claim their luggage. From a previous survey we are willing to assume that time to claim luggage is normally distributed with standard deviation of 4.24 minutes.

20)	The standard error of the average time passengers take to claim their luggage:							
	(A)	4.245	(B)	9.000	(C)	1.413	(D)	24.100
21)	A 99% upper confidence limit for the sample mean equals:							
	(A)	20.461	(B)	27.738	(C)	0.022	(D)	2.531

Question 10

A random sample of 1000 voters is selected and 250 are found to be in favor of a specific Party, then:

22)	The point estimate for the true proportion of the voters who are in favor of this Party is:							
	(A)	0.96	(B)	0.75	(C)	0.42	(D)	0.25
23)	The lower bound of the 95% confidence interval for the true proportion of the voters who are in favor of this Party is:							
	(A)	0.223	(B)	0.217	(C)	0.285	(D)	0.567
24)	The upper bound of the 95% confidence interval for the true proportion of the voters who are in favor of this Party is:							
	(A)	0.205	(B)	0.223	(C)	0.567	(D)	0.277

Questions 11

Students may choose between a 3-semester hour course on physics without labs and a 4-semester hour course with labs. The final examination is the same for each section. If 10 students in the section with labs made an average grade of 70 with standard deviation 5, and 8 students in the section without labs made an average grade of 65 with standard deviation 3. Assuming that two populations to be normally distributed with equal variances, then:

25)	The point estimate of the difference between the true grade means ($\mu_1 - \mu_2$) is:							
	(A)	4	(B)	7	(C)	5	(D)	6
26)	The maximum value of the error of estimating a 95% confidence interval of the difference between the true grade means ($\mu_1 - \mu_2$) is:							
	(A)	2.2536	(B)	4.2664	(C)	1.2584	(D)	-4.1258
27)	The upper bound of the 95% confidence interval for the difference between the true grade means ($\mu_1 - \mu_2$) is equal to:							
	(A)	2.9981	(B)	7.0225	(C)	9.2664	(D)	2.3555

Questions 12

A food company distributes two brands of milk. A random sample of 200 consumers showed that 80 consumers prefer brand A. Another independent random sample of 300 consumers showed that 90 consumers prefer brand B. Let p_A = the true proportion of consumers preferring brand (A) and p_B = the true proportion of consumers preferring brand (B)

28)	A 96% confidence interval for the true proportion of consumers preferring brand (A) is:			
	(A)	$0.388 \le p_A \le 0.375$	(B)	$0.518 \le p_A \le 0.875$
	(C)	$0.228 \le p_A \le 0.675$	(D)	$0.329 \le p_A \le 0.471$
29)	A 99% confidence interval for the difference between proportions of consumers preferring brand (A) and (B) is:			
	(A)	$-0.0023 \le p_A - p_B \le 0.012$	(B)	$0.0124 \le p_A - p_B \le 0.2124$
	(C)	$-0.0123 \le p_A - p_B \le 0.212$	(D)	$-0.2313 \le p_A - p_B \le 0.3612$

30)	At a 0.05 level of significance, the test statistic for testing H_o: $p_A = 0.3$ against H_1: $p_A \neq 0.3$, is equal to:							
	(A)	1.072	(B)	2.051	(C)	4.076	(D)	3.086

31)	If we want to test at 0.05 significance level the hypotheses H_o: $p_A = p_B$ against H_1: $p_A > p_B$, the conclusion is:					
	(A)	Don't Reject H_o	(B)	Conclusion is not	(C)	Reject H_o

Questions 13

A researcher was interested in comparing the mean score of female students, μ_f, with the mean score of male students, μ_m, in a certain test. Two independent samples gave the following results: Assume that the populations are approximately normal with standard deviations 15 and 20 respectively.

