King Saud University
Faculty of Sciences
Department of Mathematics

Final Examination Math 481 Semester II - 1443
Time: 3H

Question 1 :

1. Find the following limit $\lim\limits_{n\longrightarrow+\infty}\sum_{k=1}^{n}\frac{1}{\sqrt{4n^2-k^2}}$.

2. Study the convergence of the following improper integrals

   (a) $\displaystyle\int_0^3 \frac{e^x}{\sqrt{3-x}}dx$ (b) $\displaystyle\int_{-\infty}^{-1} e^{x^3}dx$

Question 2 :

1. State the definition of the uniform convergence of a sequence of functions $(f_n)_n$ on an interval I.

2. Study the pointwise convergence of the sequence $(f_n)_n$ defined by $f_n(x)=\dfrac{3^n x}{1+n3^n x^2}$ on the interval $[0,1]$.

3. Compute $\lim\limits_{n\to+\infty}\displaystyle\int_0^1 f_n(x)dx$ and $\displaystyle\int_0^1 \lim\limits_{n\to+\infty} f_n(x)dx$.

Question 3 :

Study the absolute and the uniform convergence of the series $\displaystyle\sum_{n\geq0}\frac{(-1)^n x}{(1+x^2)^n}$ on $[0,+\infty)$.

Question 4 :

1. State the definition of a $\sigma-$algebra on $\mathbb{R}$.
2. State the theorem of monotone convergence and the dominate convergence theorem
3. Compute the following limit $\lim\limits_{n\longrightarrow+\infty}\int_0^1 \frac{nx}{1+n^2x^2}dx$.

Question 5 :

1. Prove that the Borel $\sigma-$algebra is generated by the closed intervals $[a,b]$, with $a,b\in\mathbb{R}$.
2. (a) State the definition of the Lebesgue outer measure m^*.
   (b) Prove that m^* is monotonic.
   (c) Compute $m^*(\mathbb{Q}^c\cap[0,2])$.
3. Compute $\int_{[0,2]} f(x)dx$ if $f(x)=\begin{cases} x & x\in\mathbb{Q}\cap[0,2] \\ 3 & x\in\mathbb{Q}^c\cap[0,2]\end{cases}$.