

TIME: 3hours
M - 107

KING SAUD UNIVERSITY
DEPARTMENT OF MATHEMATICS
(SEMESTER I, 1431-1432) FINAL

FULL MARKS :100

Question: 1 . (a) For what values of λ does the system of equations have
[10+4+10] (i) unique solution, (ii) infinitely many solutions and (iii) no solution.

$$3x + \lambda z = 2$$

$$3x + 3y + 4z = 4$$

$$y + 2z = 3$$

(b) By using properties of determinant, show that

$$\begin{vmatrix} y+z & z+x & y+x \\ x & y & z \\ 1 & 1 & 1 \end{vmatrix} = 0.$$

(c) For the given system of linear equations:

$$x + 2y + 3z = 1$$

$$3x + y + 3z = 3$$

$$x + 2y + 4z = 1$$

- i. Write the system of equation in the form $AX=B$,
- ii. Find $\text{adj}(A)$,
- iii. Use $\text{adj}(A)$ to find A^{-1} , if exists, and
- iv. Use A^{-1} to solve the given system.

Question:2.(a) Find the comp_b^a and Proj_b^a

[6+6+6] if $a = -2i + j + k$ and $b = 4i - 3j + k$.

(b) Find the point at which the line

$$x = 2 + 3t, \quad y = -4t, \quad z = 5 + t, \quad t \in \mathbb{R}$$

intersects the plane $4x + 5y - 2z = 18$.

(c) Find the direction cosines and direction angles of the vector

$$a = 2i + 3j - 6k$$

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Question: 3 . (a) Show that $\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{x^2 + y^2}$ does not exist.

[6+8+6] (b) A particle starts from position $r(1) = i + j$. Its velocity is $v(t) = 2ti + 3t^2j + \sqrt{t}k$. Find its position at time t .

(c) Find the parametric equations of the tangent line to the curve with parametric equations $x = t^5, y = t^4, z = t^3$, at the point (1, 1, 1).

Question: 4 .(a) Show that the function $f(x, y) = \ln(x^2 + y^2)$ $x \neq 0, y \neq 0$

[6+6+8] satisfies the Laplace equation $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$

(b) If $3xy^3 + 3x^2y - 6y + 9xy = 8$ find $\frac{dy}{dx}$

**(c) Find the tangential and normal components of the acceleration
of a particle moving along the curve $x = t^3, y = t+2, z = 4, t \in R$
at $t=1$. Also find the curvature at $t=1$.**

Question: 5 . (a) The base radius and the height of the right circular cone are measured to be 10cm and 25cm respectively, with a possible error in measurement of 0.1cm in each. Use differentials to estimate the maximum error in the calculated volume of the cone. $\left[V = \frac{1}{3} \pi r^2 h \right]$

[6+8+6] (b) Find the directional derivative of $f(x, y) = xy$ at the point $P(3, 2)$ in the direction of $\langle 1, 2 \rangle$. In which direction is the direction of the derivative maximum? What is the maximum value of the derivative?

(c) Use Lagrange multipliers to find the maximum and minimum values of the function $f(x, y) = x + 2y$ subject to constraint $x^2 + y^2 = 5$.

**NOTE: For solution of the paper visit
<http://faculty.ksu.edu.sa/khawaja/default.aspx>**