Sample	Observations							mean
Scores of Females	89.2	81.6	79.6	80.0	82.8			82.63
Scores of Males	83.2	83.2	84.8	81.4	78.6	71.5	77.6	80.04

32)	If we want to test H_o: $\mu_f = \mu_m$ against H_1: $\mu_f \neq \mu_m$ then the test statistic equals:							
	(A)	0.256	(B)	−1.029	(C)	1.129		1.329
33)	If we want to test H_o: $\mu_f = \mu_m$ against H_1: $\mu_f \neq \mu_m$ at α=0.1, then the Acceptance Region of H_o is:							
	(A)	$(-\infty,-2.575)$	(B)	$(-2.575,2.575)$	(C)	(−3.169.3.169)	(D)	$(2.575,\infty)$

34)	If we want to test Ho: $\mu_f = \mu_m$ against H_1: $\mu_f \neq \mu_m$ at α=0.1, then the decision is:					
	(A)	not possible	(B)	not to reject H_o	(C)	to reject H_o

Question 14

A researcher wishes to compare the resistance of two types of wires. In a sample of 81 resistance readings of type A, the mean is $\overline{X}_A$= 27 ohm. In a sample of 90 resistance readings of type B, the mean is $\overline{X}_B$= 24 ohm. Assuming the two populations follow approximately two different normal distributions with standard deviations σ_A = 6.9 ohm and σ_B = 6.2 ohm.

35)	The maximum amount of error to estimate 95% C.I. for the difference between the two population means (μ_A-μ_B):							
	(A)	3.949	(B)	1.960	(C)	1.975	(D)	1.007
36)	The width of the 95% C.I. for the difference between the two population means (μ_A-μ_B):							
	(A)	3.949	(B)	1.960	(C)	1.975	(D)	1.007

Question 15

A study was conducted to compare between the proportions of smokers in two universities. Two independent random samples gave the following data:

	Univ. (1)	Univ. (2)
Sample size	200	300
Number of smokers	100	120

We wish to conduct a hypothesis test to determine if this data provide sufficient statistical evidence to indicate that the percentage of students who smoke differs for these two universities, at $\alpha=0.01$.

37)	The null and alternative hypotheses is:			
	(A)	$H_0: P_1<P_2, H_1: P_1>P_2$	(B)	$H_0: P_1=P_2, H_1: P_1\neq P_2$
	(C)	$H_0: P_1=P_2, H_1: P_1>P_2$	(D)	$H_0: P_1=P_2, H_1: P_1<P_2$

38)	The test statistic is:							
	(A)	$Z=\frac{\hat{p}_1+\hat{p}_2}{\sqrt{\frac{p_1q_1}{n_1}-\frac{p_1q_1}{n_2}}}$	(B)	$Z=\frac{\hat{p}_1-\hat{p}_2}{\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1}+\frac{\hat{p}_1\hat{q}_1}{n_2}}}$	(C)	$Z=\frac{\hat{p}_1-\hat{p}_2}{\sqrt{\frac{p_1q_1}{n_1}+\frac{p_1q_1}{n_2}}}$	(D)	$Z=\frac{\hat{p}_1-\hat{p}_2}{\sqrt{\frac{\hat{p}\hat{q}}{n_1}+\frac{\hat{p}\hat{q}}{n_2}}}$, $\hat{p}=\frac{X_1+X_2}{n_1+n_2}$

39)	The value of the test statistic equal:							
	(A)	2.207	(B)	1.026	(C)	0.527	(D)	-2.621

40)	At the 1% significance level, The decision is:					
	(A)	to Reject H_0	(B)	not possible	(C)	Not to Reject H_0

مع تمنياتنا بالتوفيق

Question 1

Assuming Normal distribution

$\mu = 80 \quad , \quad \sigma = 20$

$\rightarrow$ sample $n = 25$ $\Rightarrow$ sample mean $\bar{X} \sim N(\mu, \frac{\sigma}{\sqrt{n}})$

1) $P(\bar{X} > 75) = 1 - P(\bar{X} \leq 75)$

$= 1 - P(Z \leq \frac{75 - 80}{20/\sqrt{25}})$

$= 1 - P(Z < -1.25)$

$= 1 - 0.1056 = 0.8944$

2) Average of the 25 students is : a statistic.

Question 2 $\quad X \sim N(\mu, \sigma) \quad \mu = 100 , \sigma = 36$

$\rightarrow$ sample $n = 36$

$\bar{X} \sim N(\mu, \frac{\sigma}{\sqrt{n}})$

3) $P(95 \leq \bar{X} \leq 105) = P(\bar{X} < 105) - P(\bar{X} < 95)$

$= P(Z < \frac{105 - 100}{36/\sqrt{36}}) - P(Z < \frac{95 - 100}{36/\sqrt{36}})$

$= P(Z < 0.83) - P(Z < -0.83)$

$= 0.7967 - 0.2033 = 0.5934$

Question 3 $\quad P_1 = 0.8 , \quad P_2 = 0.5$

$n_1 = 100 \quad n_2 = 400$

$\hat{P}_1 - \hat{P}_2 \sim N(P_1 - P_2 , \sqrt{\frac{P_1 q_1}{n_1} + \frac{P_2 q_2}{n_2}})$

