General Exponential Function

Derivatives of General Exponential Function

Integration of General Exponential Functions

General Logarithm Functions

Derivative of General Logarithmic Function

The Natural Logarithm and Exponential Functions

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26 Sept 2013

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$$a^x = e^{x\ln(a)}, \quad a > 0, x \in \mathbb{R}.$$

Laws of Exponents:

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