Differential and Integral Calculus (MATH-205)

Final Exam/Fall 2023 Time Allowed: 3 Hours

Date: Tuesday, December 19, 2023 **Maximum Marks:** 40

Note: Solve all **10** questions and give **DETAILED** solutions. Make sure your solutions are clearly written and contain all necessary details.

Question 1: (4°) Determine whether the following infinite series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{2^n + 10}{n!}$$

Question 2: (4°) Find the radius of convergence of the power series given by

$$\sum_{n=0}^{\infty} \frac{(-1)^n\,(2n)!}{2^{2n+1}(n!)^2} \frac{x^{2n-1}}{(2n+1)}.$$

Question 3: (4°) Use the first three nonzero terms of a Maclaurin series to approximate $\int_0^{0.5} x\,\cos x^2\,dx$ and estimate the error in the approximation. Use 6 d.p. accuracy in your working.

Question 4: (5°) Show that the lines l_1 passing through $A(1,-2,3)$ and $B(2,0,5)$ and l_2 passing through $C(4,1,-1)$ and $D(-2,3,4)$ are skew lines. Find the shortest distance between l_1 and l_2.

Question 5: (2°) Find the limit $\displaystyle\lim_{(x,z)\to(0,-3)} \frac{x^4 - (z+3)^4}{x^2 + (z+3)^2}$.

Question 6: (3°) Find z_x and z_y if $z = f(x,y)$ is determined implicitly by the equation

$$xe^{yz} - 2ye^{xz} + 3ze^{xy} = 1$$

Give your answers in the simplest form.

—- PTO —-

Question 7: (6°) Find the local and global extrema and saddle points of $f(x, y) = x^3 + 3xy - y^3$ on its domain. Then, find the boundary extrema on the triangular region R with vertices $A(1, 2)$, $B(1, -2)$, and $C(-1, -2)$.

Question 8: (4°) The following iterated double integral represents the volume of a solid under a surface S and over a region R in the xy-plane. Describe S and sketch R. Hence, find volume of the solid.

$$\int_{-2}^{1} \int_{x-1}^{1-x^2} (x^2 + y^2)\, dydx$$

Question 9: (4°) Evaluate the double integral $\int_{-2}^{2} \int_{0}^{\sqrt{4-y^2}} \cos(x^2 + y^2)\, dxdy$. Sketch the region R.

Question 10: (4°) Find the surface area of the first-octant portion of the cylinder $y^2 + z^2 = 9$ that lies inside the cylinder $x^2 + y^2 = 9$.

—- Good Luck —-