4) $E(\hat{P}_1 - \hat{P}_2) = P_1 - P_2 = 0.8 - 0.5 = 0.3$

5) std. dev. $(\hat{p}_1 - \hat{p}_2) = \sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}}$

$= \sqrt{\frac{(0.8)(0.2)}{100} + \frac{(0.5)(0.5)}{400}} = 0.047$

6) $P(\hat{p}_1 - \hat{p}_2 < 0.3) = P(Z < \frac{0.3 - 0.3}{0.047}) = P(Z<0)$

$= 0.5$

Question 4

$X_1 \sim N(\mu_1, \sigma_1)$ $\qquad$ $X_2 \sim N(\mu_2, \sigma_2)$

$\mu_1 = 100$, $\sigma_1 = 6$ $\qquad$ $\mu_2 = 98$, $\sigma_2 = 5$

$\Downarrow$ $\qquad$ $\Downarrow$

$n_1 = 36$ $\qquad$ $n_2 = 25$

independent samples

7) $P(\bar{X}_1 - \bar{X}_2 \geq 4) = 1 - P(\bar{X}_1 - \bar{X}_2 < 4)$

$= 1 - P(Z < \frac{4-2}{1.414})$

$= 1 - P(Z < 1.41) = 0.0793$

$\bar{X}_1 - \bar{X}_2 \sim N(\mu_1 - \mu_2 , \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}})$

$E(\bar{X}_1 - \bar{X}_2) = \mu_1 - \mu_2 = 100 - 98 = 2$

std. dev. $(\bar{X}_1 - \bar{X}_2) = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} = \sqrt{\frac{6^2}{36} + \frac{5^2}{25}} = 1.414$

8) $P(\bar{X}_1 - \bar{X}_2 < 2) = P(Z < \frac{2-2}{1.414}) = P(Z<0) = 0.5$

Question 5

$X \sim N(\mu, \sigma)$ $\mu = 5$, $\sigma = 1$

9) $n = 16$ $P(4.5 < \bar{X} < 5.5)$??

$\bar{X} \sim N(\mu, \frac{\sigma}{\sqrt{n}})$ $E(\bar{X}) = \mu = 5$

$s.e.(\bar{X}) = \frac{\sigma}{\sqrt{n}} = \frac{1}{\sqrt{16}} = 0.25$

$P(4.5 < \bar{X} < 5.5) = P(\bar{X} < 5.5) - P(\bar{X} < 4.5)$

$= P(Z < \frac{5.5-5}{0.25}) - P(Z < \frac{4.5-5}{0.25})$

$= P(Z < 2) - P(Z < -2)$

$= 0.9772 - 0.0228 = 0.9544$

10) $P(\bar{X} > K) = 0.1492$

$P(\bar{X} < K) = 1 - 0.1492$

$P(Z < \frac{K-5}{0.25}) = 0.8508 \Rightarrow \frac{K-5}{0.25} = 1.04$

$K = 5.26$

Question 6

$P_1 = 0.25$, $P_2 = 0.2$

$n_1 = 50$ $n_2 = 100$

11) $E(\hat{P}_1 - \hat{P}_2) = P_1 - P_2 = 0.25 - 0.2 = 0.05$

12) $Var(\hat{P}_1 - \hat{P}_2) = \frac{P_1 q_1}{n_1} + \frac{P_2 q_2}{n_2} = \frac{(0.25)(0.75)}{50} + \frac{(0.2)(0.8)}{100}$

$= 0.00535$

$\sigma_{\hat{P}_1 - \hat{P}_2} = 0.073$

13) $\hat{P}_1 - \hat{P}_2 \sim N\left(P_1 - P_2 , \sqrt{\frac{P_1 q_1}{n_1} + \frac{P_2 q_2}{n_2}} \right)$

Approximate Normal distribution.

