

King Saud university

Second semester, 1431H

Time: 3 hours

Math 201

The final examination

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Question No.1

- (a) Find the extrema of $f(x, y, z) = x - 2y + 5z$ if (x, y, z) is restricted to

$$x^2 + y^2 + z^2 = 30$$

- (b) If $W = z \cos x + y \sin z + xy - 3$, find $W_{zxy} = \frac{\partial^3 W}{\partial y \partial x \partial z}$.

- (c) Show that the following function is continuous at $(0, 0)$:

$$f(x, y) = \begin{cases} \frac{2x^5 y}{2x^4 + 3y^4}, & (x, y) \neq (0, 0); \\ 0, & (x, y) = (0, 0). \end{cases}$$

- (d) If $W = te^{2t}$ and $t = x + y - \sin xy$, find $\frac{\partial W}{\partial y}$.
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Question No.2

- (a) Find the volume of the region Q that is bounded by $z = 2$, $z = \sqrt{x^2 + y^2}$.

- (b) Evaluate

$$\int \int \int_E x^2 dV,$$

where E lies between $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 + z^2 = 9$

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Question No.3

(a) Determine whether $\{a_n\}$ converges or diverges:

- $\left\{\frac{\sin(3n)}{n}\right\}$
- $\{\ln n - \ln(2n + 1)\}$

1. Determine whether the series is absolutely convergent, conditionally convergent, or divergent:

- $\sum_{n=1}^{\infty} \frac{3+\sin n}{\sqrt{n^5+3}}$
- $\sum_{n=1}^{\infty} (-1)^n \frac{(n+1)}{e^n}$

2. Find a Maclaurin series for $f(x) = \cos x$ and then, by using this series, prove that

$$\lim_{x \rightarrow 0} \frac{\cos x - 1}{x} = 0$$

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Question No.4

1. Let R be the region in the XY -plane bounded by the graphs of $y = x^2$, $y = 0$ and $x = 3$. Evaluate

$$\int \int_R \sin(\pi x^3) dA$$

2. Evaluate

- $\int^3 \int^{\sqrt{9-x^2}} (x^2 + y^2)^{\frac{3}{2}} dy dx$
- $\int^2 \int_x y^2 \cos(xy) dy dx$

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Good luck