14) $P(0.05 < \hat{P}_1 - \hat{P}_2 < 0.08)$

$= P(\hat{P}_1 - \hat{P}_2 < 0.08) - P(\hat{P}_1 - \hat{P}_2 < 0.05)$

$= P\left(Z < \frac{0.08 - 0.05}{0.073}\right) - P\left(Z < \frac{0.05 - 0.05}{0.073}\right)$

$= P(Z < 0.41) - P(Z < 0) = 0.6591 - 0.5$

$= \underline{\underline{0.1591}}$

Question 7

Normal distribution & $\sigma_1 = \sigma_2$ unknown

$n_N = 6$, $n_P = 6$

$\bar{X}_N = 5.7$, $\bar{X}_P = 5.4$

$S^2_N = 1.392$, $S^2_P = 1.6$

note that: $\bar{X} = \frac{\sum x_i}{n}$, $S^2 = \frac{\sum (x_i - \bar{x})^2}{n-1}$

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15) 95% C.I. for μ_N:

$(1-\alpha)100\% = 95\%$

$\alpha = 0.05$

$\frac{\alpha}{2} = 0.025$

$\boxed{t_{\frac{\alpha}{2}; n-1} = 2.571}$

$df = n - 1 = 5$

$\left(\bar{X} - t_{\frac{\alpha}{2}} \frac{S}{\sqrt{n}} , \bar{X} + t_{\frac{\alpha}{2}} \frac{S}{\sqrt{n}} \right)$

$\left(5.7 - 2.571 \times \frac{1.18}{\sqrt{6}} , 5.7 + 2.571 \times \frac{1.18}{\sqrt{6}} \right)$

$\underline{\underline{(4.462 , 6.938)}}$

16) 99% C.I. for $\mu_N - \mu_P$:

$(1-\alpha)100\% = 99\%$
$\alpha = 0.01$
$\frac{\alpha}{2} = 0.005$

$df = n_N + n_P - 2 = 10$

$t_{\frac{\alpha}{2}} = 3.169$

$$S_p^2 = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2} = \frac{5*(1.392) + 5*(1.6)}{10}$$

$$= 1.496$$

$S_p = 1.22$

$$\left[(\bar{X}_N - \bar{X}_P) - t_{\frac{\alpha}{2}} S_p \sqrt{\frac{1}{n_N} + \frac{1}{n_P}}, \; (\bar{X}_N - \bar{X}_P) + t_{\frac{\alpha}{2}} S_p \sqrt{\frac{1}{n_N} + \frac{1}{n_P}}\right]$$

$$\left[0.3 - 3.169(1.22)\sqrt{\frac{1}{6}+\frac{1}{6}}, \; 0.3 + 3.169(1.22)\sqrt{\frac{1}{6}+\frac{1}{6}}\right]$$

$$[-1.93, \; 2.53]$$

Question 8

$n = 10$, $\bar{X} = 7.575$, $S = 1.724$

17) point estimate for $\mu = \bar{X} = 7.575$

18) 95% C.I. $\Rightarrow$ $\alpha = 0.05$, $\frac{\alpha}{2} = 0.025$

$df(n-1) = 9$, $t_{\frac{\alpha}{2}} = 2.262$

Maximum error of estimation:

$$e = t_{\frac{\alpha}{2}} \cdot \frac{S}{\sqrt{n}} = 2.262 \cdot \frac{1.724}{\sqrt{10}} = 1.233$$

19) lower bound for 95% C.I. for μ:

$$\bar{X} - t_{\frac{\alpha}{2}} \frac{S}{\sqrt{n}} = 7.575 - 1.233 = 6.342$$

Q9

$n = 9$, $\bar{X} = 24.1$

$X \sim N(\mu, \sigma)$ < $\sigma = 4.24$

20) $s.e.(\bar{X}) = \frac{\sigma}{\sqrt{n}} = \frac{4.24}{\sqrt{9}} = 1.413$

$\alpha = 0.01$, $\frac{\alpha}{2} = 0.05$

$Z_{\frac{\alpha}{2}} = 1.645$

21) 99% C.I. for μ :

upper $= \bar{X} + Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} = 24.1 + 1.645 * 1.413$

$= 26.425$

Question 10

$\hat{p} = \frac{x}{n} = \frac{250}{1000} = 0.25$

22) point estimate for p is $\hat{p} = 0.25$

23) 95% C.I. for p

$\alpha = 0.05$, $\frac{\alpha}{2} = 0.025$

$Z_{\frac{\alpha}{2}} = 1.96$

$$\left(\hat{p} - Z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}\hat{q}}{n}} , \hat{p} + Z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}\hat{q}}{n}}\right)$$

$$\left(0.25 - 1.96 \sqrt{\frac{0.25*0.75}{1000}} , 0.25 + 1.96 \sqrt{\frac{0.25*0.75}{1000}}\right)$$

$(0.223 , 0.277)$

lower bound, upper bound

Question 11

$n_1 = 10$ $\quad \bar{X}_1 = 70$ $\quad\quad n_2 = 8$ $\quad \bar{X}_2 = 65$

$S_1 = 5$ $\quad\quad S_2 = 3$

$\sigma_1 = \sigma_2$ unknown

25) Point estimate for $\mu_1 - \mu_2 = \bar{X}_1 - \bar{X}_2 = 70 - 65 = 5$

95% C.I. for $(\mu_1 - \mu_2)$

max. error:

$$\boxed{e = t_{\frac{\alpha}{2}} \, S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$df = n_1 + n_2 - 2 = 16$

$t_{\frac{\alpha}{2}} = 2.12$, $S_p^2 = \dfrac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$

$$= \frac{9 * 5^2 + 7 * 3^2}{16} = 18$$

$S_p = 4.243$

6) $e = 2.12 * 4.243 * \sqrt{\frac{1}{10} + \frac{1}{8}} = 4.266$

) upper bound $= (\bar{X}_1 - \bar{X}_2) + t_{\frac{\alpha}{2}} \, S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$

$= 5 + 4.266 = 9.266$

Question 12

$n_1 = 200$, $x_1 = 80$ $\quad\quad n_2 = 300$, $x_2 = 90$

$\hat{P}_1 = \frac{80}{200} = 0.4$ $\quad\quad \hat{P}_2 = \frac{90}{300} = 0.3$

$P_1 \equiv P_A$ $\quad\quad P_2 \equiv P_B$

28) 96% C.I. for P_1 $\qquad \alpha = 0.04, \ \frac{\alpha}{2} = 0.02$

$Z_{\frac{\alpha}{2}} = 2.055$

$$\left(\hat{P}_1 - Z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{P}_1 \hat{q}_1}{n_1}}, \ \hat{P}_1 + Z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{P}_1 \hat{q}_1}{n_1}}\right)$$

$$\left(0.4 - 2.055\sqrt{\frac{(0.4)(0.6)}{200}}, \ 0.4 + 2.055\sqrt{\frac{(0.4)(0.6)}{200}}\right)$$

$$(0.329, 0.471)$$

29) 99% C.I. for $P_1 - P_2$ $\qquad \alpha = 0.01, \ \frac{\alpha}{2} = 0.005$

$Z_{\frac{\alpha}{2}} = 2.575$

$$\left((\hat{P}_1 - \hat{P}_2) - Z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{P}_1 \hat{q}_1}{n_1} + \frac{\hat{P}_2 \hat{q}_2}{n_2}}, \ (\hat{P}_1 - \hat{P}_2) + Z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{P}_1 \hat{q}_1}{n_1} + \frac{\hat{P}_2 \hat{q}_2}{n_2}}\right)$$

$$0.1 - 2.575\sqrt{\frac{(0.4)(0.6)}{200} + \frac{(0.3)(0.7)}{300}}, \ 0.1 + 2.575\sqrt{\frac{(0.4)(0.6)}{200} + \frac{(0.3)(0.7)}{300}}\Bigg)$$

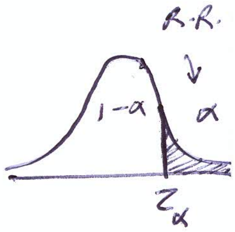
$$(-0.012, 0.212)$$

30) at $\alpha = 0.05$, Testing: $H_0: P_A = 0.3$

$H_1: P_A \neq 0.3$

Test statistic:

$$Z = \frac{\overset{0.4}{\hat{P}_1} - 0.3}{\sqrt{\frac{(0.3)(0.7)}{200}}} = \frac{0.1}{0.0324} = \underline{\underline{3.086}}$$



31) at $\alpha = 0.05$, Testing: $\boxed{\begin{matrix} H_0: & P_A = P_B \\ H_1: & P_A > P_B \end{matrix}}$ $\qquad \underline{\underline{Z_\alpha = 1.645}}$

$$\hat{P} = \frac{x_1 + x_2}{n_1 + n_2} \quad \text{or} \quad \frac{n_1 \hat{P}_1 + n_2 \hat{P}_2}{n_1 + n_2}$$

$$= \frac{80 + 90}{200 + 300} \quad \text{or} \quad \frac{200(0.4) + 300(0.3)}{200 + 300} = 0.34 \Rightarrow \hat{q} = 0.66$$

Test statistic $Z = \dfrac{\hat{P}_1 - \hat{P}_2}{\sqrt{\hat{P}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \underline{\underline{2.312}} \Rightarrow$ reject H_0

Question 13:

$\mu_f \equiv \mu_1 \quad < \quad \mu_m \equiv \mu_2$

$X_1 \sim N(\mu_1, \sigma_1) \quad < \quad X_2 \sim N(\mu_2, \sigma_2)$

$\sigma_1 = 15 \qquad \sigma_2 = 20$

$n_1 = 5, \ \bar{X}_1 = 82.63 \qquad n_2 = 7, \ \bar{X}_2 = 80.04$

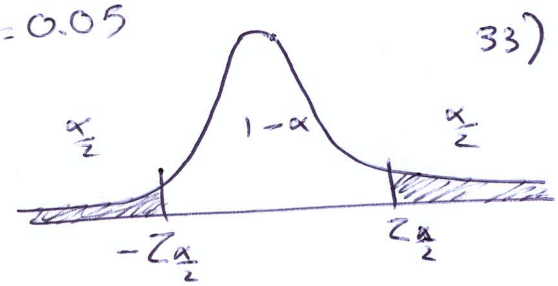
32) Testing: $H_0: \mu_1 = \mu_2 \qquad H_1: \mu_1 \neq \mu_2$

Test statistic: $Z = \dfrac{(\bar{X}_1 - \bar{X}_2) - d}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \dfrac{(82.63 - 80.04) - 0}{\sqrt{\frac{15^2}{5} + \frac{20^2}{7}}}$

$= \underline{0.256}$

$\alpha = 0.1, \ \frac{\alpha}{2} = 0.05$

$Z_{\frac{\alpha}{2}} = 2.575$



33) acceptance region $(-2.575, 2.575)$

34) Decision: accept H_0

Question 14

$n_1 = 81, \ \bar{X}_1 = 27 \qquad n_2 = 90, \ \bar{X}_2 = 24$

$X_1 \sim N(\mu_1, \sigma_1), \ \sigma_1 = 6.9 \qquad X_2 \sim N(\mu_2, \sigma_2), \ \sigma_2 = 6.2$

35) 95% C. I. for $\mu_1 - \mu_2$

Max. error $e = Z_{\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$

$= 1.96 \sqrt{\frac{6.9^2}{81} + \frac{6.2^2}{90}} = \underline{1.9745}$

36) width of C. I. $= 2e = 3.949$

Question 15

$n_1 = 200$, $x_1 = 100$ $\quad$ $n_2 = 300$, $x_2 = 120$

$\hat{P}_1 = 0.5$ $\quad$ $\hat{P}_2 = 0.4$

$\alpha = 0.01$

37) Test Hypotheses: $H_0: P_1 = P_2$

$H_1: P_1 \neq P_2$

38) Test statistic:

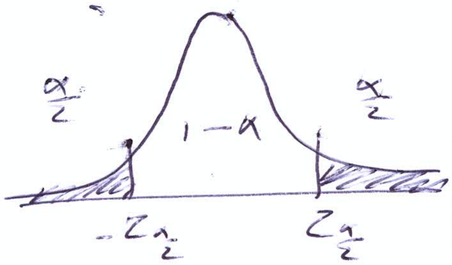
$$Z = \frac{\hat{P}_1 - \hat{P}_2}{\sqrt{\frac{\hat{p}\hat{q}}{n_1} + \frac{\hat{p}\hat{q}}{n_2}}} \quad , \quad \hat{P} = \frac{x_1 + x_2}{n_1 + n_2} = 0.44$$

$$\hat{q} = 0.56$$

39) $$Z = \frac{0.5 - 0.4}{\sqrt{(0.44)(0.56)\left(\frac{1}{200} + \frac{1}{300}\right)}} = 2.207$$

40) $\alpha = 0.01$ $\quad$ $\frac{\alpha}{2} = 0.005$

$Z_{\frac{\alpha}{2}} = 2.575$



accept Ho